GOVT. POLYTECHNIC, DHENKANAL

LEARNING MATERIALS

OF

ENGINEERING MATHEMATICS - I

PREPARED BY

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Chapter 3
Fundamentals of Trigonometry

3.1 Introduction:

The word "trigonometry" is a Greek word. Its mean "measurement of a triangle". Therefore trigonometry is that branch of mathematics concerned with the measurement of sides and angle of a plane triangle and the investigations of the various relations which exist among them. Today the subject of trigonometry also includes another distinct branch which concerns itself with properties relations between and behavior of trigonometric functions.

The importance of trigonometry will be immediately realized when its applications in solving problem of mensuration, mechanics physics, surveying and astronomy are encountered.

3.2 Types of Trigonometry:

There are two types of trigonometry

(1) Plane Trigonometry (2) Spherical Trigonometry

1. Plane Trigonometry

Plane trigonometry is concerned with angles, triangles and other figures which lie in a plane.

2. Spherical Trigonometry

Spherical Trigonometry is concerned with the spherical triangles, that is, triangles lies on a sphere and sides of which are circular arcs.

3.3 Angle:

An angle is defined as the union of two non-collinear rays which have a common end-points.

An angle is also defined as it measures the rotation of a line from one position to another about a fixed point on it.

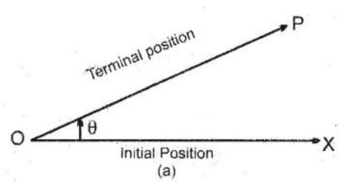


Fig. 4.1

In figure 5.1(a)the first position OX is called initial line (position)and second position OP is called terminal line or generating line(position) of $\angle XOP$.

If the terminal side resolves in anticlockwise direction the angle described is positive as shown in figure (i)

If terminal side resolves in clockwise direction, the angle described is negative as shown in figure (ii)

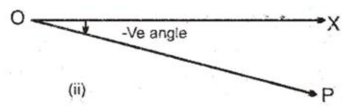


Fig. 4.3

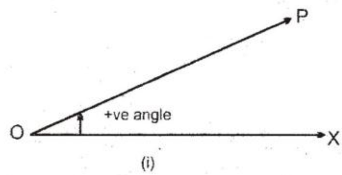


Fig. 4.2

3.4 Quadrants:

Two mutually perpendiculars straight lines xox□ and yoy□ divide the plane into four equal parts, each part is called quadrant.

Thus XOY, X′OY, X′OY′ and XOY′ are called the Ist, IInd, IIIrd and IVth quadrants respectively.

In first quadrant the angle vary from 0^o to 90^o in anti-clockwise direction and from -270^o to -360^o in clockwise direction.

In second quadrant the angle vary from 90^o to 180^o in anti-clockwise direction and -180^o to -270^o in clockwise direction.

In third quadrant the angle vary from 180^o to 270^o in anti-clockwise direction and from -90^o to -180^o in clockwise direction.

In fourth quadrant the angle vary from 270^o to 360^o in anti-clockwise direction and from -0^o to -90^o in clockwise direction.

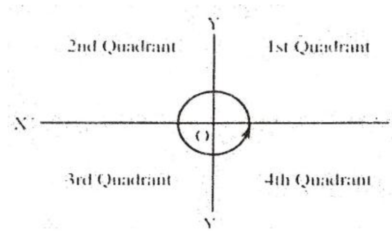


Fig. 4.4

3.5 Measurement of Angles:

The size of any angle is determined by the amount of rotations. In trigonometry two systems of measuring angles are used.

(i) Sexagesimal or English system (Degree)

(ii) Circular measure system (Radian)

(i) Sexagesimal or English System (Degree)

The sexagesimal system is older and is more commonly used. The name derive from the Latin for "sixty". The fundamental unit of angle measure in the sexagesimal system is the degree of arc. By definition, when a circle is divided into 360 equal parts, then

One degree = $\frac{1}{360}$th part of a circle.

Therefore, one full circle = 360 degrees.

The symbol of degrees is denoted by $(\ )^0$.

Thus an angle of 20 degrees may be written as 20^o.
Since there are four right angles in a complete circle .

One right angle = $\frac{1}{4}$ circle = $\frac{1}{4}(360^o) = 90^o$

The degree is further subdivided in two ways, depending upon whether we work in the common sexagesimal system or the decimal sexagesimal system. In the common sexagesimal system, the degree is subdivided into 60 equal parts, called minutes, denoted by the symbol$(\ )'$, and the minute is further subdivided into 60 equal parts, called second, indicated by the symbol $(\ )''$. Therefore

1 minute = 60 seconds
1 degree = 60 minutes = 3600 seconds
1 circle = 360 degrees = 21600 minutes = 12,96,000 sec.

In the decimal sexagesimal system, angles smaller than 1^o are expressed as decimal fractions of a degree. Thus one-tenth $\left(\frac{1}{10}\right)$ of a degree is expressed as 0.1^o in the decimal sexagesimal system and as $6'$ in the common sexagesimal system; one-hundredth $\left(\frac{1}{100}\right)$ of a degree is 0.01^o in the decimal system and 36" in the common system; and $47\frac{1}{9}$ degrees comes out $(47.111\ \ldots)^o$ in the decimal system and $47^o6'40''$ in the common system.

(ii) Circular measure system (Radian)

This system is comparatively recent. The unit used in this system is called a Radian. The Radian is define "The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle." As shown in fig., Arc AB is equal in length to the radius $\overline{OB}$ of the circle. The subtended, $\angle AOB$ is then one radian.

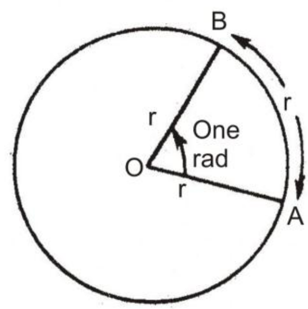


Fig. 4.5

i.e. m $\angle AOB$ = 1 radian.

3.6 Relation between Degree and Radian Measure:

Consider a circle of radius r, then the circumference of the circle is $2\pi r$. By definition of radian,

An arc of length ′r′ subtends an angle = 1 radian

$\therefore$ An arc of length $2\pi r$ subtends an angle = 2π radian

Also an arc of length $2\pi r$ subtends an angle = 360^o

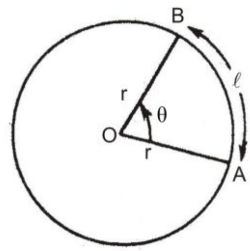


Fig. 4.6

Then 2π radians = 360^o

Or π radians = 180^o

$$1 \text{ radians} = \frac{180^o}{\pi}$$

$$1 \text{ radians} = \frac{180}{3.1416}$$

Or 1 radians = 57.3^o

Therefore to convert radians into degree,

we multiply the number of radians by $\frac{180^o}{\pi}$ or 57.3.

Now Again, $360^o = 2\pi$ radians

$$1^o = \frac{2\pi r}{360^o} \text{ radians}$$

$$1^o = \frac{\pi}{180^o}$$

Or $$1^o = \frac{3.1416}{180^o}$$

$1^o = 0.01745$ radians

Therefore, to convert degree into radians, we multiply the number of degrees by $\frac{\pi}{180}$ or 0.0175.

Note: One complete revolution = $360^o = 2\pi$ radius.

3.7 Relation between Length of a Circular Arc and the adian Measure of its Central Angle:

Let "l" be the length of a circular arc $\overline{AB}$ of a circle of radius r, and θ be its central angle measure in radians. Then the ratio of l to the circumference $2\pi r$ of the circle is the same as the ratio of θ to 2π.

Therefore

$$l : 2\pi r = \theta : 2\pi$$

Or $$\frac{l}{2\pi r} = \frac{\theta}{2\pi}$$

$$\frac{l}{r} = \theta$$

$l = \theta r$, where θ is in radian

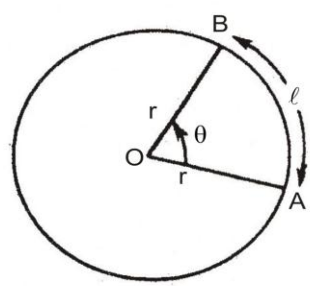


Fig. 4.7

Note:

If the angle will be given in degree measure we have to convert it into Radian measure before applying the formula.

