LECTURE NOTES ON

CONTROL SYSTEM AND COMPONENTS

6TH SEMESTER ETC

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FUNDAMENTAL OF CONTROL SYSTEM

control system plays a very important role in the advancement & important improvements of engeneering skills.

SYSTEM

→ It is an arrangement or combigation of different physical component that are connected together to forman entire unit to archive contain objective.

→ It can be of any time that is physical biological ecological etc.

CONTROL :-

→ Control means to regulate derect or command a system so that a desered objective is obtained. Example - switching ON & OFF a Lamp using a switch.

PLANT :

It is defined as a person of a system which is to be controlled or regulated it is also called a process.

CONTROLLER :

It is the element the system itself which controls the entire process.

→ It may be external to the system & or internal to the system.

INPUT :-

The applied signal or excitation signal that is applied to a control system to obtained a specified 'O/P'.

OUTPUT

It is the actual response that is obtained from a control system due to the application of the input.

Disturbance

It is the signal that has some adverse dificult on the value of the output of a system.

→ It may be two types
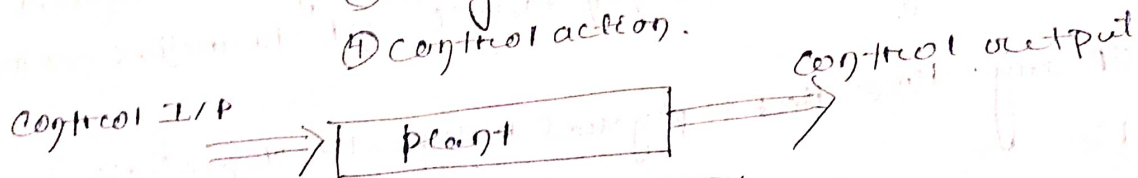(i) Internal disturbance
(ii) external disturbance

CONTROL SYSTEM

→ It is an arrangement of different physical elements linked in such a manner so as to regulate, direct or command, it self to obtained a certain objective.

→ It must have
1. input
2. output
3. may to archive input & o/p objective
4. control action.

Control I/P → ⟶ [plant] ⟶ Control output
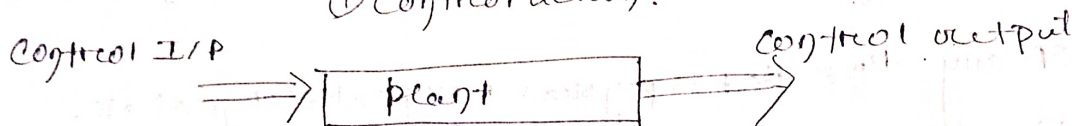
CLASSIFICATION OF CONTROL SYSTEM :-

1 NATURAL CONTROL SYSTEM:-

The system inside a biological or occur in a natural way is called Natural control system.

MAN MAIN CONTROL SYSTEM :-

The system inside a biological or occur in a natural way is called Natural control system.

It is a type of control system that has designed & developed by man.

ex. automobile system.

AUTOMATIC CONTROL SYSTEM :

It is a theoritical base for mechanization & automation which uses the method from mathmetics engeneering it has three component sensor responderce & aulticaufory.

COMBINATIONAL CONTROL SYSTEM :

It is the combination of natural control system & man-main control system. Example :- Driver Driving a car.

Time varient control system :-

It is a type of control system in which the parameter vary with time.

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② output
③ may to archive input & o/p objective
④ control action.

Control I/P   Control output

⟹ | plant | ⟹

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Time invariant control system :-

It is type of control system in which the parameters vary with time.

Time invariant control system

It is type of control system in which the parameters vary with time.

Time invariant control system

It is a type of control system in which the parameters doesn't vary with time.

LINEAR CONTROL SYSTEM

If the control system satisfied the additive property as well as homogeneous property then it is known as linear control system.

$$Additive = f(x+y) = f(x) + f(y)$$

$$Homogeneous = \pi(\beta x) = \beta f(x)$$

$$\beta = constant.$$

NON - LINEAR :-

If a control system does not satisfy the additive property as well as the homogeneous property then it is known as non-linear control system.

Ex: ALL physical system.

Continuous time control system :-

If all the system variables of a control system are function of time then is known as continuous time control system

Ex: - Speed control of DC motor with tappo generator feedback.

Discrete Time control system :

If one or more system variables of a control system known at a certain descrete time than it is called as discrete time control system.

Ex :- microprocessor Based system

Deterministic control system :-

If the response to the I/P & the external disturbances of control system is predictable & repeatative than it is known as deterministic control system.

Stochastic control system :-

If the response of a system is not predictable then it is known as stochastic control system.

Lumped parameter control system :-

→ If the control system can be represented by ordinary differential equation the it is known as lumped parameter control system.

→ In a electrical network parameter such as inductance, Resistance, capacitance.

Distributed Parameter control System :-

If a control system can be described by partial differential equation then it is known as distributed parameter control system.

EX: transmission line characteristics parameter.

SISO (single input single output)

If a control system has one input and one output then it is known as SISO.

MIMO (Multiple input multiple output)

If a control system has multiple input and multiple output then it is known as MIMO.

→ MIMO also known as multivariable control system.

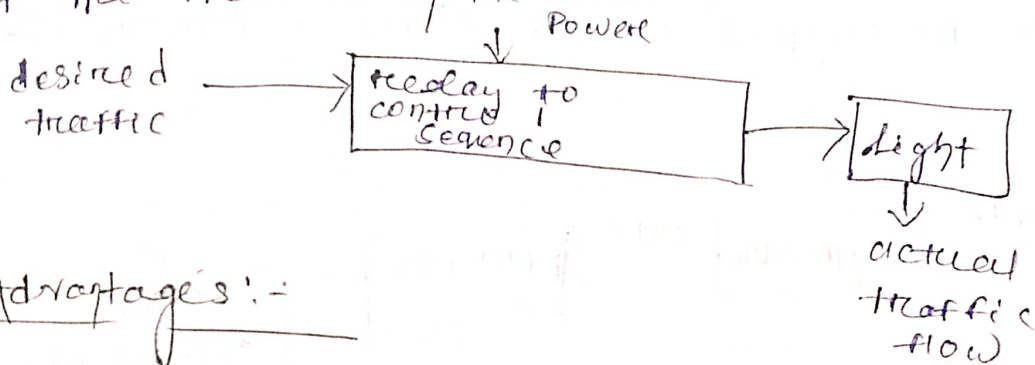## OPEN LOOPED CONTROL SYSTEM :-

A system in which control action does not depends on output it is known as open looped control system.

EX: Automatic washing machine. Bread toaster, driver etc.

