LECTURE NOTES ON

DIGITAL SIGNAL PROCESSING

6TH SEMESTER ETC

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GOVERNMENT POLYTECHNIC, DHENKANAL

Signal
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Introduction

Anything that carries some information can be called a signal. Common examples of signals are electrocardiogram (ECG) that provides information about the health of a person's heart & Electroencephalogram (EEG) signal that provides information about brain's activity.

→ A signal is also defined as any physical quantity that varies with time, space or any other independent variable.

Examples of signals

→ Speech signal
→ ECG signal
→ Electroencephalogram (EEG) signal.

Classification of signals

Continuous - time signals :-

The signals that are defined for every instant of time are known as continuous - time signals.

Discrete - time signal

The signals that are defined at discrete instant of time are known as discrete - time signals. The discrete - time signals are continuous in amplitude & discrete in time. They are denoted by $x(n)$.

Digital Signals

The signals that are discrete in time & quantized in amplitude are digital signals.

Deterministic & Random Signals

Deterministic Signal :-

A deterministic signal is a signal exhibiting no uncertainty of value at any given instant of time.

Random Signal :-

A random signal is a signal characterized by uncertainty before its actual occurrence.

Ex:- noise.

System :-

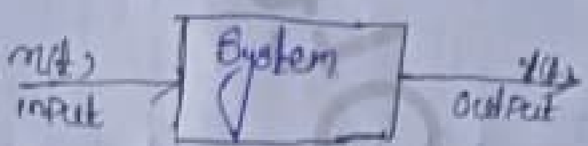
A system is defined as a physical device that generates a response or output signal, for a given input signal.

A system is an interconnection of components, a physical device that performs an operation on an input signal & produces another signal as output. It is a cause & effect relation between two or more signal. The concept of system is applicable to the fields of electrical, electronics, communication, channel chemical biological manufacturing etc.

Some common examples -

- In a communication system the inputs are signals to be transmitted & the outputs are signals that are received.

- In a manufacturing system the inputs are flow the rate of materials & the outputs are rate of production of a finished product.



- The relationship betn the input x(t) & corresponding output y(t) of a system has the form.

$$y(t) = \text{operation on } x(t)$$

mathematically.

$$y(t) = T[x(t)]$$

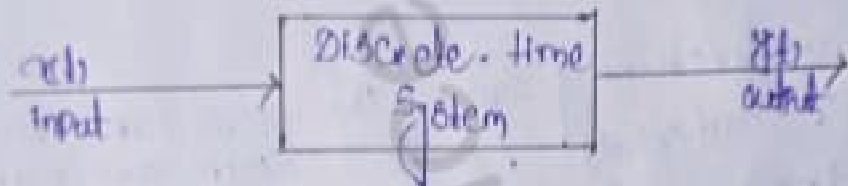
which represent that the input x(t) is transformed to y(t). In other words y(t) is the transformed form of x(t).

The systems can be classified as continuous time systems & discrete - time systems

Discrete time System:-

A discrete time system is one which operates on discrete-time signal & produces a discrete time output signal. If the input & output of discrete time system are $x(n)$ & $y(n)$, then we can write.

$$y(n) = T[x(n)]$$



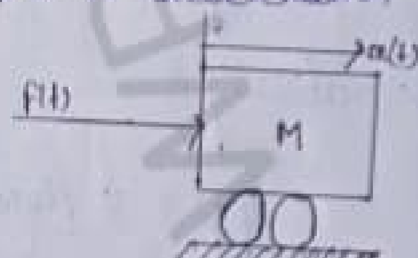
Mechanical Elements

Mass

The dynamics of a mass element are described by Newton's second law of motion.

$$F(t) = ma = m\frac{d^2x}{dt^2}$$

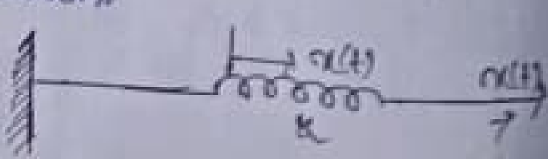
where M is mass in kg, & a is acceleration due to gravity.



The Spring:-

Over its linear region, the spring satisfies Hooke's law which relates the linear force to the displacement by the expression.

$$F(t) = K x(t)$$



Where k is the spring constant with units N/m.

Electrical Elements :-

The Resistor :-

It is an energy dissipative element. For a linear resistor, the relationship bet^n current & voltage is given by

$$v(t) = R\,i(t)$$

Where R = Resistance in ohm

$v(t)$ = The voltage across the resistor

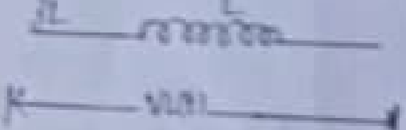
$i(t)$ = The current through the resistor.

The Inductor :-

An inductor stores energy in magnetic field. The relationship bet^n $v_L(t)$ & $i_L(t)$ for an inductor is.

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$$

$$v_L(t) = \frac{L\,di_L(t)}{dt}$$



The Capacitor :-

If $v_c(t)$ is the voltage across capacitance & $i_c(t)$ is the current through the capacitance then the relation bet^n $v_c(t)$ & $i_c(t)$ is given by