Example 1:

Convert 120^o into Radian Measure.

Solution:

120^o

$$120^o = 120 \text{ x } \frac{\pi}{180}$$

$$= \frac{2\pi}{3} = \frac{2(3.1416)}{3} = 2.09 \text{ rad.}$$

Example 2:

Convert 37^o 25′ 38" into Radian measure.

Solution:

$$37^o\ 25'\ 28'' = 37^o + \frac{25}{60} + \frac{28}{3600}$$

$$= 37^o + \frac{5^o}{12} + \frac{7^o}{900}$$

$$= 37^o + \frac{382^o}{900}$$

$$= 37 + \frac{181^o}{450} = \frac{16831^o}{450} = \frac{16831}{450} \text{ x } \frac{\pi}{180}$$

$$= \frac{16831(3.14160)}{81000} = \frac{52876.26}{81000}$$

$$37^o\ 25'\ 28'' = 0.65 \text{ radians}$$

Example 3:

Express in Degrees:

(i) $\frac{5\pi}{3}$ rad (ii) 2.5793 rad (iii) $\frac{\pi}{6}$ rad (iv) $\frac{\pi}{3}$ rad

Solution:

(i) Since 1 rad $= \frac{180}{\pi}$ deg

$$\therefore \frac{5\pi}{3} \text{ rad} = \frac{5\pi}{3} \text{ x } \frac{180}{\pi} \text{ deg} = 5(60) \text{ deg} = 300^o$$

(ii) 2.5793 rad $= 2.5793\ (57^o\ .29578)\ \therefore\ 1 \text{ rad} = 57^o\ .295778$

$= 147^o\ .78301 = 147.78$ (two decimal places)

(iii) $\frac{\pi}{6}$ rad $= \frac{\pi}{6} \text{ x } \frac{12}{\pi}$ deg $= 30^o$

(iv) $\frac{\pi}{3}$ rad $= \frac{\pi}{3} \times \frac{12}{\pi}$ deg = 60^o

Example 4:

What is the length of an arc of a circle of radius 5 cm. whose central angle is of 140^o?

Solution: l = length of an arc = ?

r = radius = 5 cm

θ = 140^o

Since 1 deg = 0.01745 rad

$\therefore$ θ = 140 x 0.01745 rad = 2.443 rad

$\therefore$ l = $r\theta$

l = (5)(2.443) = 12.215 cm

Example 5:

A curve on a highway is laid out an arc of a circle of radius 620m. How long is the arc that subtends a central angle of 32^o?

Solution: r = 620m l = ? $\theta = 32^o = 32 \times \frac{\pi}{180}$ rad

$l = 620 \times 32 \times \frac{\pi}{180}$ = 346.41 m

Example 6:

A railway Train is traveling on a curve of half a kilometer radius at the rate of 20 km per hour through what angle had it turned in 10 seconds?

Solution:

Radius = r = $\frac{1}{2}$km, θ = ?

We know s = vt

v = velocity of Train = 20km/hour = $\frac{20}{3600}$ km/sec.

v = $\frac{1}{180}$ km/sec

l = Distance traveled by train in 10 seconds = $\frac{1}{180}$ x10 km/sec

$l = \frac{1}{18}$

Since $l = r\theta$

$\Rightarrow$ $\frac{1}{18} = \frac{1}{2}\theta$

$$\theta = \frac{2}{18} = \frac{1}{9} \text{rad}$$

Example7

The moon subtends an angle of 0.5^o as observed from the Earth. Its distance from the earth is 384400 km. Find the length of the diameter of the Moon.

Solution:

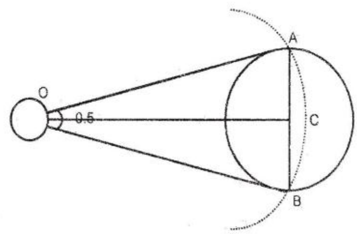


Fig. 4.8

l = AB = diameter of the Moon = ?

as angle 0.5 is very small.

i.e. AB (arc length) consider as a straight line AB

θ = $0.5^o = 0.5 \times 0.01745$ rad = 0.008725 rad

r = OC = d = distance between the earth and the moon

r = OC = 3844000 km

Since $l = r\theta$

l= 384400 x 0.008725 = 3353.89 km

Exercise 3.1

Q1. Convert the following to Radian measure

(i) 210^o (ii) 540^o (iii) $42^o\ 36'\ 12''$ (iv) $24^o32'30''$

Q2. Convert the following to degree measure:

(i) $\frac{5\pi}{4}$ rad (ii) $\frac{2\pi}{3}$ rad (iii) 5.52 rad (iv) 1.30 rad

Q3. Find the missing element l, r, θ when:

(i) $l = 8.4$ cm, $\theta = 2.8$ rad

(ii) $l = 12.2$ cm, r = 5cm

(iii) r = 620m, $\theta = 32^o$

Q4. How far a part are two cities on the equator whose longitudes are 10^oE and 50^oW? (Radius of the Earth is 6400km)

Q5. A space man land on the moon and observes that the Earth´s diameter subtends an angle of $1^o\ 54'$ at his place of landing. If the Earth´s radius is 6400km, find the distance between the Earth and the Moon.

Q6. The sun is about 1.496 x 10^8 km away from the Earth. If the angle subtended by the sun on the surface of the earth is 9.3 x 10^{-3} radians approximately. What is the diameter of the sun?

Q7. A horse moves in a circle, at one end of a rope 27cm long, the other end being fixed. How far does the horse move when the rope traces an angle of 70^o at the centre.

Q8. Lahore is 68km from Gujranwala. Find the angle subtended at the centre of the earth by the road. Joining these two cities, earth being regarded as a sphere of 6400km radius.

Q9. A circular wire of radius 6 cm is cut straightened and then bend so as to lie along the circumference of a hoop of radius 24 cm. find measure of the angle which it subtend at the centre of the hoop

Q10. A pendulum 5 meters long swings through an angle of 4.5^0. through what distance does the bob moves ?

Q11. A flywheel rotates at 300 rev/min. If the radius is 6 cm. through what total distance does a point on the rim travel in 30 seconds ?

Answers 3.1

Q1	(i) 3.66 rad	(ii) 3π	(iii) 0.74 rad	(iv) 0.42 rad
Q2.	(i) 225^o	(ii) 120°	(iii) $316^o\,16'\,19''$	(iv) $74^o\,29'\,4''$
Q3.	(i) r = 3cm	(ii) θ = 2.443 rad	(iii) l = 346.4 meters	
Q4.	6704.76 km	Q 5. 386240 km		
Q6.	1.39x10^6 km	Q7. 33 m	Q8. $36'\,43''$	Q9. $\pi/2$ or 90^0
Q10.	0.39 m	Q11. 5657 cm		

3.8 Trigonometric Function and Ratios:

Let the initial line OX revolves and trace out an angle θ. Take a point P on the final line. Draw perpendicular PM from P on OX:

$\angle$XOP = θ, where θ may be in degree or radians.

Now OMP is a right angled triangle,

We can form the six ratios as follows:

$\frac{a}{c}, \frac{b}{c}, \frac{a}{b}, \frac{b}{a}, \frac{c}{b}, \frac{c}{a}$

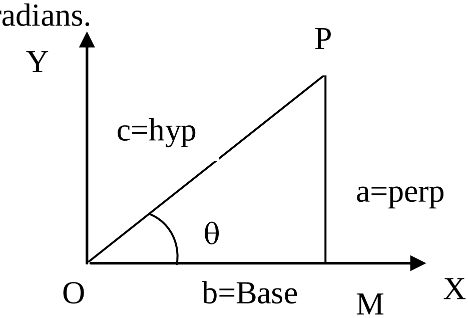


Fig. 3.10

In fact these ratios depend only on the size of the angle and not on the triangle formed. Therefore these ratios called **Trigonometric ratios or trigonometric functions** of angle θ

and defined as below: θ

$$\textbf{Sin } \theta = \frac{a}{c} = \frac{MP}{OP} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Cos } \theta = \frac{b}{c} = \frac{OM}{OP} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{a}{b} = \frac{MP}{OM} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\text{Cot } \theta = \frac{b}{a} = \frac{OM}{MP} = \frac{\text{Base}}{\text{Perpendicular}}$$

$$\text{Sec } \theta = \frac{c}{b} = \frac{OP}{OM} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\text{Cosec } \theta = \frac{c}{a} = \frac{OP}{PM} = \frac{\text{Hyponenuse}}{\text{Perpendicular}}$$

3.9 Reciprocal Functions:

From the above definition of trigonometric functions, we observe that

(i) $\text{Sin } \theta = \dfrac{1}{\text{Cosec } \theta}$ or, $\text{Cosec } \theta = \dfrac{1}{\text{Sin } \theta}$ i.e. Sin θ and Cosec θ are reciprocal of each other.