Input → [Controller] → [process] → control output
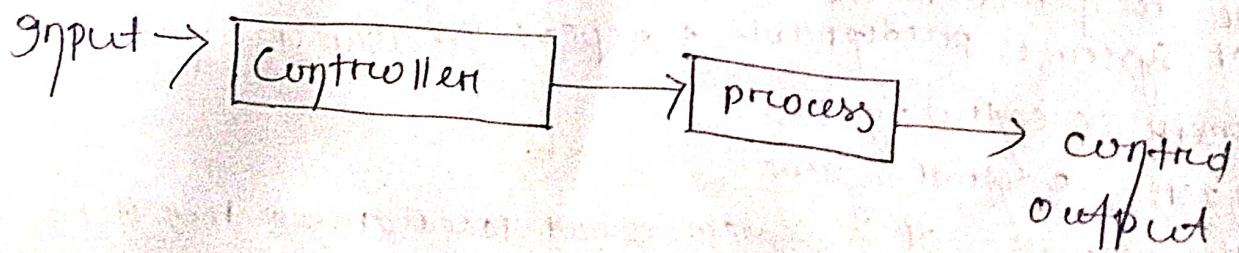
Traffic Light controller

→ Any traffic light control system used on the road is a type of open looped control system which is time depended. The traffic on the road can be either mobile or stationary & depends on the duration & sequence of the light glow, which is time depended & is controlled by the relay.

→ These are pre-determined & donot depend on the volume of the traffic on the road.

```
                                Power
                                  ↓
desired ─────────→ ┌─────────────────┐
traffic            │ relay to        │
                   │ control         │──────→ ┌───────┐
                   │ sequence        │        │ Light │
                   └─────────────────┘        └───────┘
                                                  ↓
                                               actual
                                               traffic
                                               flow
```

Advantages :-

→ It is simple & in construction & designed.

→ It is economic

→ It is easy for maintance pointer view.

→ This system are not much two buled with problem or stability.

→ The system are confident to use.

Dis - Advantage

→ This system are not accurate & reliceble

→ So this system in internal disturbance doccures.

→ Re-colibraction of the controller & are required from time-to-time for maintaining quality & accuracy.

Closed - Loop control System :-

If a controlling action of a system once some how dependened on output on changes in output then it is known as closed loop control system.

→ The main property of system is permiting the comprission of output with the input.

→ So that appropreiate controlling the action can be taken which is known as feedback of a system.

→ The parel of the output is fed to Input for comparcison.
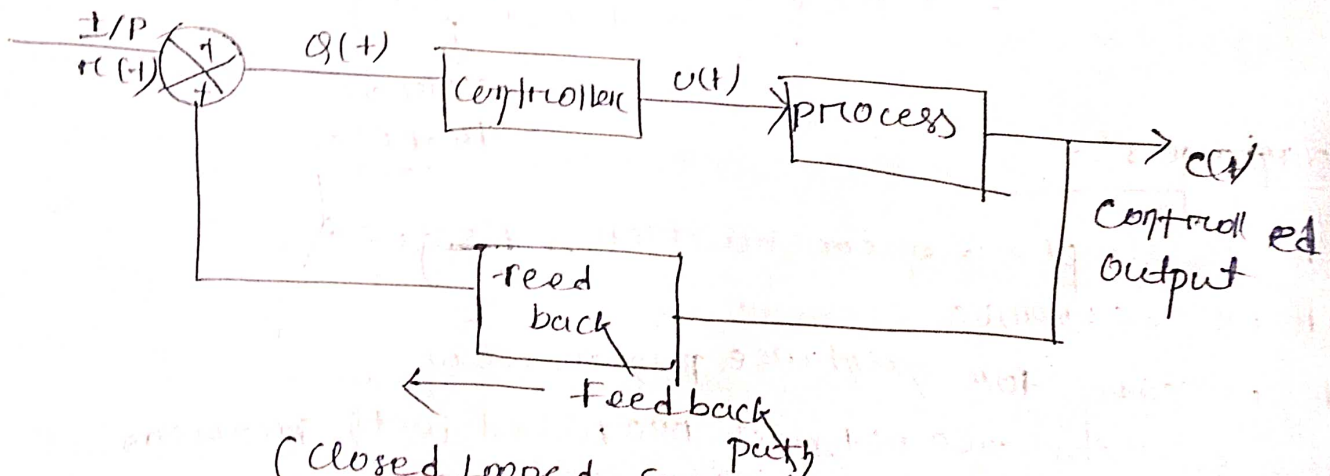
reference

(Closed Looped System)

$e(t) = r(t) + b(t)$ ——→ positive feedback
$e(t) = r(t) - b(t)$ ——→ Negative feed back

where,

$r(t)$ = reference I/P
$e(t)$ = error signal
$v(t)$ = actuating signal
$c(t)$ = control output
$b(t)$ = feedback signal.

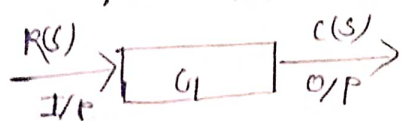EX→ Speed control of a DC shunt motor.
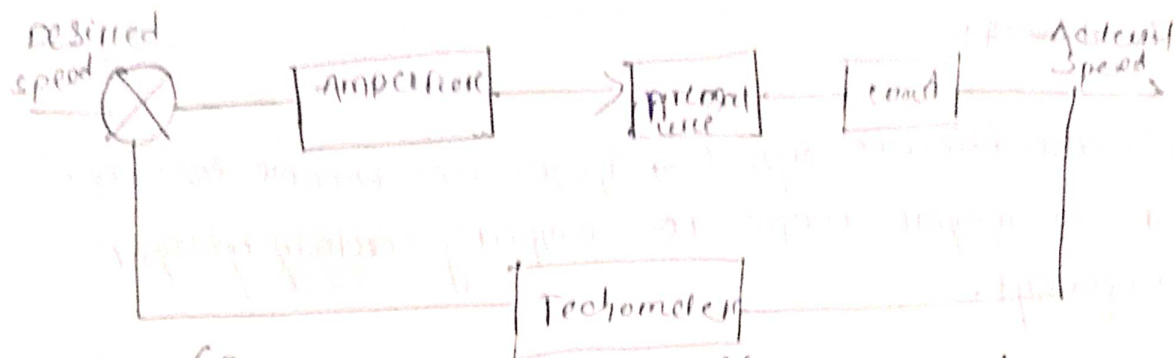
(Block diagram of Dc shunt motore)
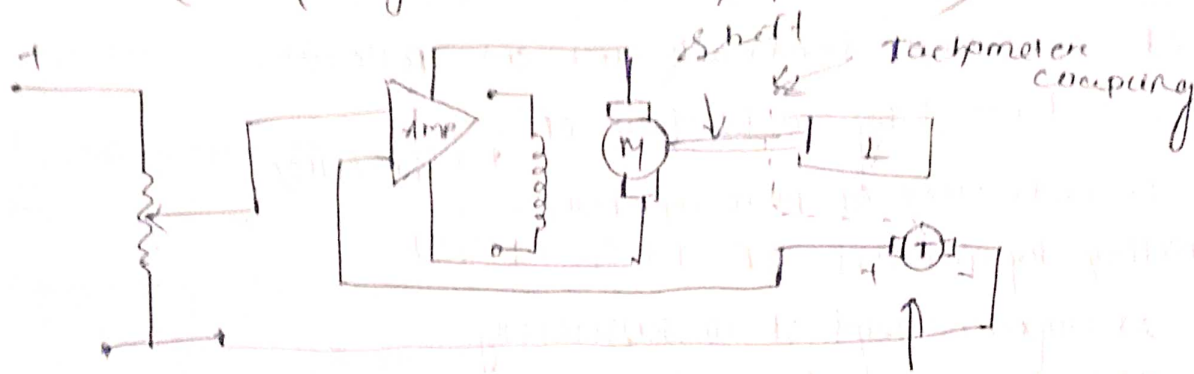
(Circuit diagream of Dc shunt motore)