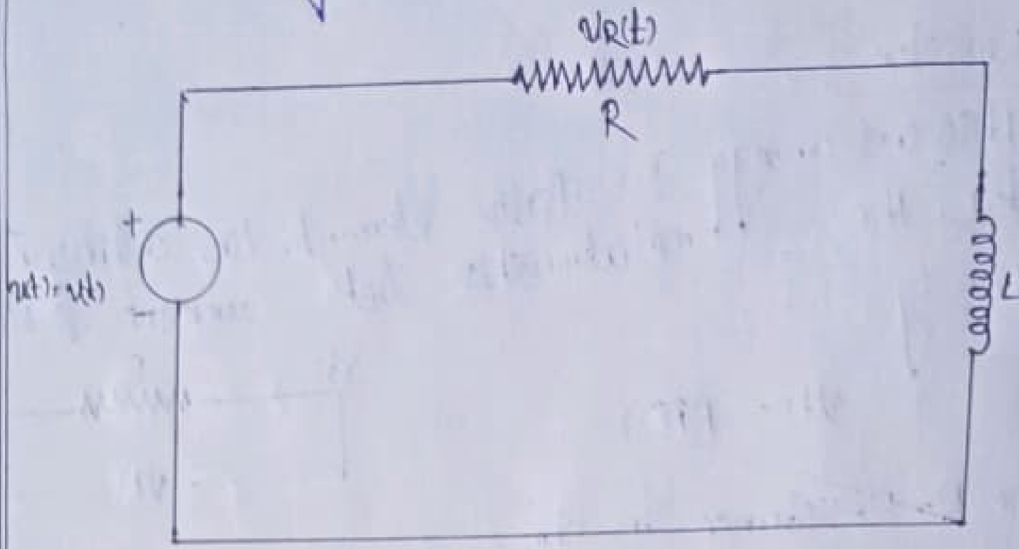
$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\,dt$$

$$i_c(t) = C\,\frac{dv_c(t)}{dt}$$

RL Circuit

Consider the RL circuit. The RL circuit can be viewed as a single input & single output system with output ...

equal to the applied voltage $v(t)$ & with output $y(t)$ equal to voltage across inductance.



By Kirchhoff's voltage law we have

$$v(t) = v_R(t) + v_L(t)$$
$$= Ri(t) + L\frac{di(t)}{dt}$$

$$\Rightarrow \frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{v(t)}{L} = \frac{x(t)}{L}$$

$$y(t) = L\frac{di(t)}{dt}$$

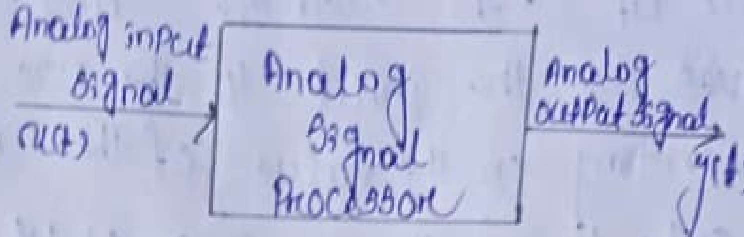
Signal Processing :-

A system is defined as a physical device that performs an operation on a signal. For example, a filter used to reduce the noise corrupting a desired information bearing signal is called a system.

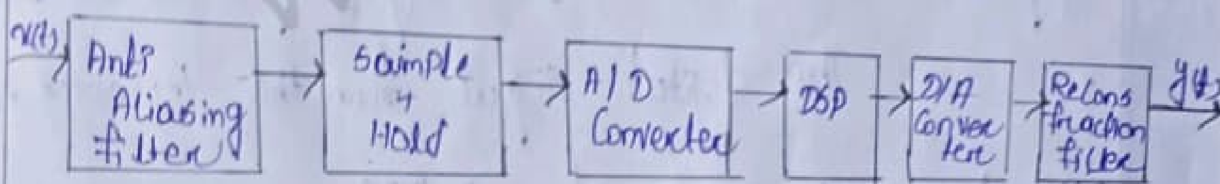
Signal Processing is any operation that changes the characteristics of a signal. These characteristics ~~can~~ include the amplitude, shape, phase & frequency

Content of the signal.

The system that processed the analog signal is known as analog signal processing system. The block diagram of an analog processing system.



Present block diagram of a typical digital signal processing system, where x(t) is the analog input signal & y(t) is the divided into six sections.

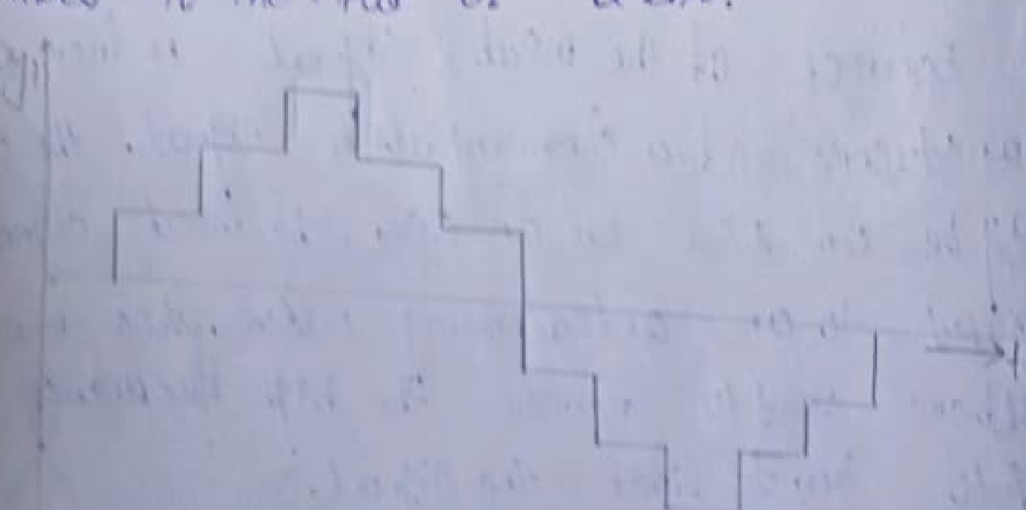


The source of the input signal is from a transducer of a communication signal, the signal may be an EEG or an ECG. The input signal is applied to an antialiasing filter. This is a low pass filter used to remove the high frequency noise & to band limit the signal.

The amplifier may be used to bring the signal upto the voltage range that is required by the i/p of the analog to digital conversion unit. The sample & hold device provides the input to the ADC & will be required if the input signal must remain relatively constant during the conversion of the analog signal to digital format. The output of the sample and hold circuit serves as the input to the ADC. number depending on the value of the analog signal

The output of the ADC is an N-bit binary number depending on the value of the analog signal at its input. The precoding amplifier provide a signal in this range. once converted to digital form the signal can be processed using digital techniques.