(ii) $\text{Cos } \theta = \dfrac{1}{\text{Sec } \theta}$ or, $\text{Sec } \theta = \dfrac{1}{\text{Cos } \theta}$ i.e. Cos θ and Sec θ are reciprocals of each other.

(iii) $\tan \theta = \dfrac{1}{\text{Cot } \theta}$ or, $\text{Cot } \theta = \dfrac{1}{\tan \theta}$ i.e. tan θ and Cot θ are reciprocals of each other.

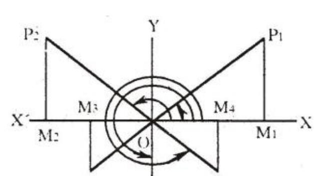
We can also see that;

$$\tan \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta} \quad \text{and} \quad \text{Cot } \theta = \frac{\text{Cos } \theta}{\text{Sin } \theta}$$

3.10 Rectangular Co-ordinates and Sign Convention:

In plane geometry the position of a point can be fixed by measuring its perpendicular distance from each of two perpendicular called co-ordinate axes. The horizontal line (x-axis) is also called abscissa and the vertical line(y-axis) is called as ordinate.

Distance measured from the point O in the direction OX and OY are regarded as positive, while in the direction of OX′ and OY′ are



considered negative.

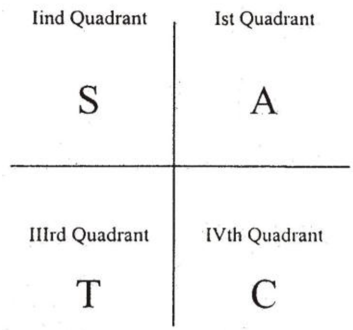
Thus in the given figure OM_1, OM_4, MP_1 and M_2P_2 are positive, while OM_2, OM_3, M_3P_3 and M_4P_4 are negative.

The terminal line i.e., OP_1, OP_2, OP_3, and OP_4 are positive in all the quadrants.

Fig. 3.11

3.11 Signs of Trigonometric Functions:

The trigonometric ratios discussed above have different signs in different quadrants. Also from the above discussion we see that OM and MP changes their sign in different quadrants. We can remember the sign of trigonometric function by "ACTS" Rule or CAST rule. In "CAST" C stands for cosine A stands for All and S stands for Sine and T stands for Tangent.



First Quadrant:

In first quadrant sign of all the trigonometric functions are positive i.e., sin, cos, tan, Cot, Sec, Cosec all are positive.

Second Quadrant:

In second quadrant sine and its inverse cosec are positive. The remaining four trigonometric function i.e., cos, tan, cot, sec are negative.

Third Quadrant:

In third quadrant tan and its reciprocal cot are positive the remaining four function i.e., Sin, cos, sec and cosec are negative.

Fourth Quadrant:

In fourth quadrant cos and its reciprocal sec are positive, the remaining four functions i.e., sin, tan, cot and cosec are negative.

3.12 Trigonometric Ratios of Particular Angles:

1. Trigonometric Ratios of 30^o or $\frac{\pi}{6}$:

Let the initial line OX revolve and trace out an angle of 30^o. Take a point P on the final line. Draw PQ perpendicular from P on OX. In 30^o right angled triangle, the side opposite to the 30^o angle is one-half the length of the hypotenuse, i.e., if PQ = 1 unit then OP will be 2 units.

From fig. OPQ is a right angled triangle

$\therefore$ By Pythagorean theorem, we have

$$(OP)^2 = (OQ)^2 + (PQ)^2$$
$$(2)^2 = (OQ)^2 + (1)^2$$
$$4 = (OQ)^2 + 1$$

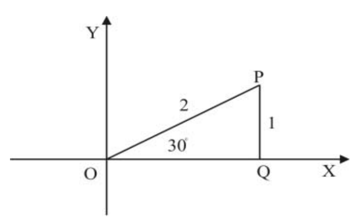


Fig. 4.12

$(OQ)^2 = 3$

$(OQ) = \sqrt{3}$

Fig. 3.12

Therefore $\text{Sin } 30^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{1}{2}$

$$\text{Cos } 30^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{\sqrt{3}}{2}$$

$$\tan 30^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{1}{\sqrt{3}}$$

$$\text{Cot } 30^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\text{Sec } 30^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{2}{\sqrt{3}}$$

$$\text{Cosec } 30^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{2}{1} = 2$$

2. Trigonometric ratios of 45^o Or $\frac{\pi}{4}$

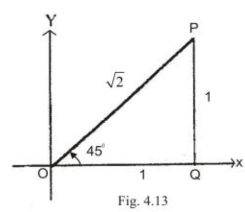


Fig. 4.13

Let the initial line OX revolve and trace out an angle of 45^o. Take a point P on the final line. Draw PQ perpendicular from P on OX . In 45^o right angled triangle the length of the perpendicular is equal to the length of the base

i.e., if PQ = 1 unit. then OQ = 1 unit

From figure by Pythagorean theorem.

$(OP)^2 = (OQ)^2 + (PQ)^2$

$(OP)^2 = (1)^2 + (1)^2 = 1 + 1 = 2$

$OP = \sqrt{2}$

Therefore $\text{Sin } 45^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{1}{\sqrt{2}}$

$$\text{Cos } 45^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{1}{\sqrt{2}}$$

$$\tan 45^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{1}{1} = 1$$

$$\text{Cot } 45^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{1}{1} = 1$$

$$\text{Sec } 45^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\text{Cosec } 45^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

3. Trigonometric Ratios of 60^o or $\frac{\pi}{3}$

Let the initial line OX revolve and trace out an angle of 60^o. Take a point P on the final line. Draw PQ perpendicular from P on OX. In 60^o right angle triangle the length of the base is one-half of the Hypotenuse.

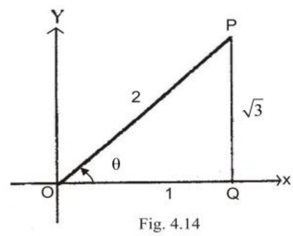


Fig. 4.14

i.e., OQ = Base = 1 unit

then, OP= Hyp = 2 units

from figure by Pythagorean Theorem:

$(OP)^2 = (OQ)^2 + (PQ)^2$

$(2)^2 = (1)^2 + (PQ)^2$

$4 = 1 (PQ)^2$

$(PQ)^2 = 3$

$PQ = \sqrt{3}$

Therefore $$\text{Sin } 60^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{\sqrt{3}}{2}$$

$$\text{Cos } 60^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{1}{2}$$

$$\tan 60^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\text{Cot } 60^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{1}{\sqrt{3}}$$

$$\text{Sec } 60^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\text{Cosec } 60^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{2}{\sqrt{3}} =$$

Trigonometric ratios of 0^o

Let the initial line revolve and trace out a small angle nearly equal to zero 0^o. Take a point P on the final line. Draw PM perpen-dicular on OX.

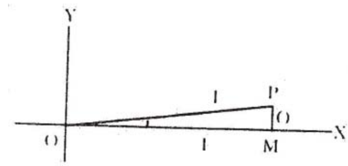


Fig. 4.15

PM = 0

and OP = 1, OM = 1

(Because they just coincide x-axis)

Therefore from figure.

$$\text{Sin } 0^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PM}{OP} = \frac{0}{1} = 0$$

$$\text{Cos } 0^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OM}{OP} = \frac{1}{1} = 1$$

$$\tan 0^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PM}{OM} = \frac{0}{1} = 0$$

$$\text{Cot } 0^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OM}{PM} = \frac{1}{0} = \infty$$

$$\text{Sec } 0^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OM} = \frac{1}{1} = 1$$

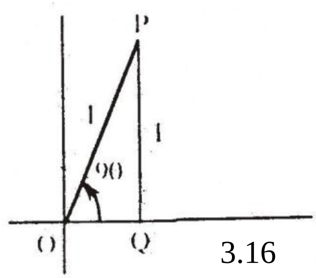
$$\text{Cosec } 0^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PM} = \frac{1}{0} = \infty$$

Trigonometric Ratio of 90^o

Let initial line revolve and trace out an angle nearly equal to 90^o. Take a point P on the final line. Draw PQ perpendicular from P on OX.