→ from the above figure Dc shunt motore is used & field current is kept constant to control the speed of the Dc shunt motore.

effect feedback:-

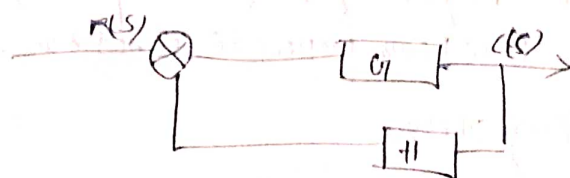The error between the system input and output can be reduced by using a feed back system.

Open Looped

the gain of the system is expressed by

$$G(s) = \frac{C(s)}{R(s)}$$

Closed Looped

The transfere function of the system is expression by

$$T(s) = \frac{C(s)}{R(s)}$$

$$= \frac{G(s)}{1 + G(s) H(s)}$$

Positive feedback:-

It enhance the errore signal & hence o/p become instable it is used in mayor loops to amplify certain external signal or component.

Negative feedback:-

The effect of (-ve) feedback are as follows
① Gain is reduced by a factor of $\dfrac{1}{1+G(s)/H(s)}$
② there is reduction of parameters.
  Variation by factor of $1+G(s)/H(s)$
→ There is improvement of in sensitivity
→ There may by reduction of stability
→ Accuracy in tracking steady state value.
→ Reduction of distrubance signal
→ The measure dis-advantages of reduction of gain & reduction of stability which can be over come by gain amplification & good design.

Standard test signal:-

Standard test signal can be physically religeble by using difficult function like step function, Ramp function, parabolic function, impulse function.
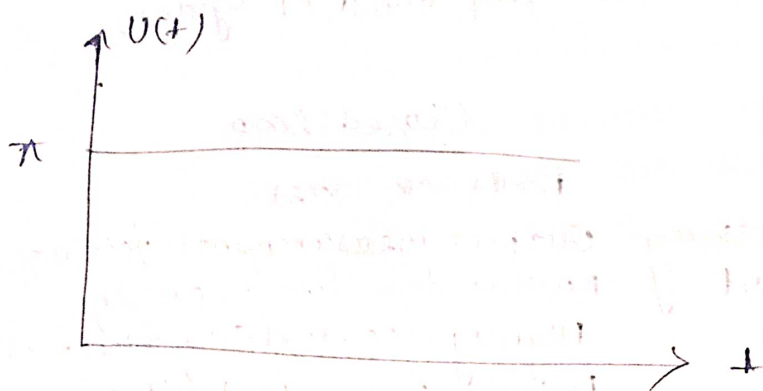
Step function

the value of the function is (0) for $t<0$ & its value is A for $t>0$
→ if $A=1$ then function $U(t)=1$ & it is known as unit step function
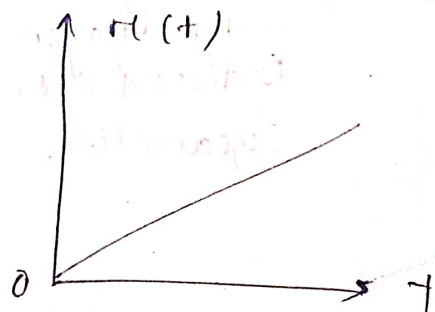$$U(t) = 0 \text{ for } t<0$$
$$U(t) = A \text{ for } t>0$$

(step-function)

Ramp-function

the value of ramp-function is '0' for $t < 0$ & after $t > 0$ it linearly increases with time.

(Ramp function)

$$r(t) = 0 \text{ for } t < 0$$
$$r(t) = At \text{ for } t > 0$$

Parabolic-function :-
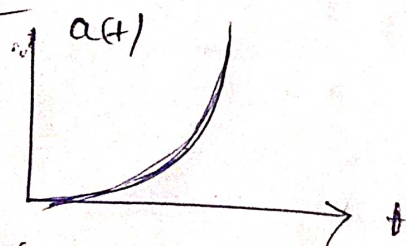
the value of parabolic-function is 0 for $t < 0$ & it is equal to $\frac{At^2}{2}$ for $t > 0$

$$a(t) = 0 \text{ for } t < 0$$
$$a(t) = \frac{At^2}{2} \text{ for } t > 0$$


(parabolic function)

Comparsion of open - loop and closed - loop control system.

Open loop	Closed loop
Feedback does not exist.	Feedback exists
Output measurement is not necessary	Output measurement is necessary
Erreor detector is not present	Erreor detector is present
Not accurate and reliable	Highly accurate and reliable
Highly sensitive to disturbance.	Less sensitive to disturbance
Very sensitive to environmental changes.	Less-sensitive to enveronmental changes.
Bandwidth is small	Bandwidth is large
Design is simple and cheap	Design is complicated and costly.
Usually stable in nature	During designing, stability is the major consideration
Highly affected by non-linearities	Reduced effect of non - linearities.