The digital signal from the processor is applied to the input of a DAC.



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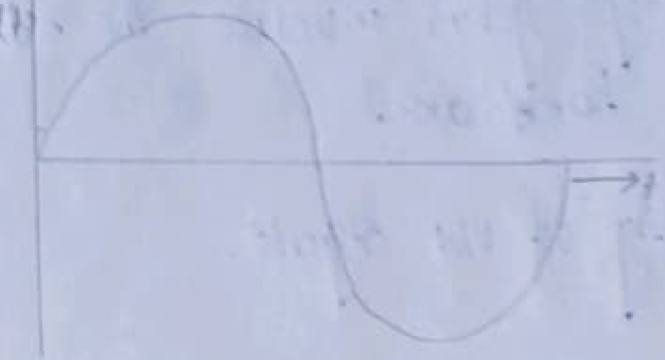

The output of DAC is continuous but not smooth, the signal contains high frequency components that are unwanted.

To eliminate high frequency components the output of DAC is applied to a reconstruction filter.

Advantages & Limitations of Digital Signal Processing :-

Advantages

1. Greater Accuracy
2. Cheaper
3. Ease of Data storage
4. Implementation of sophisticated algorithms when compared to its analog counter part.
5. Flexibility in configuration a DSP system can be easily reconfigured by changing the program. Reconfigure

of an analog system involves the redesign of the system hardware.

6) Applicationbility of VLF signals.

Limitations

~~1) System Complexity.~~

~~1) System Complexity~~
~~digital Processing~~

1) System Complexity
It increased in the digital Processing of an analog signal because of the devices such as A/D & D/A Converters & their associated filters

2) Bandwidth limited by sampling rate,

3) Power Consumption.

Application of DSP

1) Telecommunication

2) Consumer Electronics

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5) Instrumentation & Control

4) Image Processing

5) medicine

6) Speech Processing

7) Seismology

8) military

Elementary Continuous - Time signal Unit step function

The unit step function is ~~electrical~~ defined as,

$u(t) = 1$ for $t \geq 0$

$= 0$ for $t < 0$

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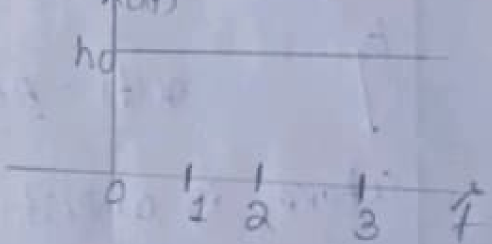
Here unit step means that the amplitude of $u(t)$ is equal to one



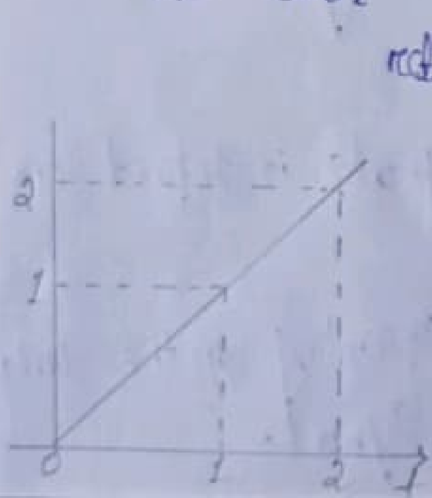
Unit Ramp function

The unit ramp function is defined as

$r(t) = t$ for $t \geq 0$

$= 0$ for $t < 0$

$r(t) = t\,u(t)$



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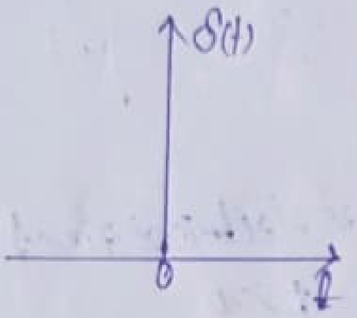
Impulse Function

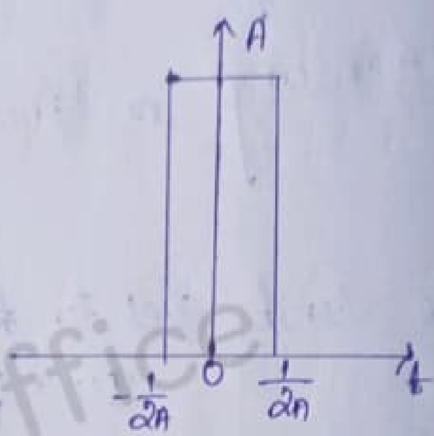
The unit impulse function δ(t) is defined as

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

&

$\delta(t) = 0$ for $t \neq 0$





Sinusoidal Signal

A continuous-time sinusoidal signal is given by

$$x(t) = A \sin(\Omega t + \theta)$$

where is amplitude, Ω is the frequency in radians per second & θ is the phase angle in radians.

Real Exponential Signals

A real exponential signals is defined as

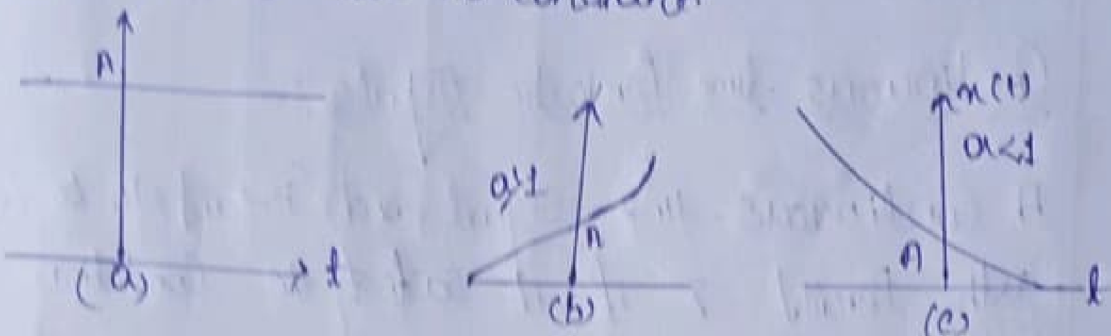
$$x(t) = Ae^{at}$$

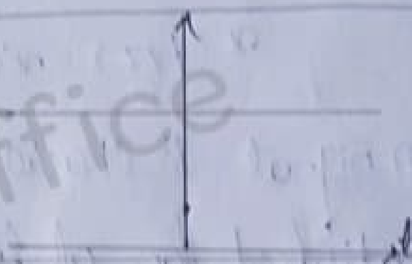
where both A & a real. Depending on the value of 'a' we get different signal.