OQ = 0, OP = 1, PQ = 1 (Because they just coincide y-axis).

Therefore $$\text{Sin } 90^o = \frac{\text{Prep.}}{\text{Hyp}} = \frac{PQ}{OP} = \frac{1}{1} = 1$$



3.16

Fig. 4.16

$$\text{Cos } 90^o = \frac{\text{Base}}{\text{Hyp}} = \frac{OQ}{OP} = \frac{0}{1} = 0$$

$$\tan 90^o = \frac{\text{Prep.}}{\text{Base}} = \frac{PQ}{OQ} = \frac{1}{0} = \infty$$

$$\text{Cot } 90^o = \frac{\text{Base}}{\text{Prep.}} = \frac{OQ}{PQ} = \frac{0}{1} = 0$$

$$\text{Sec } 90^o = \frac{\text{Hyp.}}{\text{Base}} = \frac{OP}{OQ} = \frac{1}{0} = \infty$$

$$\text{Cosec } 90^o = \frac{\text{Hyp.}}{\text{Prep.}} = \frac{OP}{PQ} = \frac{1}{1} = 1$$

Table for Trigonometrical Ratios of Special angle

$\frac{\text{Angles}}{\text{Ratios}}$	0^o	30^o Or $\frac{\pi}{6}$	45^o Or $\frac{\pi}{4}$	60^o Or $\frac{\pi}{3}$	90^o Or $\frac{\pi}{2}$
Sin θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
Cos θ	$\sqrt{\frac{4}{4}} = 1$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{0}{4}} = 0$
Tan θ	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{2}} = 1$	$\sqrt{\frac{3}{1}} = \sqrt{3}$	$\sqrt{\frac{4}{0}} = \propto$

Example 1:

If cos $\theta = \frac{5}{13}$ and the terminal side of the angle lies in the first quadrant find the values of the other five trigonometric ratio of θ.

Solution:

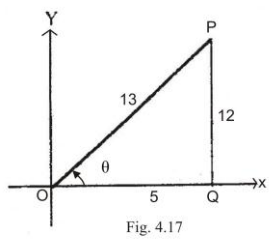


Fig. 4.17

In this cause $\quad \cos\theta = \frac{5}{13}$

$$\cos\theta = \frac{\text{Base}}{\text{Hyp}} = \frac{5}{13}$$

From Fig. $\quad (OP)^2 = (OQ)^2 + (PQ)^2$

$(13)^2 = (5)^2 + (PQ)^2$

$169 = 25 + (PQ)^2$

$(PQ)^2 = 169 - 25$

$= 144$

$PQ = \pm 12$

Because θ lies in the first quadrant

i.e., $\quad \sin\theta = \frac{12}{13} \quad \because$ All the trigonometric ratios will be positive.

$$\text{Cos } \theta = \frac{5}{13}$$

$$\tan\theta = \frac{12}{5}, \qquad \sec\theta = \frac{13}{5}$$

$$\cot\theta = \frac{5}{12}, \qquad \operatorname{cosec}\theta = \frac{13}{12}$$

Example 2:

Prove that $\cos 90^o - \cos 30^o = -2\sin 60^o \sin 30^o$

Solution:

$$\text{L.H.S} = \cos 90^o - \cos 30^o$$

$$= 0 - \frac{\sqrt{3}}{2}$$

$$\text{L.H.S} = -\frac{\sqrt{3}}{2}$$

$$\text{R.H.S} = -2\sin 60^o \sin 30^o$$

$$= -2.\frac{\sqrt{3}}{2} \text{ x } \frac{1}{2}$$

$$\text{R.H.S} = -\frac{\sqrt{3}}{2}$$

Hence L.H.S = R.H.S

Example 3:

Verify that $\sin^2 30^o + \sin^2 60^o + \tan^2 45^o = 2$

Solution:

$$\text{L.H.S} = \sin^2 30^o + \sin^2 60^o + \tan^2 45^o$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$$

$$= \frac{1}{4} + \frac{3}{4} + 1$$

$$= \frac{1+3+4}{4}$$

$$= \frac{8}{4}$$

L.H.S = 2 = R.H.S

Exercise 3.2

Q.1 If $\sin\theta = \frac{2}{3}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

Q.2 If $\sin\theta = \frac{3}{8}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios.

Q.3 If $\cos\theta = -\frac{\sqrt{3}}{2}$, and the terminal side of the angle lies in the third quadrant, find the remaining trigonometric ratios of θ.

Q.4 If $\tan\theta = \frac{3}{4}$, and the terminal side of the angle lies in the third quadrant, find the remaining trigonometric ratios of θ.

Q.5 If $\tan\theta = -\frac{1}{3}$, and the terminal side of the angle lies in the second quadrant, find the remaining trigonometric ratios of θ.

Q.6 If $\cot\theta = \frac{4}{3}$, and the terminal side of the angle is not in the first quadrant, find the trigonometric ratios of 0.

Q.7 If $\cot\theta = \frac{2}{3}$, and the terminal side of the angle does not lies in the first quadrant, find the trigonometric ratios of θ.

Q.8 If $\sin\theta = \frac{4}{5}$,and $\frac{\pi}{2} < \theta < \pi$ find the trigonometric ratios of θ

Q.9 If $\sin\theta = \frac{7}{25}$, find $\cos\theta$, if angle θ is an acute angle.

Q.10 If $\sin\theta = \frac{5}{6}$, find $\cos\theta$, if angle θ is an obtuse angle.

Q.11 Prove that:

(i) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6} = \sin\frac{\pi}{2}$

(ii) $4\tan 60^o \tan 30^o \tan 45^o \sin 30^o \cos 60^o = 1$

(iii) $2\sin 45^o + \frac{1}{2}\operatorname{cosec} 45^o = \frac{3}{\sqrt{2}}$

(iv) $\cos 90^o - \cos 30^o = -2\sin 60^o \sin 30^o$

(v) $\sin^2\frac{\pi}{6}+\sin^2\frac{\pi}{3}+\tan^2\frac{\pi}{4}=2$

Q.12 $\sin^2\frac{\pi}{6}:\sin^2\frac{\pi}{4}:\sin^2\frac{\pi}{3}:\sin^2\frac{\pi}{2}=1:2:3:4$

Q.13 Evaluate

(i) $\cos 30^o \cos 60^o - \sin 30^o \sin 60^o$

(ii) $\dfrac{\tan 60^o - \tan 30^o}{1+\tan 60^o \tan 30^o}$

Answers 3.2

Q.1 $\text{Sin }\theta = \frac{2}{3}$ $\text{Cot }\theta = -\frac{\sqrt{5}}{2}$

$COS\theta = -\frac{\sqrt{5}}{3}$ $\sec\theta = -\frac{3}{\sqrt{5}}$

$\tan\theta = -\frac{2}{\sqrt{5}}$ $\text{Cosec }\theta = \frac{3}{2}$

Q.2 $\text{Sin }\theta = \frac{3}{8}$ $\text{Cot }\theta = -\frac{\sqrt{55}}{3}$

$\text{Cos }\theta = -\frac{\sqrt{55}}{8}$ $\text{Sec }\theta = -\frac{8}{\sqrt{55}}$

$\tan\theta = -\frac{3}{\sqrt{55}}$ $\text{Cosec }\theta = \frac{8}{3}$

Q.3 $\text{Sin }\theta = \frac{-1}{2}$ $\text{Cot }\theta = \sqrt{3}$

$\text{Cos }\theta = -\frac{\sqrt{3}}{2}$ $\text{Sec }\theta = -\frac{2}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\text{Cosec }\theta = -2$

Q.4 $\text{Sin }\theta = -\frac{3}{5}$ $\text{Cot }\theta = \frac{4}{3}$

$\text{Cos }\theta = -\frac{4}{5}$ $\text{Sec }\theta = -\frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\text{Cosec }\theta = -\frac{5}{3}$