Servomechanism

A servomechanism is a power-amplifying feedback control system in which the controlled variable or the output is a mechanical position or it's time derivatives such as velocity or acceleration. A positional control system is a simple example of servomechanism. A load is considered here which requires a constant position in its application where the position is detected and converted to voltage using feedback potentiometer. This is compared with input potentiometer voltage to generate the error signal, which is amplified and fed into the controller now regulates the voltage applicable to the motor due to which it changes its position. Figure 1.21 shows a schematic diagram of servomechanism.
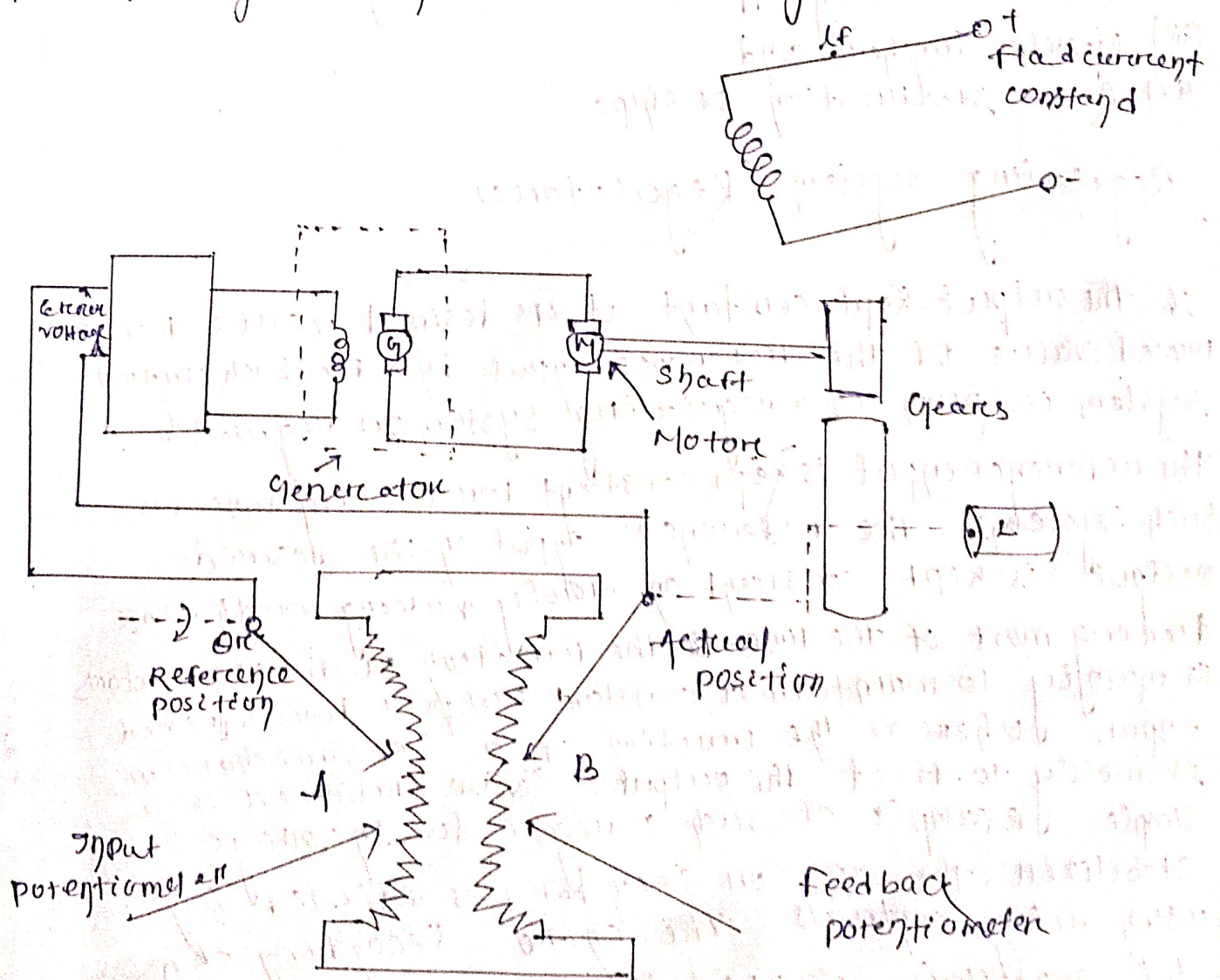
(Schematic diagram of a servomechanism)
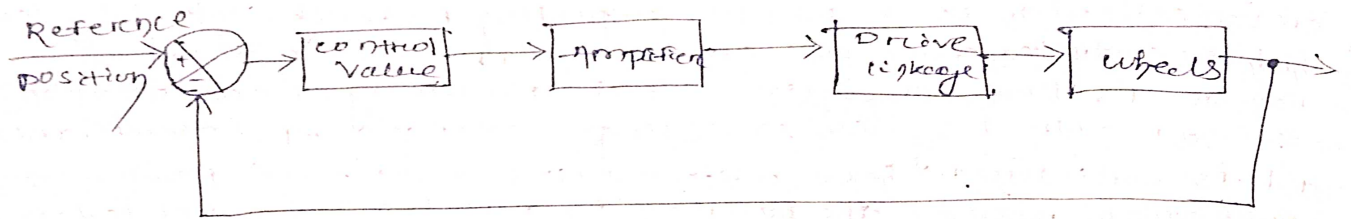
Servomechanism or an automatic steering system.

The following are the other examples of servomechanism:

(i) Machine tool position control
(ii) Power-steering apparatus for an automobile
(iii) Missile launchers and
(iv) Roll stabilization of ships

Regulating system (Regulators)

If the output kept constant at its desired value for a preset value of the reference input in a feedback control system is known as a regulating system or regulators. The reference input is kept constant for a long time in such systems. The reference input on the desired output is kept constant or slowly varying width time during most of the time. The function of the regulators is mainly to maintain a constant output for a fixed input, whereas the function of a servomechanism is mostly to effect the output. Servo stabilizer is a simple example of such a regulator or voltage stabilizer. The top or secondary is adjusted by using relay controls. The entire secondary can be smoothly tapped using a servomotor drive instead of a fixed tap. The shaft is driven by the servomotor and hence the