* If a is positive the signal $x(t)$ is a growing exponential.

* If a is negative, the signal $x(t)$ is a decaying exponential.

For $a = 0$, $x(t)$ is constant.



$s = 0$	
$s = -\sigma$	
$s = \sigma$	
$s = \pm j\Omega$	
$s = \sigma \pm j\Omega,\ \sigma < 0$	

$s = \sigma \pm j\Omega ; \sigma > 0$	

Continuous-time Periodic Signals :-

A Continuous-time signal $x(t)$ is said to be periodic with Period T if it satisfy the Condition

$$x(t+T) = x(t). \text{ For all } t \quad -\infty < t < \infty \quad \cdots (1)$$

→ A signal is aperiodic if the above Condition is not satisfied for at last one value of T.

→ The smallest value of T that satisfy the above Condition is known as fundamental Period.

→ Complex exponential & sinusiodal signals are examples for Continuous-time Periodic signals. Consider a sinusoidal signal.

$$x(t) = A \sin(\Omega_0 t + \theta) \quad \cdots (2)$$

→ where A is the amplitude, Ω_0 is the frequency in radians per second (rad/sec), & θ the phase in radians. The frequency f_0 in hertz is give by

$$f_0 = \frac{\Omega_0}{2\pi} \quad \cdots (3)$$

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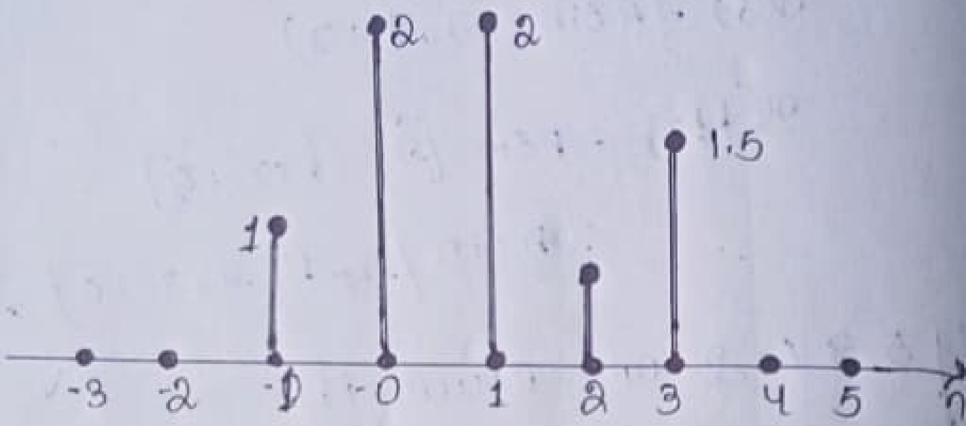
Graphically.



Functional Representation

The distance time signal can be represented using functional representation as below.

$$x(n) = \begin{cases} 1 & \text{for } n = -1 \\ 2 & \text{for } n = 0, 1 \\ 0.5 & \text{for } n = 2 \\ 1.3 & \text{for } n = 3 \\ 0 & \text{otherwise.} \end{cases}$$

Tabular Representation

The distance time signal can also be represented as

n	-1	0	1	2	3
$x(n)$	1	2	2	0.5	1.5

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Sequence Representation

A Finite duration sequence with time origin (n=0) indicated by the symbol ↑ is represented as

$$x(n) = \{1, \underset{\uparrow}{2}, 2, 0.5, 1.3\}$$

An infinite duration sequence can be represented as

$$x(n) = \{\cdots 0.2, \underset{\uparrow}{1}, -1, 3, 2 \cdots\}$$

A finite duration sequence that satisfies the Condition $x(n) = 0$ for $n<0$ can be represented