Q.5 $\text{Sin }\theta = \dfrac{1}{\sqrt{10}}$ $\text{Cot }\theta = -3$

$\text{Cos }\theta = -\dfrac{3}{\sqrt{10}}$ $\text{Sec }\theta = -\dfrac{\sqrt{10}}{3}$

$\tan\theta = -\dfrac{1}{3}$ $\text{Cosec }\theta = \sqrt{10}$

Q.6 $\text{Sin }\theta = -\dfrac{3}{5}$ $\text{Cot }\theta = \dfrac{4}{3}$

$\text{Cos }\theta = -\dfrac{4}{5}$ $\text{Sec }\theta = -\dfrac{5}{4}$

$\tan\theta = \dfrac{3}{4}$ $\text{Cosec }\theta = -\dfrac{5}{3}$

Q.7 $\text{Sin }\theta = \dfrac{-3}{\sqrt{10}}$ $\text{Cot }\theta = \dfrac{2}{3}$

$\text{Cos }\theta = -\dfrac{2}{\sqrt{13}}$ $\text{Sec }\theta = -\dfrac{\sqrt{13}}{2}$

$\tan\theta = \dfrac{3}{2}$ $\text{Cosec }\theta = -\dfrac{\sqrt{13}}{3}$

Q.8 $\text{Sin }\theta = \dfrac{4}{5}$ $\text{Cot }\theta = -\dfrac{3}{4}$

$\text{Cos }\theta = -\dfrac{3}{4}$ $\text{Sec }\theta = -\dfrac{5}{3}$

$\tan\theta = -\dfrac{4}{3}$ $\text{Cosec }\theta = \dfrac{5}{4}$

Q.9 $\cos\theta = \dfrac{24}{25}$

Q.10 $\cos\theta - \dfrac{\sqrt{11}}{6}$

Q.13 (i) 0 (ii) $\dfrac{1}{\sqrt{3}}$

3.13 Fundamental Identities:

For any real number θ, we shall derive the following three fundamental identities

(i) $\text{Cos}^2\theta + \text{Sin}^2\theta = 1$

(ii) $\text{Sec}^2\theta = 1 + \tan^2\theta$

(iii) $\text{Cosec}^2\theta = 1 + \text{Cot}^2\theta$

Proof :

Consider an angle $\angle XOP = \theta$ in the standard position. Take a point P on the terminal line of the angle θ. Draw PQ perpendicular from P on OX.

From fig., ΔOPQ is a right angled triangle. By pythagoruse theorem

$$(OP)^2 = (OQ)^2 + (PQ)^2$$

Or, $z^2 = x^2 + y^2$

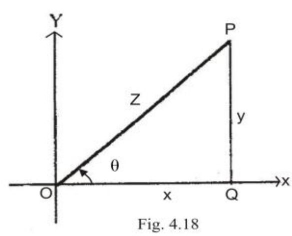


Fig. 4.18

(i) Dividing both sides by z^2

then $$\frac{z^2}{z^2} = \frac{x^2}{z^2} + \frac{y^2}{z^2}$$

$$1 = \left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2$$

$$1 = (\text{Cos}\,\theta)^2 + (\text{Sin}\,\theta)^2$$

$$1 = \text{Cos}^2\theta + \text{Sin}^2\theta$$

or, $$\text{Cos}^2\theta + \text{Sin}^2\theta = 1$$

(ii) Dividing both sides of Eq. (i) by x^2, we have

$$\frac{z^2}{x^2} = \frac{x^2}{x^2} + \frac{y^2}{x^2}$$

$$\left(\frac{z}{x}\right)^2 = 1 + \left(\frac{y}{x}\right)^2$$

$$(\text{Sec}\,\theta)^2 = 1 + (\tan\theta)^2$$

$$\text{Sec}^2\theta = 1 + \tan^2\theta$$

(iii) Again, dividing both sides of Eq (i) by y^2, we have

$$\frac{z^2}{y^2} = \frac{x^2}{y^2} + \frac{y^2}{y^2}$$

$$\left(\frac{z}{y}\right)^2 = \left(\frac{x}{y}\right)^2 + 1$$

$$(\text{Cosec } \theta)^2 = (\text{Cot } \theta)^2 + 1$$

$$\text{Cosec}^2 \theta = \text{Cot}^2 \theta + 1$$

$$\text{Cosec}^2 \theta = 1 + \text{Cot}^2 \theta$$

Example 1:

Prove that $\dfrac{\text{Sin x}}{\text{Cosec x}} + \dfrac{\text{Cos x}}{\text{sec x}} = 1$

Solution:

$$\text{L.S.H.} = \frac{\text{Sin x}}{\text{Cosec x}} + \frac{\text{Cos x}}{\text{sec x}}$$

$$= \text{Sin x.}\frac{1}{\text{Cosec x}} + \text{Cos x.}\frac{1}{\text{sec x}} \quad \because \quad \frac{1}{\text{Cosec x}} = \text{Sin x}$$

$$= \text{Sin x. Sin x + Cos x. Cos x} \quad \because \quad \frac{1}{\text{Sec x}} = \text{Cos x}$$

$$= \text{Sin}^2 \text{ x} + \text{Cos}^2 \text{ x}$$

$$= 1$$

$$= \text{R.H.S}$$

Example 2:

Prove that $\dfrac{\text{Sec x} - \text{Cos x}}{1 + \text{Cos x}} = \text{Sec x} - 1$

Solution:

$$\text{L.H.S} = \frac{\text{Sec x} - \text{Cos x}}{1 + \text{Cos x}}$$

$$= \frac{\dfrac{1}{\text{Cos x}} - \text{Cos x}}{1 + \text{Cos x}}$$

$$= \frac{\dfrac{1 - \text{Cos}^2 \text{ x}}{\text{Cos x}}}{1 + \text{Cos x}} = \frac{1 - \text{Cos}^2 \text{ x}}{\text{Cos x}(1 + \text{Cos x})}$$

$$= \frac{(1 - \text{Cos x})(1 + \text{Cos x})}{\text{Cos x}(1 + \text{Cos x})}$$

$$= \frac{1 - \text{Cos x}}{\text{Cos x}} = \frac{1}{\text{Cos x}} - \frac{\text{Cos x}}{\text{Cos x}}$$

$$= \text{Sex x} - 1 = \text{R.H.S.}$$

Example 3:

prove that $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

Solution: L.H.S. $= \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{(1-\sin\theta)\,(1-\sin\theta)}{(1+\sin\theta)\,(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} \qquad = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{(1-\sin\theta)}{\cos\theta} \qquad = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta \qquad = \text{R.H.S.}$$

Exercise 3.3

Prove the following Identities:

Q.1 $1 - 2\,\text{Sin}^2\,\theta = 2\,\text{Cos}^2\,\theta - 1$

Q.2 $\text{Cos}^4\,\theta - \text{Sin}^4\,\theta = 1 - 2\,\text{Sin}^2\theta$

Q.3 $\dfrac{1}{\text{Cosec}^2\,\theta} + \dfrac{1}{\text{Sec}^2\,\theta} = 1$

Q.4 $\dfrac{1}{\tan\theta + \text{Cot}\,\theta} = \text{Sin}\,\theta.\text{Cos}\,\theta$

Q.5 $(\text{Sec}\,\theta - \tan\theta)^2 = \dfrac{1-\text{Sin}\,\theta}{1+\text{Sin}\,\theta}$

Q.6 $(\text{Cosec}\,\theta - \text{Cot}\,\theta)^2 = \dfrac{1-\text{Cos}\,\theta}{1+\text{Cos}\,\theta}$

Q.7 $(1-\text{Sin}^2\theta)(1+\tan^2\theta)=1$

Q.8 $\dfrac{1}{1+\text{Sin}\,\theta} + \dfrac{1}{1-\text{Sin}\,\theta} = 2\text{Sec}^2\theta$

Q.9 $\sqrt{\dfrac{1-\text{Sin}\,\theta}{1+\text{Sin}\,\theta}} = \text{Sec}\theta - \tan\theta$

Q.10 $\sqrt{\dfrac{1+\text{Cos}\,\theta}{1-\text{Cos}\,\theta}} = \text{Cosec}\,\theta + \cot\theta$