position of the tap on the secondary is controlled as per the control signal.
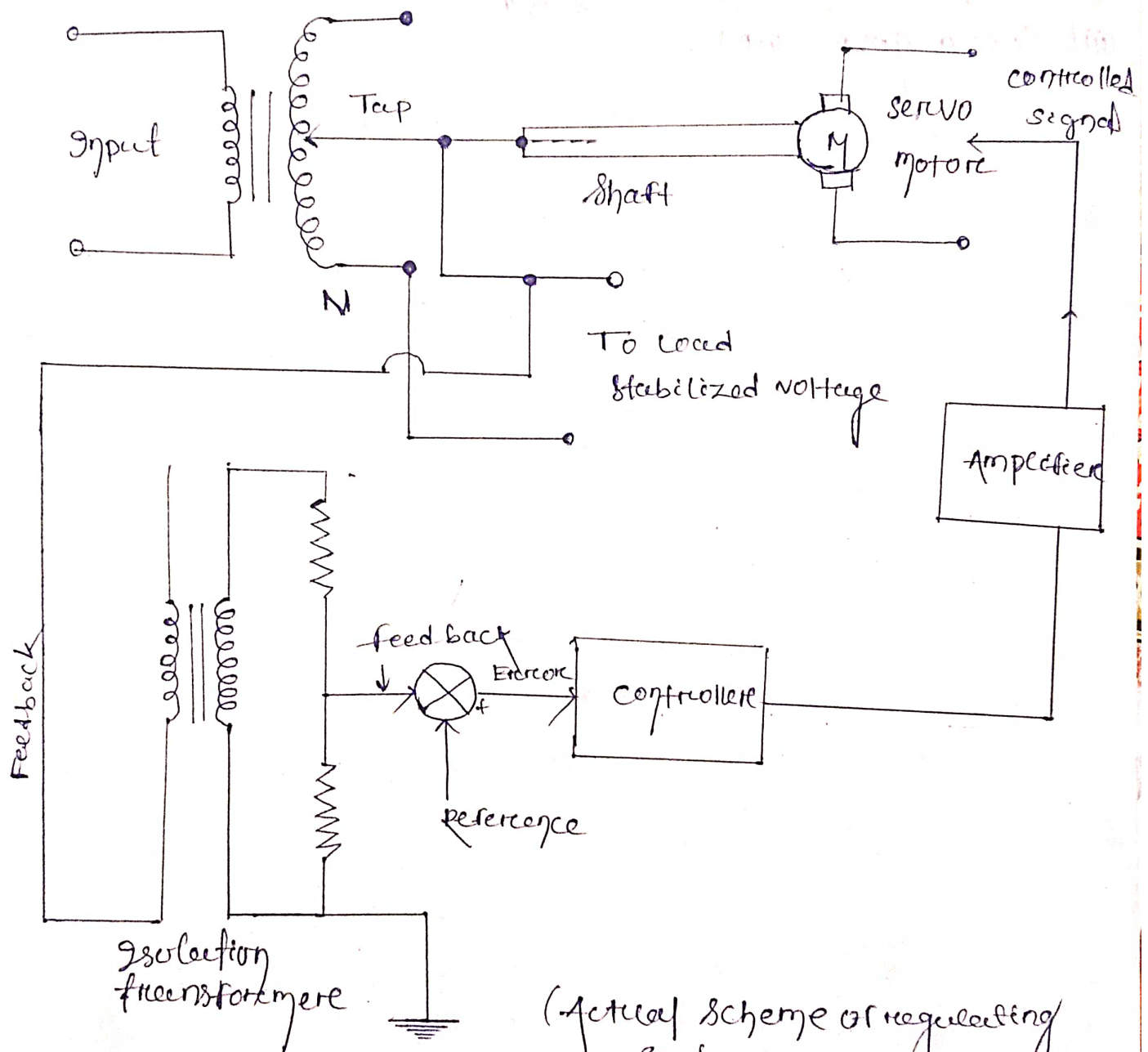
(Actual Scheme of regulating System)
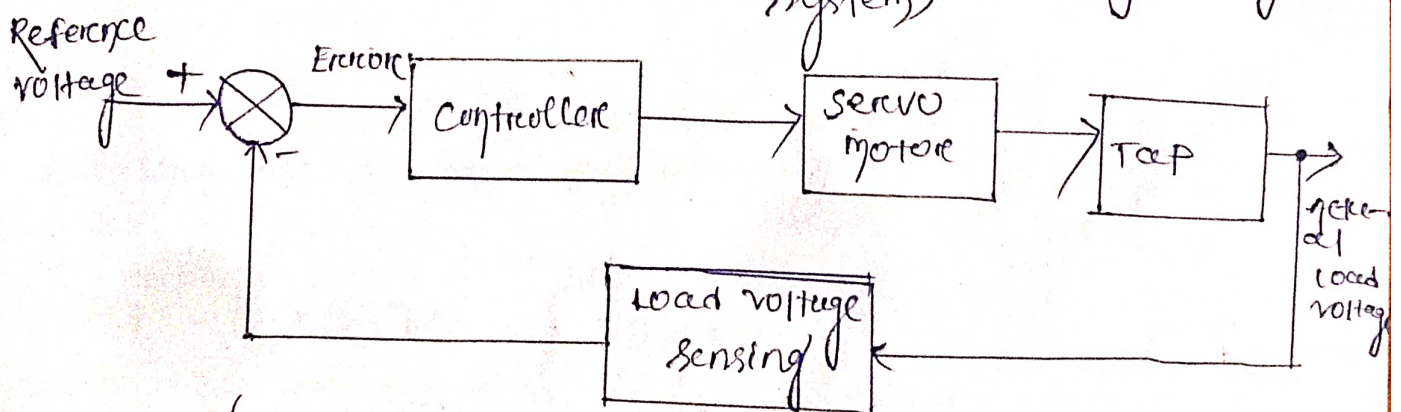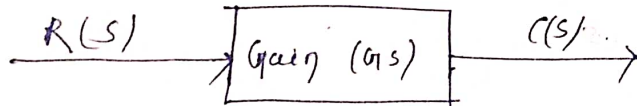

(Block diagram of regulating system)

The following are the other examples of a regulating system:

(i) temperature regulators,
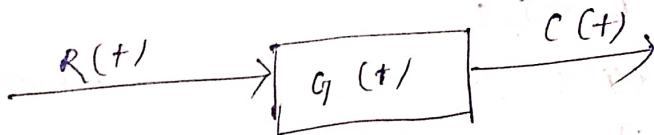(ii) frequency regulators, and
(iii) speed governors.

Transfere function

→ Transfere function is the ratio of the laplasy transfere of output to the laplash trcansfere of input.
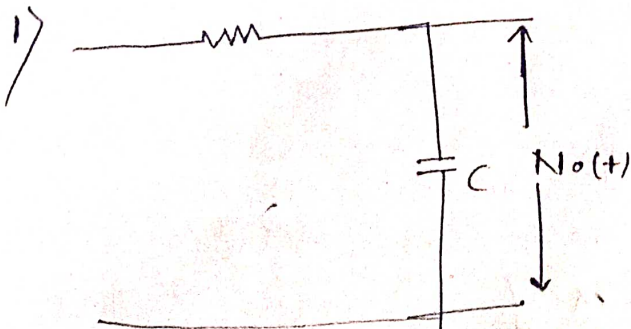
→ Assuming all the initial condition to be 0.