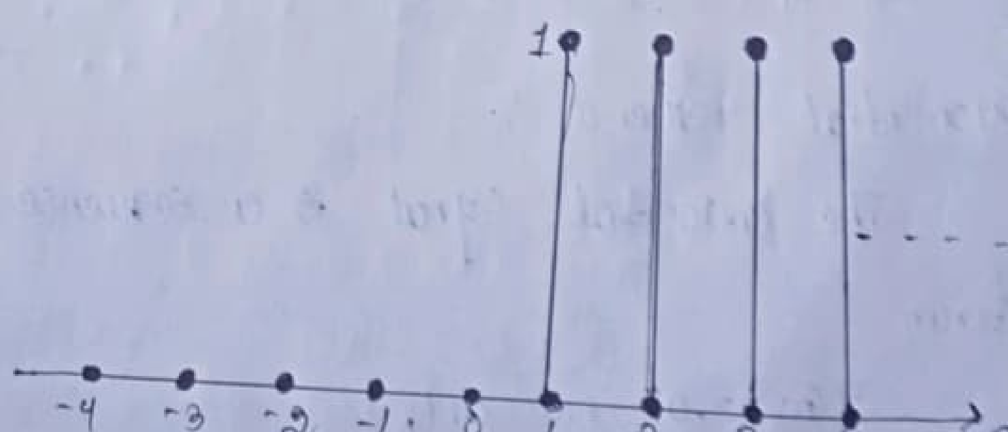
$$x(n) = \{2, 4, 6, 8, -3\}$$

Elementary Discrete - time signals

unit step sequence :-

The unit step sequence is defined as

$$x(n) = 1 \text{ for } n \geq 0$$
$$= 0 \text{ for } n < 0$$



Unit Ramp sequence :-

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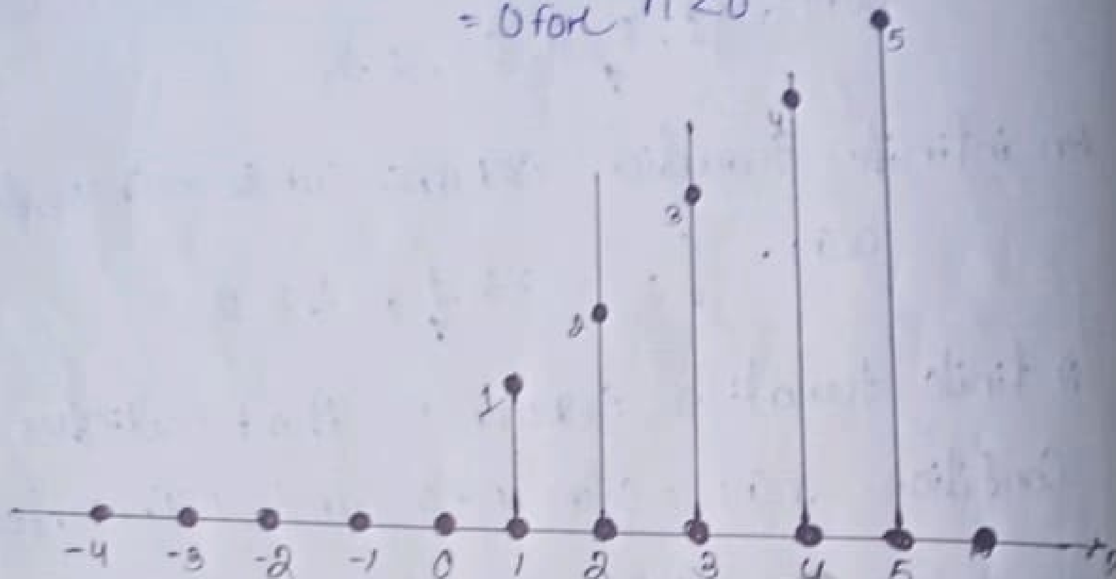
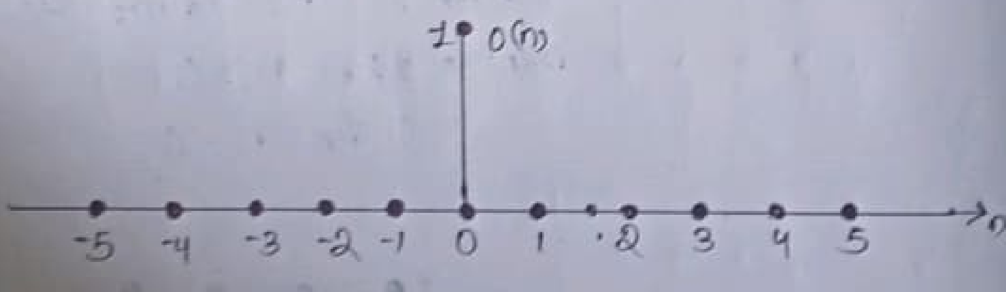


Unit -: sample sequence (unit impulse sequence

The unit sample sequence is defined as

$$\delta(n) = 1 \text{ for } n = 0$$
$$= 0 \text{ for } n \neq 0$$



Exponential sequence :-

The exponential signal is a sequence of the form

$$x(n) = a^n \text{ for all } n$$

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Sinusoidal Signal

The discrete time sinusoidal signal is given by $x(n) = A\cos(\omega_0 n + \phi)$.

where ω_0 is the frequency (in radians per sample & ϕ is the phase (in radians) using Euler's identity, we can write.

$$A\cos(\omega_0 n + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 n} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 n}$$

Complex Exponential signal

The discrete-time complex exponential signal is given by

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$= a^n \cos(\omega_0 n + \phi) + j a^n \sin(\omega_0 n + \phi)$$

Classification of Discrete-Time Signals

Energy signals & power signals.

For a discrete-time signal $x(n)$, the energy E is defined as.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{--- (1)}$$

The average power of a discrete-time signal $x(n)$ is defined as.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \quad \text{--- (2)}$$

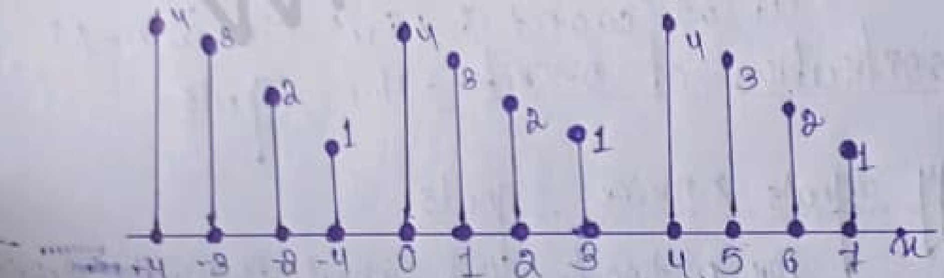
A signal is an energy signal, if & only if the total energy of the signal is finite. For an energy signal $P=0$ similarly the signal is said to be power signal. If the average power of the signal is finite. For a power signal $E=\infty$. The signals that does not satisfy above properties are neither energy nor power signals.