Q.11 $\dfrac{1-\tan A}{1+\tan A}=\dfrac{\text{Cot } A-1}{\text{Cot } A+1}$

Q.12 $\dfrac{\text{Cot}^2\theta-1}{\text{Cot}^2\theta+1}=2\text{Cos}^2\theta-1$

Q.13 $\dfrac{\tan\theta}{1-\text{Cot }\theta}+\dfrac{\text{Cot }\theta}{1-\tan\theta}=\text{Sec }\theta\text{ Cosec }\theta+1$

Q.14 $\dfrac{\text{Sec }\theta-\tan\theta}{\text{Sec }\theta+\tan\theta}=1-2\text{Sec }\theta\tan\theta+2\tan^2\theta$

Q.15 $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}=\dfrac{(1-\tan\theta)^2}{(1-\cot\theta)^2}$

Q.16 $\text{Cosec } A+\text{Cot } A=\dfrac{1}{\text{Cosec } A-\text{Cot } A}$

Q.17 $\dfrac{1}{\text{Sec }\theta+\tan\theta}=\dfrac{1-\text{Sin }\theta}{\text{Cos }\theta}=\text{Sec } x-\tan x$

Q.18 $(1-\tan\theta)^2+(1-\text{Cot }\theta)^2=(\text{Sec }\theta-\text{Cosec }\theta)^2$

Q.19 $\dfrac{\text{Cos}^3 t-\text{Sin}^3 t}{\text{Cos } t-\text{Sin } t}=1+\text{Sin } t\text{ Cos } t$

Q.20 $\text{Sec}^2 A+\tan^2 A=(1-\text{Sin}^4 A)\text{ Sec}^4 A$

Q.21 $\dfrac{\text{Sec } x-\text{Cos } x}{1+\text{Cos } x}=\text{Sec } x-1$

Q.22 $\dfrac{1+\text{Sin }\theta+\text{Cos }\theta}{1+\text{Sin }\theta-\text{Cos }\theta}=\dfrac{\text{Sin }\theta}{1-\text{Cos }\theta}$

Q.23 $\dfrac{\text{Sin } x+\text{Cos } x}{\tan^2 x-1}=\dfrac{\text{Cos}^2 x}{\text{Sin } x-\text{Cos } x}$

Q.24 $(1+\text{Sin }\theta)(1-\text{Sin }\theta)=\dfrac{1}{\text{Sec}^2\theta}$

Q.25 $\dfrac{\tan\theta}{\text{Sec }\theta-1}+\dfrac{\tan\theta}{\text{Sec }\theta+1}=2\text{Cosec }\theta$

Q.26 $\dfrac{\cot\theta\cos\theta}{\cot\theta+\cos\theta}=\dfrac{\cot\theta-\cos\theta}{\cot\theta\cos\theta}$

Q.27 If $m=\tan\theta+\text{Sin }\theta$ and $n=\tan\theta-\text{Sin }\theta$ than prove that
$m^2-n^2=4\sqrt{m\,n}$

3.14 Graph of Trigonometric Functions:

In order to graph a function = f(x), we give number of values of x and obtain the corresponding values of y. The several ordered pairs (x, y) are obtained we plotted these points by a curve we get the required graph.

3.14.1 Graph of Sine Let, y = Sinx where, $0^o \leq x \leq 360^o$

Or where, $0 \leq x \leq 2\pi$

1. Variations

Quadrants

	1st	2nd	3^{rd}	4^{th}
x Sinx	0 to 90^o + ve, Increase from 0 to 1	90^o to 180^o + ve, decrease from 1 to 0	180^o to 270^o $-$ve, decrease from 0 to -1	270^o to 360^o $-$ve, Increases from -1 to 0

2. Table:

x	0	30^o	60^o	90^o	120^o	150^o	180^o
Sinx	0	0.5	0.87	1	0.87	0.5	0

x	210^o	240^o	270^o	300^o	330^o	360^o
Sinx	-0.50	$-.87$	-1	$-.87$	$-.5$	0

3. Graph in Figure (3.19):

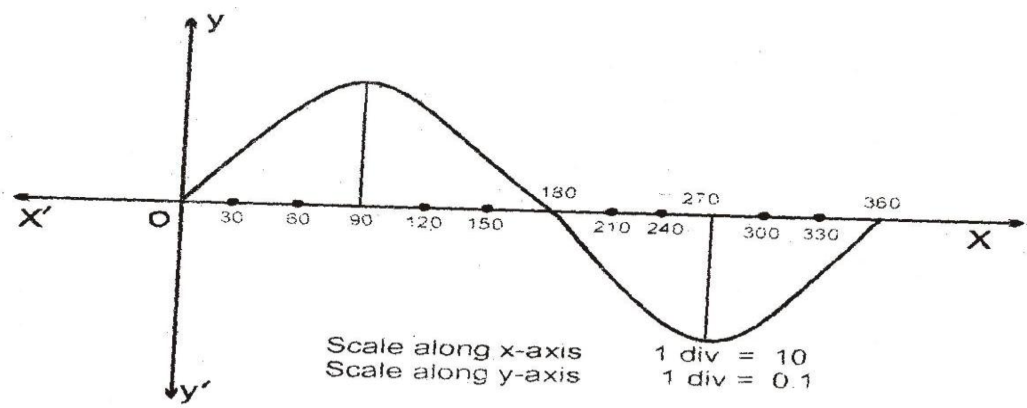


Fig. 4.19

3.14.2 Graph of Cosine

Let, y = Cosx where, $0^o \leq x \leq 360^o$

Or where, $0 \leq x \leq 2\pi$

1. Variations

Quadrants

	1st	2nd	3^{rd}	4^{th}
x y = Cosx	0 to 90^o + ve, decrease from 1 to 0	90^o to 180^o $-$ ve, decrease from 0 to -1	180^o to 270^o $-$ve, increase from -1 to 0	270^o to 360^o + ve, increases from 0 to 1

2. Table:

x	0	30^o	60^o	90^o	120^o	150^o	180^o
y = Cosx	1	0.87	0.5	0	-0.5	-0.87	-1

x	210^o	240^o	270^o	300^o	330^o	360^o
y = Cosx	-0.87	-0.5	0	0.5	0.87	1

3. Graph in Figure (3.20):

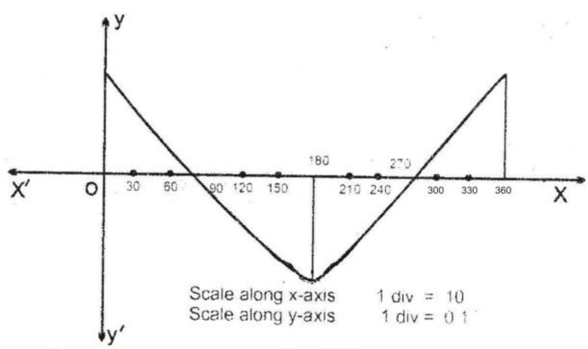


Fig. 4.20

Fig. 3.20

3.14.3 Graph of tanx

Let, y = tanx where, $0^o \leq x \leq 360^o$

Or where, $0 \leq x \leq 2\pi$

1. Variations

Quadrants

	1^{st}	2nd	3rd	4^{th}
x y = tanx	0 to 90^o + ve, Increase from 0 to $\propto$	90^o to 180^o $-$ve, increase from $-\propto$ to 0	180^o to 270^o + ve, increase from 0 to $\propto$	270^o to 360^o $-$ve, increases from $-\propto$ to 0

2. Table:

x	0	30^o	60^o	90^o	120^o	150^o	180^o
y = tanx	0	0.58	1.73	$\propto$	-1.73	-0.58	0

x	210^o	240^o	270^o	300^o	330^o	360^o
y = tanx	+ .58	1.73	$-\propto$, $+\propto$	-1.73	-2.58	0