R(S) → [Gain (Gs)] → C(S)

systematic laplash domain

R(t) → [G (t)] → C(t)

systematic time domain

$$\boxed{(\text{Transfere function}) = \frac{\text{Laplase trcansfere of output}}{\text{Laplase trcansfere of input}} = \frac{CS}{RS} \text{ all initial condition are zero}}$$

1)

$$C \quad No(t)$$

Determin the transfere fuction of fig

$$Vi t = R e(t) + \frac{1}{C} \iint i(t) d(t)$$

If $i(t)$ is the current $v_i(t) = Ri(t) + L\frac{di}{dt}$

then by applying KVL

we found

$$v_i(t) = Ri(t) + \frac{1}{C}\int i(t)\,d(t)$$

$$v_0(t) = \frac{1}{C}\int i(t)\,d(t)$$

$$V_{Is} = R(s) + \frac{1}{sc} + I(s)$$

$$V_0(s) = \frac{1}{sc}I(s)$$

$$I_s = V_0(s).sc$$

$$V_i(s) = RI(s) + \frac{1}{C}\times\frac{1}{s}\times I.s.$$

$$= RI(s) + \frac{1}{cs}\times Is$$

$$= RI(s) + \frac{I(s)}{sc}$$

$$\therefore V_0(s) = \frac{Is}{sc}$$

$$V_{Is} = R(s) + \frac{1}{sc}.v_0(s).sc$$
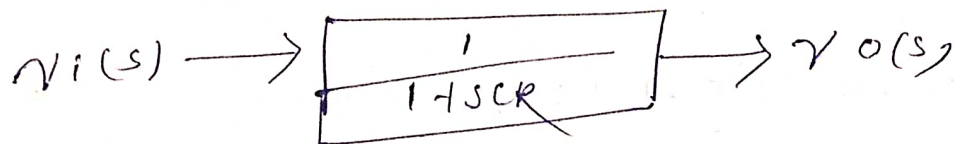
$$= R.v_0(s).sc + \frac{1}{C}.\frac{1}{s} + v_0(s).sc$$
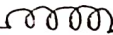
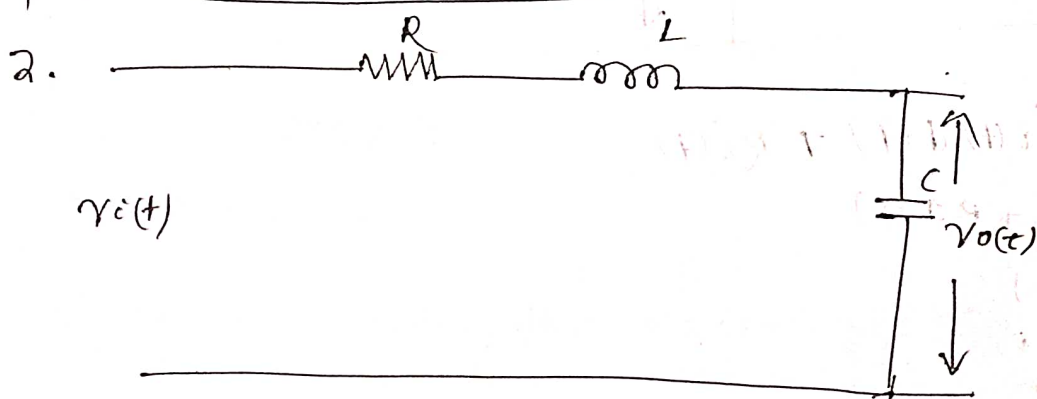$$= v_0(s).sc\left(R+\frac{1}{C}\times\frac{1}{s}\right)$$

$$\frac{v_0(s)}{v_i(s)}$$

$$= v_0(s)\,sc\times\left[\frac{RCs+1}{cs}\right]$$

$$v_i(s) = v_0(s)\times\left[1+sRC\right]$$

$$G(s) = \frac{v_0(s)}{v_i(s)} = \frac{1}{1+sRC}$$

$$V_i(s) \longrightarrow \boxed{\frac{1}{1+SCR}} \longrightarrow V_0(s)$$

	time domain	laplace
—wwv—	$\frac{R\,I(t)}{}$	$\frac{R\,I(s)}{}$
—||—	$\frac{1}{c}\int i(t)\,d(t)$	$\frac{I_s}{sc}$
—oooo—	$\frac{L\,i(t)}{dt}$ =	$SL\,I(s)$

2.

$$\underset{}{\overset{R \quad\quad L}{—wwv—\,—oooo—}}$$

$V_i(t)$

C

$V_0(t)$

$$V_i(t) = R\,i(t) + \frac{1}{c}\int i(t)\,d(t) + L\frac{d\,i(t)}{d(t)}$$

$$V_0(t) = \frac{1}{c}\int i(t)\,d(t)$$

Laplace

$$V_i(s) = R\,I(s) + SL\,I(s) + \frac{I(s)}{sc}$$

$$V_0(s) = \frac{I(s)}{sc}$$

$$I(s) = V_0 s \cdot sc$$

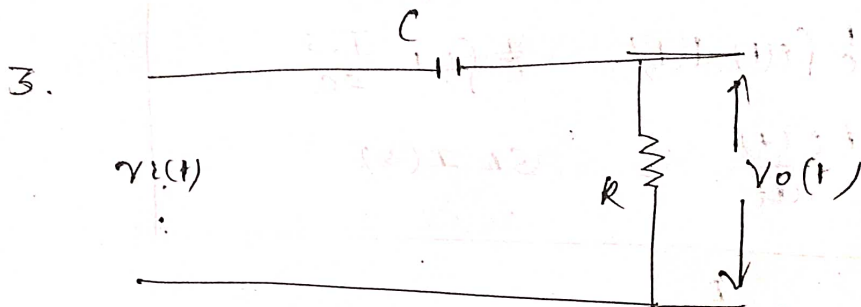$$V_i(s) = R \cdot V_0 \cdot sc + sL \cdot V_0 \cdot sc + \frac{V_0 \cdot sc}{sc}$$

$$V_i(s) = V_0(s)\,sc\left(R + sL + \frac{1}{sc}\right)$$

$$V_i(s) = V_0(s) \cdot sc\left(\frac{sc \cdot R + sc \cdot sL + sc}{sc}\right)$$

$$= V_0(s)\left(sc \cdot R + s^2 c \cdot L + sc\right)$$

$$VI(s) = Vo(s) \left(sc \cdot R + s^2 c \cdot L + sc \right)$$

$$\frac{1}{sc \cdot R + s^2 c \cdot L + sc} = \frac{Vo(s)}{VI(s)}$$

3.