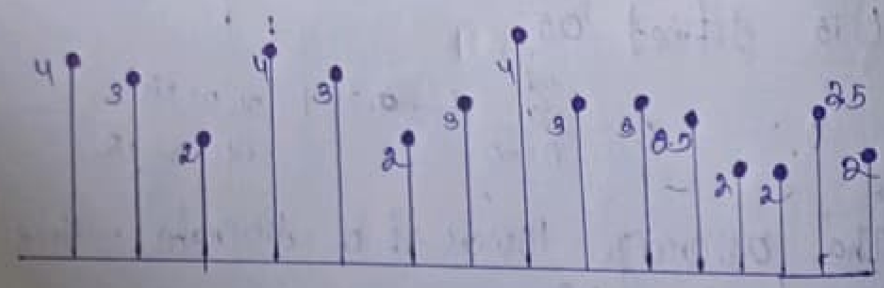
Periodic & aperiodic signals :-

A discrete-time signal $x(n)$ is said to be periodic with period with period N if & only if.

$$x(N+n) = x(n) \text{ for all } n. \quad -(3)$$

The smallest value of N for which Eq. 2 holds is known as fundamental period. If eq. 3 does not satisfy even for one value of n then the discrete-time signal is aperiodic.





$$x(n) = A\sin(\omega_0 n + \theta)$$

$$x(n+N) = x(n)$$

$$x(n+N) = A\sin(\omega_0(n+N) + \theta).$$

$$= A\sin(\omega_0 n + \omega_0 N + \theta)$$

Example :- $x(n) = \cos \omega n$

The signal is said to be an odd signal if it satisfies the condition.

Example :- $A \sin \omega n$

if $x(n)$ is odd then $x(0) = 0$.

Causal & Non Causal Signals

A signal $x(n)$ is said to be Causal if it's value is zero for $n<0$. otherwise the signal is non Causal.

Eg - Causal : $x_1(n) = a^n u(n)$

$x_2(n) = \{\underset{\uparrow}{1}, 2, -3, -1, 2\}$

Eg non-causal = $x_1(n) = a^n u(-n+1)$

$x_2(n) = (1, -2, \underset{\uparrow}{1}, 4, 3)$

A signal that is zero for all $n \geq 0$ is called an anti Causal signal.

Operation on signals :-

Signal Processing is a group of basic operations applied to an input signal resulting in another signal as the output.

→ The mathematical transformation from one signal to another is represented as $y(n) = T[x(n)]$

The basic set of operation are :-

1) Shifting
2) Time reversal
3) Time scaling

4) scalar multiplication
5) signal multiplier
6) signal addition

1) **Shifting :-**

The shift operation takes the input sequence & shift the values by an integer increment of the independent variable.

- The shifting may delays or advances the sequences in time.
- mathematically this can be represented as

$$y(n) = x(n-k)$$

where $x(n)$ is the input & $y(n)$ is the output.

2) **Time reversal :-**

- Time reversal of sequence $x(n)$ can be obtained by folding the sequence about $n=0$. It is denoted as $x(-n)$.

- The signal $x(-n+2)$ is $x(n)$ delayed by two units of time & $x(-n+2)$ is $x(-n)$ advanced by two units of time.

3) **Time scalling**

This is accomplished by replacing n by αn in the sequence $x(n)$.

let $x(n)$ is a sequence

If $\alpha = 2$ we get a new sequence

$$y(n) = x(2n).$$

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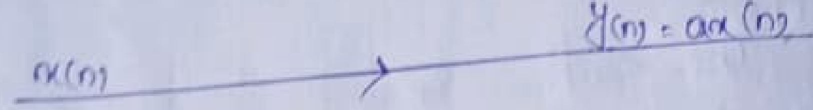
4. Scalar multiplication :-

A scalar multiplier, there the signal $x(n)$ is multiplied by a scale factor a.



For example if $x(n) = \{1, 2, 1, -1\}$ & $a = 2$

then the signal $ax(n) = \{2, 4, 2, -2\}$

5. Signal Multiplier :-

The multiplication of two signal sequences to form another sequence.

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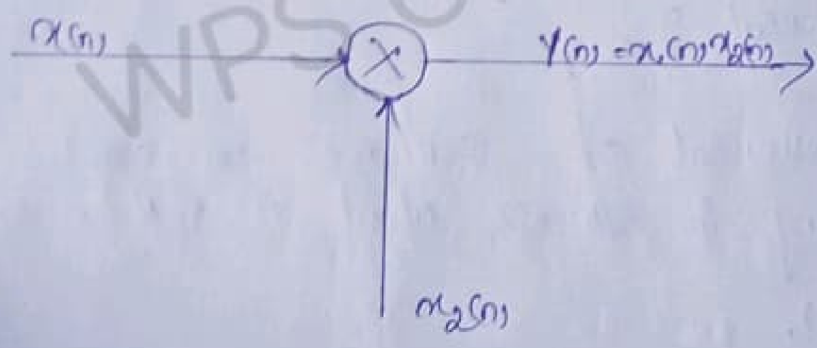
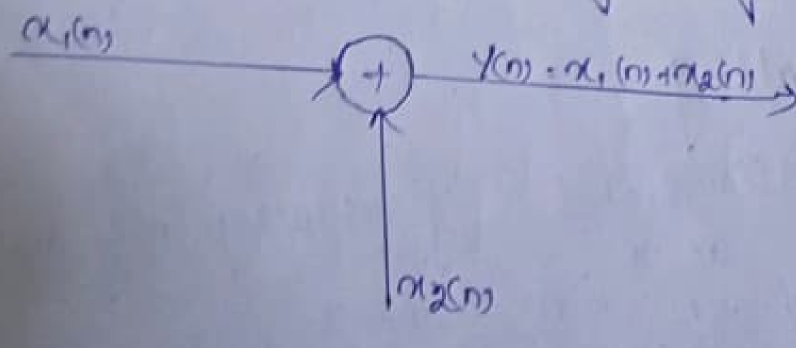


Addition Operation

Two signals can be added by using an adder.



For example of

$$x_1(n) = \{1, 2, 3, 4\} \;\&$$
$$x_2(n) = \{4, 3, 2, 1\}$$

Then

$$x_1(n) + x_2(n) = \{1+4, 2+3, 3+2, 4+1\}$$
$$= \{5, 5, 5, 5\}$$

Sampling & Aliasing

In Practice, sampling is Performed by applying a Continuous-time signal to an ADC whose output is a series of digital values.