3. Graph in Figure (3.21):

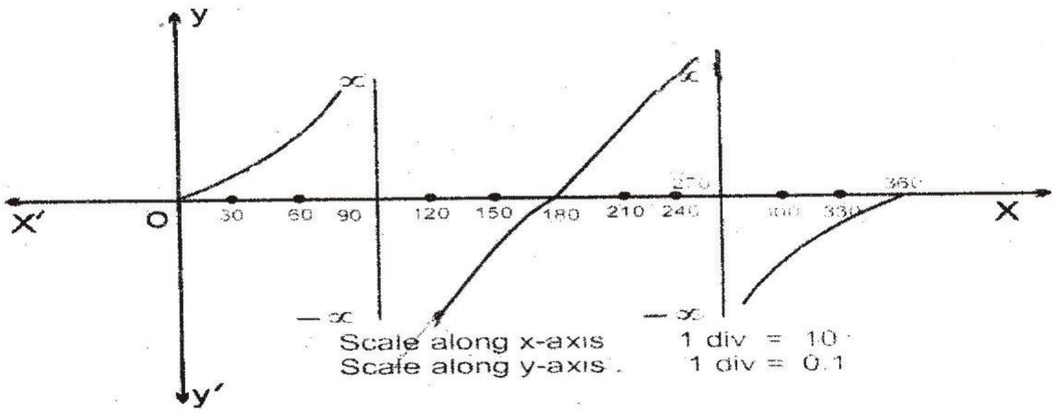


Fig. 4.21

Fig. 3.21

3.14.4 Graph of Cotx:

Let, y = cotx where, $0^o \leq x \leq 360^o$

1. Variations

Quadrants

	1^{st}	2nd	3rd	4^{th}
x y = Cotx	0 to 90^o + ve, Increase from $\propto$ to 0	90^o to 180^o $-$ve, increase from 0 to $-\propto$	180^o to 270^o + ve, increase from $\propto$ to 0	270^o to 360^o $-$ve, increases from 0 to $-\propto$

2. Table:

x	0	30^o	60^o	90^o	120^o	150^o	180^o
y = Cotx	$\propto$	1.73	0.58	0	-0.58	-1.73	$-\propto$

x	210^o	240^o	270^o	300^o	330^o	360^o
y = Cotx	1.73	0.58	0	-0.58	-1.73	$\propto$

3. Graph in Figure (3.22):

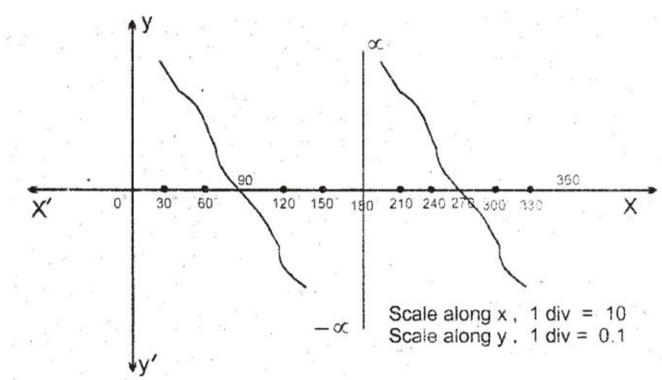


Fig. 4.22

Fig. 3.22

3.14.5 Graph of Secx:

Let, y = secx where, $0^o \le x \le 360^o$

1. Variations

Quadrants

	1^{st}	2nd	3rd	4^{th}
x y = Secx	0 to 90^o + ve, Increase from 1 to $\propto$	90^o to 180^o $-$ve, increase from $-\propto$ to -1	180^o to 270^o $-$ve, increase from -1 to $-\propto$	270^o to 360^o + ve, increases from $-\propto$ to 1

2. Table:

x	0	30^o	60^o	90^o	120^o	150^o	180^o
y = Secx	1	1.15	2	$+\propto$	-2	1.15	1

x	210^o	240^o	270^o	300^o	330^o	360^o
y = Secx	-1.15	-2	$\propto$	2	1.15	1

3. Graph in Figure (3.23):

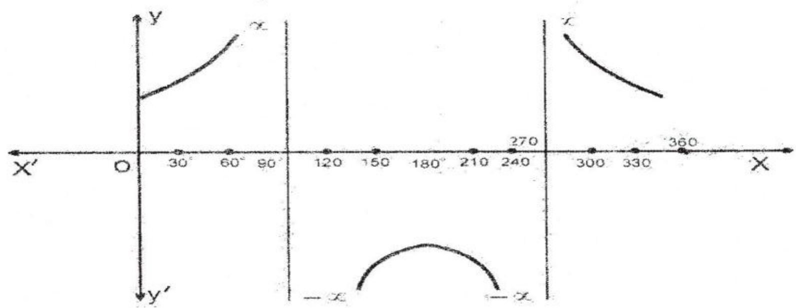


Fig. 4.23

3.14.6 Graph of Cosecx: Let, y = Cosecx ,where $0^o \le x \le 360^o$

1. Variations

Quadrants

	1^{st}	2nd	3rd	4^{th}
x y = Cosecx	0 to 90^o + ve, Increase from $\propto$ to 1	90^o to 180^o + ve, increase from 1 to $\propto$	180^o to 270^o $-$ve, increase from $-\propto$ to -1	270^o to 360^o $-$ ve, increases from -1 to $-\propto$

2. Table:

x	0	30^o	60^o	90^o	120^o	150^o	180^o
y = Cosecx	$\propto$	2	1.15	1	1.15	2	$\propto$

x	210^o	240^o	270^o	300^o	330^o	360^o
y = Cosecx	-2	-1.15	-1	-1.15	-2	$-\propto$

3. Graph in Figure (3.24):

Fig.3.2

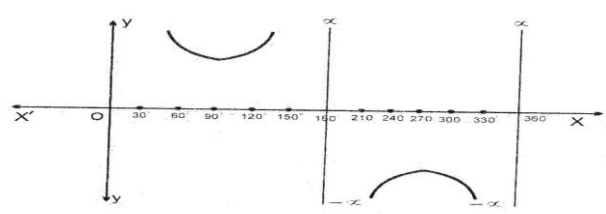


Fig. 4.24

Exercise 3.4

Q.1 Draw the graph of tan 2A as A varies from 0 to π .

Q.2 Plot the graph of 1 – Sin x as x varies from 0 to 2π .

Q.3 Draw the graphs for its complete period.

(i) $y = \frac{1}{2}\text{Sin } 2x$ (ii) $y = \text{Sin } 2x$

(iii) $y = \frac{1}{2}\cos x$

Summary

Trigonometry means measurement of triangles.

1. Radian is an angle subtended at the center of a circle by an arc of the circle equal in length to its radius.

   i.e. π Radian = 180 degree

   1 rad = $57^{o}\ 17'\ 45''$

   1 degree = 0.01745 radian

2. Length of arc of the circle, $l = s = r\theta$

3. Trigonometric functions are defined as:

$$\text{Sin}\theta = \frac{AP}{OP}, \text{Cosec}\theta = \frac{OP}{AP}$$

$$\text{Cos}\theta = \frac{OA}{OP}, \text{Sec}\theta = \frac{OP}{OA}$$

$$\tan\theta = \frac{AP}{OA}, \text{Cot}\theta = \frac{OA}{AP}$$

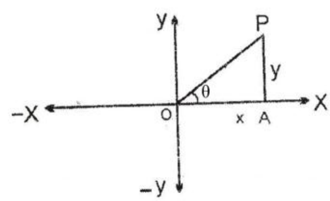


Fig. 4.25

4. Relation between trigonometric ratios:

(i) $\text{Sec}\theta = \frac{1}{\text{Cos}\theta}$ (ii) $\text{Cosec}\theta = \frac{1}{\text{Sin}\theta}$

(iii) $\text{Cot}\theta = \frac{1}{\tan\theta}$ (iv) $\text{Cos}\theta = \frac{1}{\text{Sec}\theta}$

(v) $\text{Sin}\theta = \frac{1}{\text{Cosec}\theta}$ (vi) $\tan\theta = \frac{1}{\text{Cot}\theta}$

(vii) $\text{Sin}^2\theta + \text{Cos}^2\theta = 1$ (viii) $\text{Sec}^2\theta = 1 + \tan^2\theta$

(ix) $\text{Cosec}^2\theta = 1 + \text{Cot}^2\theta$

5. Signs of the trigonometric functions in the Four Quadrants.

Quadrant	I	II	III	IV
Positive	All +ve	$\text{Sin}\,\theta$, $\text{Cosec}\,\theta$	$\tan\theta$, $\cot\theta$	$\text{Cos}\,\theta$, $\text{Sec}\,\theta$
Negative	Nil	$\text{Cos}\,\theta$ $\text{Sec}\,\theta$ $\tan\theta$ $\text{Cot}\,\theta$	$\text{Cos}\,\theta$ $\text{Sec}\,\theta$ $\text{Sin}\,\theta$ $\text{Cosec}\,\theta$	$\text{Sin}\,\theta$ $\text{Cosec}\,\theta$ $\tan\theta$ $\text{Cot}\,\theta$

Short Questions

Write the short answers of the following:

Q.1: **Define degree and radians measure**

Q.2: Convert into radius measure.