$$Vl(t) = \frac{1}{c} \int i(t) \, d(t) + R i(t)$$

$$VI(s) = \frac{I(s)}{sc} + R I(s)$$

$$Vo(t) = R i(t)$$

$$Vo(s) = R I(s)$$

$$I(s) = \frac{Vo(s)}{R}$$
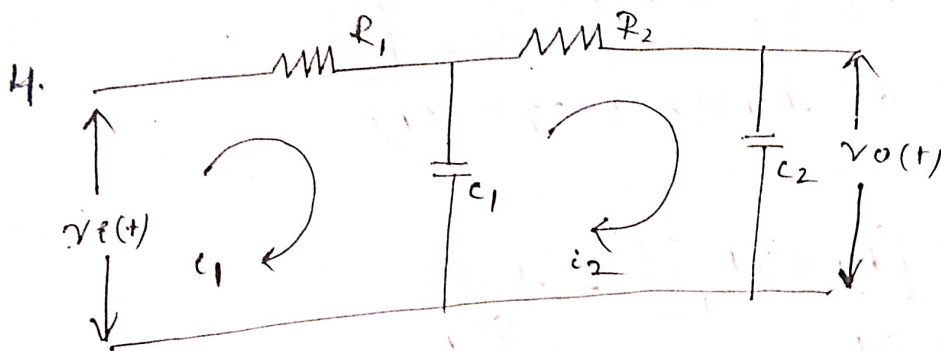
$$VI(s) = \frac{Vo(s)}{R}{sc} + R \cdot Vo(s)$$

$$= \frac{Vo(s)}{R} \times \frac{1}{sc} + Vo(s)$$

$$= \frac{Vo(s)}{R \cdot sc} + \frac{Vo(s)}{1}$$

$$= Vo(s) \left(\frac{1}{R \cdot sc} + 1 \right)$$

$$VI(s) = Vo(s) \left(\frac{1 + R \cdot sc}{R \cdot sc} \right)$$

$$\frac{R \cdot sc}{1 + R \cdot sc} = \frac{Vo(s)}{VI(s)}$$

4.

For loop - 1

$$v_i(t) = R_1 i_1(t) + \frac{1}{C_1} \int [i_1(t) - i_2(t)] \, dt$$

$$v_i(t) = R_1 i_1(t) + \frac{1}{C_1} \int i_1(t) - \frac{1}{C_1} i_2(t) \cdot dt$$

$$v_i(S) = R_1 I_1(S) + \frac{I_1(S)}{SC_1} - \frac{I_2(S)}{SC_1}$$

$$v_i(S) = I_1(S) \left[R_1 + \frac{1}{SC_1} \right] - \frac{I_2(S)}{SC_1} \qquad \text{———— (1)}$$

for loop 2 :-

$$\frac{1}{C_1} \left[\int (i_2(t) - i_1(t)) \, dt \right] + R_2 i_2(t) + \frac{1}{C_2} \int i_2(t) \cdot dt = 0$$

$$\Rightarrow \frac{1}{C_1} \int i_2(t) \cdot dt - \frac{1}{C_1} \int i_1(t) \, dt + R_2 i_2(t) + \frac{1}{C} \int i_2(t) \cdot dt = 0$$

$$\Rightarrow \frac{I_2(S)}{SC_1} - \frac{I_1(S)}{SC_1} + R_2 I_2(S) + \frac{I_2(S)}{SC_2} = 0$$

$$\Rightarrow I_2(S) \left[\frac{1}{SC_1} + R_2 + \frac{1}{SC_2} \right] - \frac{I_1(S)}{SC_1} = 0 \qquad \text{———— (II)}$$

for 0/P voltage

$$v_0(t) = \frac{1}{C_2} \int i_2(t) \, dt$$

$$v_0(S) = \frac{I_2(S)}{SC_2} \qquad \text{———— (3)}$$

from eqⁿ (2) we found the value of $I_1(S)$

$$\frac{I_1(S)}{SC_1} = I_2(S) \left[\frac{1}{SC_1} + R_2 + \frac{1}{SC_2} \right]$$

$$I_1(s) = SI_2(s)C_1\left[\frac{1}{sc_1} + R_2 + \frac{1}{sc_2}\right]$$

put the value of $I_1(s)$ in eqn (1)

we know that

$$vi(s) = I_1(s)\left[R_1 + \frac{1}{sc_1}\right] = \frac{I_2(s)}{sc_1}$$

$$vi(s) = SI_2(s)C_1\left(\frac{1}{sc_1} + R_2 + \frac{1}{sc_2}\right)\left(R_1 + \frac{1}{sc_1}\right) - \frac{I_2(s)}{sc_1}$$

$$= I_2(s)\left[(sc_1)\left(\frac{1}{sc_1} + R_2 + \frac{1}{sc_2}\right)\left(R_1 + \frac{1}{sc_1}\right) - \frac{1}{sc_1}\right]$$

$$I_2(s)\left[sc_1\left[\frac{1 + sc_1 R_2}{sc_1} + \frac{1}{sc_2}\right]\left[\frac{R_1 sc_1 + 1}{sc_1} - \frac{1}{sc_1}\right]\right]$$

$$I_2(s)\left[\frac{(c_2 + c_2 sc_1 R_2 + c_1)(R_1 sc_1 + 1)}{sc_1 c_2} - \frac{1}{sc_1}\right]$$

$$I_2(s)\left[\frac{R_1 sc_1 c_2 + R_1 R_2 s^2 c_1^2 c_2 + R_1 sc_1^2 + \frac{c_2 + c_2 sc_1 R_2 + c_1}{sc_1 c_2}}{\frac{1}{sc_1}}\right]$$

$$vi(s) = I_2(s)\left[\frac{(sc_1 R_1 + 1)(s R_2 c_1 c_2 + c_1 + c_2) - c_2}{sc_1 c_2}\right] - \textcircled{3}$$

$$vo(s) = \frac{I_2(s)}{sc_2}$$

$$T.F = \frac{vo(s)}{vI(s)}$$

$$T.f = \cfrac{\cfrac{I_2(s)}{sc_2}}{I_2(s)\left[\cfrac{(sc_1 R_1 + 1)(s R_2 c_1 c_2 + c_1 + c_2) - c_2}{sc_1 c_2}\right]}$$