- The Primary Concern in sampling is to select the sampling rate that Preserve the information Contained in Continuous time signal.
- If T is the time interval between successive samples, then the discrete time signal can be represented by the relation. $x(nT) = x(n) \quad -\infty < n < \infty$

The time interval between successive sample are Called as the sampling Period & its reciprocal $\frac{1}{T} = F_s$ is called sampling rate.

Classification of Discrete-Time Systems:-

Discrete time systems are classified according to their general Properties & characteristics. They are:-

1) Static & Dynamic System
2) Causal & Non-Causal Systems.
3) Linear & non-linear Systems.
4) Time-variant & time-invariant Systems.

5) FIR & IIR systems

6) Stable & unstable systems.

Static & Dynamic Systems :-

- A discrete - time system is called static or memory less if its output at any instant n depends on the input samples at the same time, but not on past or future sample of the input.

- In any other case, the system is said to be dynamic or to have memory.

The system described by the following equation

$$y(n) = a x(n)$$

$$y(n) = a x^2(n)$$

are static, on the other hand, the systems described by the following equations.

$$y(n) = x(n-1) + x(n-2)$$

$$y(n) = x(n+1) + x(n)$$

are dynamic systems

Causal & Non-Causal System :-

- A system is said to be causal if the output of the system at any time n depends only at present a past inputs, but does not depend on future inputs.

- This can be represented mathematically as

$$y(n) = F[x(n), x(n-1), x(n-2) \cdots]$$

If the output of a system depends on future inputs, the system is said to be non-causal or anticipatory.

Ex:- $y(n) = x(n) + x(n-1)$ Causal system

$y(n) = x(2n)$ non-causal system.

Linear & Non-Linear Systems:-

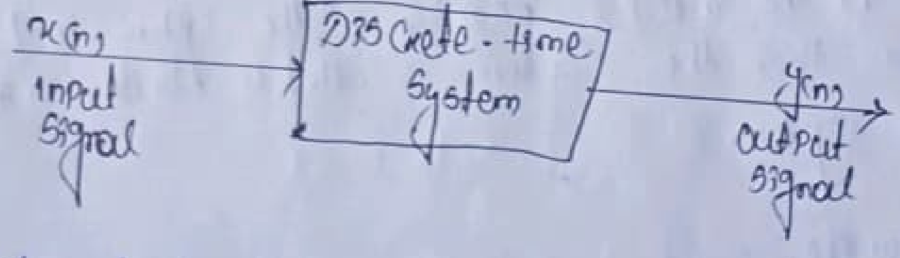
→ A systems that satisfies the super position principle is said to be a linear system.

→ Super position principle states that the response of the system to a weighted sum of signals be equal to a weighted sum of signals be equal to the corresponding weighted sum of the outputs of the system to each of the individual input signals.

A system is linear if & only if

$$T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

For any arbitrary constants a_1 & a_2



A relaxed system that does not satisfy the supper position principle is called non-linear.

Time variant & Time - Invariant System:-

A system is said to be time-variant if the characteristics of the system does not change with time.

- For a time-invariant system if $y(n)$ is the response of the system to the input $x(n)$ then the response of a system to the input $x(n-k)$ is $y(n-k)$.

- In other words if the input sequence is shifted by k samples, the generated output sequence is the original sequence shifted by k samples.

FIR & IIR Systems :-

Linear time invariant systems can be classified according to the type of impulse response, They are

1. FIR System
2. IIR System

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FIR System

If the impulse response of the system is of finite duration then the system is called a finite impulse response system.

Example :-

$$h(n) = \begin{cases} 1 & \text{for } n = -1, 2 \\ 2 & \text{for } n = 1 \\ 3 & \text{for } n = 0, 3 \\ 0 & \text{Otherwise} \end{cases}$$

2) IIR System

An infinite impulse response system has an impulse response for infinite duration.

Example :- $h(n) = a^n u(n)$

Stable & unstable Systems

→ An LTI system is stable if it produces a bounded output sequence $x(n)$, the output is unbounded (infinite) the system is classified as unstable.

Inter Connection of LTI System

(i) Parallel Connection of Systems

Consider two LTI systems with impulse responses $h_1(n)$ & $h_2(n)$ connected in parallel.

The output of system 1 is

$$y_1(n) = x(n) * h_1(n)$$

The output system 2 is

$$y_2(n) = x(n) * h_2(n)$$

The output $y(n) = y_1(n) + y_2(n)$

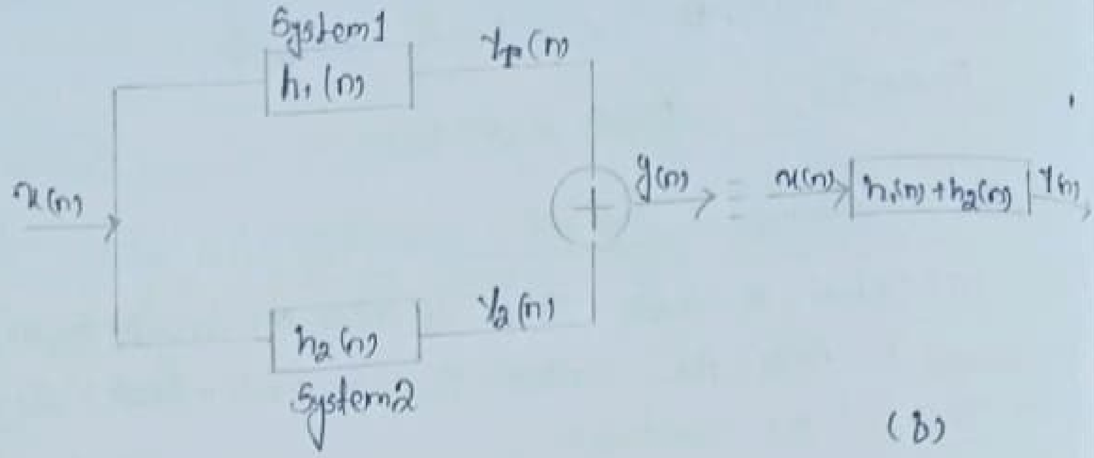
$= x(n) * h(n)$

where $h(n) = h_1(n) + h_2(n)$

→ Thus if the two-Systems has ~~an impulse response for~~ infinite duration.