(a) 120^{o}, (b) $22\frac{1}{2}^{o}$, (c) $12^{o}\ 40'$, (d) $42^{o}\ 36'\ 12''$

Q.3: Convert into degree measure

(a) $\frac{\pi}{2}$ rad, (b) 0.726 rad. (c) $\frac{2\pi}{3}$ rad.

Q.4: Prove that $\ell = r\theta$

Q.5: What is the length of an arc of a circle of radius 5 cm whose central angle is 140^{o} ?

Q.6: Find the length of the equatorial arc subtending an angle 1^{o} at the centre of the earth taking the radius of earth as 6400 KM.

Q.7: Find the length of the arc cut off on a circle of radius 3 cm by a central angle of 2 radius.

Q.8: Find the radius of the circle when $\ell = 8.4$ cm, $\theta = 2.8$ rad

Q.9: If a minute hand of a clock is 10 cm long, how far does the tip of the hand move in 30 minutes?

Q.10 Find x, if $\tan^2 45^{o} - \cos^2 60^{o} = x \sin 45^{o} \cos 45^{o}. \tan 60^{o}$.

Q.11: Find r when $l = 33$ cm. $\theta = 6$ radian

Q12: Prove that $2 \sin 45^{o} + \frac{1}{2} \text{ cosec } 45^{o} = \frac{3}{\sqrt{2}}$

Q.13: Prove that $\tan^2 30^{o} + \tan^2 45^{o} + \tan^2 60^{o} = \frac{13}{3}$

Q.12: Prove that $\dfrac{2\tan\frac{\bar{\wedge}}{6}}{1-\tan^2\frac{\bar{\wedge}}{6}} = \sqrt{3}$

Q.13: prove that $\cos 30^o \, \text{Cos}\, 60^o - \sin 30^o \sin 60^o = 0$

Q.14: Prove that $\text{Cos}\, 90^o - \cos 30^o = -2 \sin 60^o \sin 30^o$

Q.15: Prove that $\text{Sin}^2\theta + \cos^2\theta = 1$

Q.16: Prove that: $1 + \tan^2\theta = \sec^2\theta$

Q.17: Prove that $1 + \cot^2\theta = \text{Cosec}^2\theta$

Q.18: Prove that: $(1 + \text{Sin}\,\theta)(1 - \text{Sin}\,\theta) = \dfrac{1}{\text{Sec}^2\theta}$

Q.19: Show that: $\text{Cot}^4\theta + \text{Cot}^2\theta = \text{Cosec}^4\theta - \text{cosec}^2\theta$

Q.20: Prove that: $\text{Cos}\theta + \tan\theta\, \text{Sin}\theta = \text{Sec}\,\theta$

Q.21: Prove that $1 - 2\,\text{Sin}^2\theta = 2\,\text{Cos}^2\theta - 1$

Q.22: $\text{Cos}^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$

Q.23: $\dfrac{1}{1 + \text{Sin}\,\theta} + \dfrac{1}{1 - \text{SIn}\theta} = 2\,\text{Sec}2\theta$

Answers

2. (a) 2.09 rad (b) 0.39 rad (c) 0.22 rad (d) 0.74 radius

3. (a) 90^o (b) $41^0\, 35'\, 48''$ (c) 120 degree

5. 12.21 cm. 6. 111.7 Km 7. 6 cm

8. 3cm. 9. 31.4 cm 10. $\dfrac{\sqrt{3}}{2}$ 11. 5.5 cm.

Objective Type Questions

Q.1 Each questions has four possible answers. Choose the correct answer and encircle it.

__1. One degree is equal to:

(a) π rad (b) $\frac{\pi}{180}$ rad

(c) $\frac{180}{\pi}$ rad (d) $\frac{\pi}{360}$ rad

__2. 15^o is equal to:

(a) $\frac{\pi}{6}$ rad (b) $\frac{\pi}{3}$ rad

(c) $\frac{\pi}{12}$ rad (d) $\frac{\pi}{15}$ rad

__3. 75^o is equal to

(a) $\frac{\pi}{12}$ rad (b) $\frac{2\pi}{3}$ rad

(c) $\frac{4\pi}{3}$ rad (d) $\frac{5\pi}{12}$ rad

__4. One radian is equal to:

(a) 90^o (b) $\left(\frac{90}{\pi}\right)^o$

(c) 180^o (d) $\left(\frac{180}{\pi}\right)^o$

__5. The degree measure of one radian is approximately equal to:

(a) 57.3 (b) 57.2

(c) 57.1 (d) 57.0

__6. $\frac{2\pi}{3}$ radians are equal to:

(a) 60^o (b) 90^o

(c) 120^o (d) 150^o

__7. The terminal side of θ lies in 4^{th} quadrant, sign of the sin θ will be:

(a) Positive (b) Negative

(c) Both +ve and – ve (d) None of these

__8. The terminal side of θ lies in 4^{th} quadrant, both $\sin\theta$ and $\tan\theta$ are:

(a) $\sin\theta > 0, \tan\theta > 0$ (b) $\sin\theta > 0, \tan\theta < 0$

(c) $\sin\theta < 0, \tan\theta < 0$ (d) $\sin\theta < 0, \tan\theta > 0$

__9. A circle is equal to 2π rad and also to 360^o, then:

(a) $360^o = 2\pi$ rad (b) $360^o = \frac{3}{4}\pi$ rad

(c) $360^o = \frac{\pi}{6}$rad (d) None of a, b & c

__10. π rad is equal to:

(a) 360^o (b) 270^o

(c) 180^o (d) 90^o

__11. The relation between are l, radius r and central angle θ rad is:

(a) $l = \frac{\theta}{r}$ (b) $l = \frac{r}{\theta}$

(c) $l = r\,\theta$ (d) $l = r^2\,\theta$

__12. If l = 12cm and r = 3 cm, then θ is equal to:

(a) 36 rad (b) 4 rad

(c) $\frac{1}{4}$rad (d) 18 rad

__13. An angle subtended at the centre of a circle by an arc equal to the radius of the circle is called:

(a) Right angle (b) Degree

(c) Radian (d) Acute angle

__14. The radian measure of the angle described by a wheel in 5 revolution is:

(a) 5π (b) 10π

(c) 15π (d) 20π

__15. If an are of a circle has length l and subtends an angle θ, then radius ′r′ will be:

(a) $\frac{\theta}{l}$ (b) $\frac{l}{\theta}$

(c) $l\,\theta$ (d) $l + \theta$

__16. If sin x = $\frac{\sqrt{3}}{2}$ and the terminal ray of x lies in 1st quadrant, then cosx is equal to:

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) $-\frac{1}{\sqrt{2}}$

__17. If sin $\theta = \frac{3}{5}$ and the terminal side of the angle lies in 2nd quadrant, then tan θ is equal to:

(a) $\frac{4}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $-\frac{3}{4}$

__18. If sin θ is +ve and cos θ is –ve, then the terminal side of the angle lies in:

(a) 1st quad (b) 2nd quad (c) 3rd quad (d) 4th quad

__19. If sin θ is +ve and tan θ is-ve, then the terminal side of the angle lies in

(a) 1st quad (b) 2nd quad
(c) 3rd quad (4) 4th quad

__20. If sin $\theta = \frac{2}{\sqrt{7}}$ and cos $\theta = -\frac{1}{\sqrt{7}}$, then cot θ is equal to:

(a) 2 (b) -1
(c) $-\frac{1}{2}$ (d) –2

21. $\sec^2\theta + \text{cosec}^2\theta$ is equal to:

(a) $\sec^2\theta\,\text{cosec}^2\theta$ (b) $\sin\theta\ \cos\theta$
(c) $2\sec^2\theta$ (d) $2\,\text{cosec}^2\theta$

Answers

1.	b	2.	c	3.	d	4.	d	5.	a
6.	c	7.	b	8.	c	9.	a	10.	c
11.	c	12.	b	13.	c	14.	b	15.	b
16.	c	17.	b	18.	b	19.	d	20.	c
21.	a								