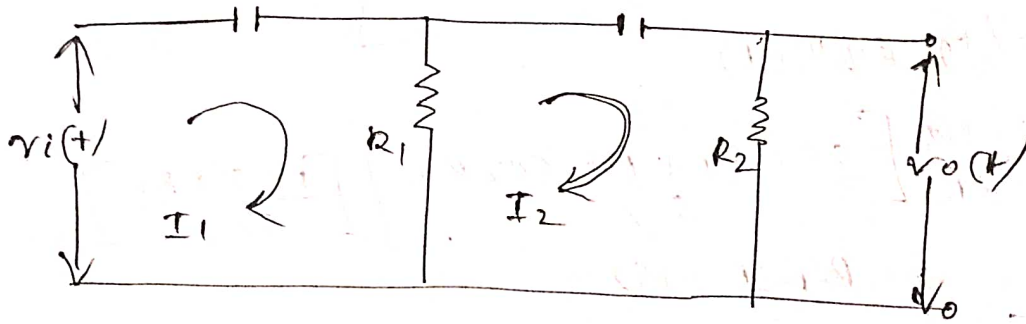$$= \cfrac{c_1}{(sc_1 R_1 + 1)(s R_2 c_1 c_2 + c_1 + c_2) - c_2}$$

$$= \frac{c_1}{(s^2 c_1^2 R_1 R_2 c_2 + sc_1^2 R_1 + sc_1 R_1 c_2 + s R_2 c_1 c_2 + c_1}{+ c_2 - c_2)}$$

$$\frac{C_1}{C_1\left(S^2 C_1 R_1 R_2 C_2 + S C_1 R_1 + S R_1 C_2 + S R_2 C_2 + 1\right)}$$

$$TF = \frac{1}{\left(S^2 C_1 R_1 R_2 C_2 + S C_1 R_1 + S R_1 C_2 + S R_2 C_2 + 1\right)}$$

5.

$\underline{\text{for loop 1}}$:-

$$r_i(t) = \frac{1}{C_1}\int i(t)\,dt + R_1\left[i_1(t) - i_2(t)\right]$$

$$r_i(s) = \frac{I_1(s)}{S C_1} + R_1\left[I_1(s) - I_2(s)\right]$$

$$r_i(s) = I_1(s)\left[\frac{1}{S C_1} + R_1\right] - R_1 I_2(s) \quad\quad\quad (1)$$

$\underline{\text{for loop 2}}$:-

$$R_1\left[I_2(t) - i_1(t)\right] + \frac{1}{C_2}\int i_2(t)\,dt + R_2 i_2(t)\,dt = 0$$

$$\Rightarrow R_1 I_2(s) - R_1 I_1(s) + \frac{I_2(s)}{S C_2} + R_2 I_2(s) = 0$$

$$\Rightarrow I_2(s)\left[R_1 + \frac{1}{S C_2} + R_2\right] - R_1 I_1(s) = 0$$

$$r_o(t) = R_2\, i_2(t)$$

$$r_o(s) = R_2 I_2(s) \quad\quad\quad (3)$$

from eqn (2) find $I_1(s)$

$$R_1 I_1(s) = I_2(s)\left[R_1 + \frac{1}{S C_2} + R_2\right]$$

$$I_2(S) = \frac{I_2(S)}{R_1}\left[R_1 + \frac{1}{SC_2} + R_2\right]$$

$$= \frac{I_2(S)}{R_1}\left[\frac{SC_2 R_1 + 1}{SC_2} + R_2\right]$$

$$I_1(S) = \frac{I_2(S)}{R_1}\left[\frac{SC_2 R_1 + 1 + SC_2 R_2}{SC_2}\right]$$

put $I_1(S)$ in eqn (1)

$$Vi(S) = \frac{I_2(S)}{R_1}\left[\frac{SC_2 R_1 + 1 + SC_2 R_2}{SC_2}\right]\left[\frac{1 + SC_1 R_1}{SC_1}\right]$$

$$+ R_1 I_2(S)$$

$$= I_2(S)\left[\frac{(SC_2 R_1 + 1 + SC_2 R_2)(1 + SC_1 R_1)}{R_1 S^2 C_1 C_2}\right] - R_1 I_2(S)$$

$$Vi(S) = I_2(S)\left[\frac{SC_2 R_1 + 1 + SC_2 R_2 + S^2 C_1 C_2 R_1^2 + SC_1 R_1 + S^2 C_1 C_2 R_1 R_2}{S^2 R_1 C_1 C_2}\right]$$

$$- R_1 I_2(S)$$

$$= I_2(S)\left[\frac{SC_2 R_1 + 1 + SC_2 R_2 + S^2 C_1 C_2 R_1^2 + SC_1 R_1 + S^2 C_1 C_2 R_1 R_2 - S^2 R_1^2 C_1 C_2}{S^2 R_1 C_1 C_2}\right]$$

$$Vc(S) = I_2(S)\left[\frac{SR_1 C_2 + 1 + SR_2 C_2 + SR_1 C_1 + S^2 R_1 R_2 C_1 C_2}{S^2 R_1 C_1 C_2}\right]$$

$$T.F = \frac{Vo(S)}{Vi(S)}$$

$$T.F = \frac{S^2 R_1 R_2 C_1 C_2}{SR_1 C_2 + 1 + SR_2 C_2 + SR_1 C_1 + S^2 R_1 R_2 C_1 C_2}$$

$C_1 = 2 \times 10^{-6}$

$C_2 = 2 \times 10^{-6}$

$R_1 = \frac{1}{2} \times 10^6$

$R_2 = 1 \times 10^6$

$$T.F = \frac{S^2 \times \frac{1}{2} \times 10^6 \times 1 \times 10^6 \times 2 \times 10^{-6} \times 2 \times 10^{-6}}{S \times \frac{1}{2} \times 10^6 \times 2 \times 10^{-6} + 1 + S \times 1 \times 10^6 \times 2 \times 10^{-6} + S \times \frac{1}{2} \times 10^6 \times 2 \times 10^{-6} + S^2 \times \frac{1}{2} \times 10^6 \times 1 \times 10^6 \times 2 \times 10^{-6} \times 2 \times 10^{-6}}$$

$$\Rightarrow \frac{S^2 \times \frac{1}{2} \times 1 \times 2 \times 2}{S \times \frac{1}{2} \times 2 + 1 + S \times 1 \times 2 + S \times \frac{1}{2} \times 2 + S^2 \times \frac{1}{2} \times 1 \times 2 \times 2}$$

$$= \frac{2S^2}{S + 1 + 2S + S + 2S^2}$$

$$= \frac{2S^2}{1 + 4S + 2S^2}$$