Example :- $h(n)$

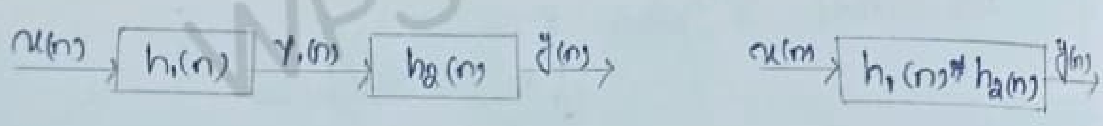
→ Thus if the two Systems are Connected in Parallel then the Over all impulse response is equal to sum of two impulse response.



(a)   (b)

ii) <u>Cascade Connection of Two Systems</u>

Consider two LTI Systems with impulse responses $h_1(n)$ & $h_2(n)$ Connected in Cascade let



Then output,

$$y(n) = y_1(k) * h_2(k)$$
$$= x(n) * h(n)$$

where $h(n) = \sum_{k=-\infty}^{\infty} h_1(k)\, h_2(n-k)$

$$= h_1(n) * h_2(n)$$

Hence the impulse response of two LTI Systems Connected in Cascade is the Convolution of the individual impulse responses.

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Correlation of two sequences

Correlation is measure of degree to which two signals is divided into

i) Cross - Correlation.
ii) Auto - Correlation.

i) Cross - Correlation

The Cross Correlation between a pair of signals $x(n)$ & $y(n)$ is given by

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) y(n-l) \quad l = 0, \pm 1, \pm 2, \pm 3 \ldots$$

→ The index is the shift (Lag) Parameter.

→ The order of subscripts xy indicates that $x(n)$ is the reference sequence that remains unshifted in time where as the sequency(n) is shifted units in time with respect to $x(n)$

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) y[-(l-n)]$$

$$= x(l) * y(-l)$$

Auto Correlation

The auto Correlation of a sequence is Correlation of a sequence with itself.

→ The auto Correlation of a sequence $x(n)$ is defined by

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n-l)$$

or equivalently

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n+l) x(n)$$

If the time shift $l = 0$, then we have

$$V_{xx}(0) = \sum_{n=-\infty}^{\infty} x^2(n)$$

Frequency Analysis of Discrete - time signals

→ Any Continuous - time Periodic signal $x(t)$ with Period T can be represented as a weighted sum of harmonically related sinusoids or complex exponentials.

→ If the signal is represented as weighted sum of harmonically related sine & cosine functions then the series of sine & cosine terms is known as trigonometric series which can be written as.

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\Omega_0 t) + b_n \sin(n\Omega_0 t)]$$

→ where the constants $a_0, a_1, a_2, \cdots, a_n, b_1, b_2, b_3 \cdots b_n$ are called as fourier coefficients & Ω_0 is the fundamental frequency equal to $\frac{2\pi}{T}$.

→ Similarly if the signal $x(t)$ is represented by weighted sum of harmonically related complex exponential, then the series is known as exponential fourier series which can be expressed as.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\Omega_0 t} \quad \Omega_0 = \frac{2\pi}{T}$$

→ Here C_n known as exponential fourier series Cofficients

→ In general the fourier series coefficient C_n are complex valued. That is if

$$C_n = |C_n| e^{j\theta n}$$

then

$$C_{-n} = |C_n| e^{-j\theta n}$$

The Power of a Periodic signal $x(t)$ is given by

$$P = \frac{1}{T}\int_T |x(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

which is called Parseval's relation for Power signal.

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad k = 0, 1, \dots N-1$$

$$x(n) = \sum_{k=0}^{N-1} C_k e^{j2\pi nk/N} \quad n = 0, 1, \dots N-1$$

Discrete Frequency Spectrum & frequency Range

Consider a Periodic sequence $x(n)$ with Period N.

> This sequence can be expressed in the discrete Fourier series as

$$x(n) = \sum_{k=0}^{N-1} C_k e^{j2\pi kn/N}$$

> The values of C_k, $k = 0, 1 \dots N-1$ are called the discrete or line spectra of $x(n)$.

Property	Periodic signal	Fourier series Cofficients
	$x(n)$ with Period N $y(n)$ with Period N $\omega_0 = \frac{2\pi}{N}$	C_k with Period N D_k with Period N
Linearity	$a_1 x(n) + a_2 y(n)$	$a_1 C_k + a_2 D_k$
Time Shifting	$x(n-n_0)$	$C_k e^{-j2\pi k n_0/N}$
frequency Shifting	$e^{j2\pi mn/N} x(n)$	C_{k-M}
Time Reversal	$x(-n)$	C_{-k}
Conjugation	$x^*(n)$	C^*_{-k}
Convolution	$\sum_{r=0}^{N-1} x(r) y(n-r)$	$N C_k D_k$
Multiplication	$x(n)\, y(n)$	$\sum_{l=0}^{N-1} C_l D_{k-l}$
Conjugate symmetry For real signals	$x(n)$ real	$C_k = C^*_{-k}$

		$Re\{C_k\} = Re\{C_{-k}\}$ $Im\{C_k\} = -Im\{C_{-k}\}$ $\lvert C_k \rvert = \lvert C_{-k} \rvert$ $\angle C_k = -\angle C_{-k}$
Real & even signals Real & odd signals	$x(n)$ real & even $x(n)$ real & odd $x_e(n)$ $x_o(n)$	C_k real & even C_k imaginary & odd $Re\{C_k\}$ $jIm\{C_k\}$

Parseval's relation for Periodic signals.

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x(n)|^2 = \sum_{k=\langle N\rangle} |C_k|^2$$

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