LECTURE NOTE

On

STRUCTURAL DESIGN-I

Faculty Name- Sr. Lect. Swetapadma Sahoo

Department Of Civil Engg.

21/4/21

Working Stress Method (WSM)

Introduction

Code - IS 456 : 2000

* Plane cement concrete (PCC) :- (used in floor)

It is the mixture of cement, fine aggregates (sand), coarse aggregates (gravel) and water in definite proportion.

* Reinforced cement concrete (RCC) :- (used in roof slab)

It is the mixture of cement, coarse aggregate, water and reinforcement.

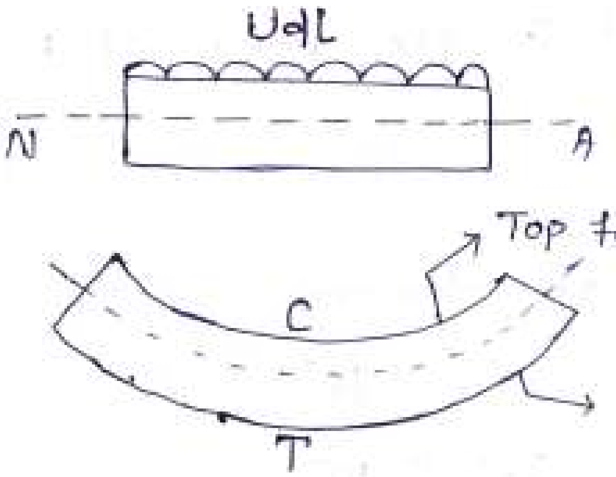
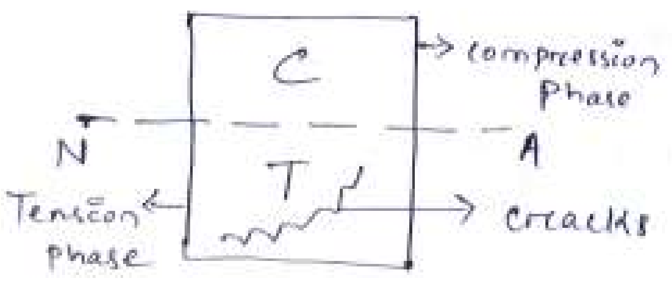


Fig. 1

* By testing the strength of concrete we found that :-

  - Concrete is good in compression. But weak in tension.

If we take a cross section of fig. 1



In a structure, cracks are always form in the lower part of the neutral axis.

→ This is ~~because~~ because concrete is weak in tension.

* By testing the strength of steel we found that:-
  - Steel is strong in tension as well as compression.

Q: Why steel is provided in RCC?

Ans:- As steel is strong in tension but concrete is weak in tension, steel is provided to prevent the weakness of tensile strength of concrete.

It is provided in the phase of tensile.

Q: Why we preffered steel in RCC?

Ans:-

Coefficient of thermal expansion

Concrete = 1.2×10^{-5}

Steel = 1.1×10^{-5}

Alluminium = 2.4×10^{-5}

Thermal expansion of concrete

Concrete expand in high temperature. (summer)

But it contract in cold temperature (winter).

→ By expansion its volume increases.

→ By contraction its volume decreases.

But in case of aluminium,

Aluminium expands more as compared to concrete.

But in steel and concrete,
Both of them expands, in same proportion,
that's why we preffered steel for RCC work.

The composite action of steel and concrete depends upon following factors :-

1. The bond between steel and concrete.
2. Prevention of corrosion of steel bars embedded in the concrete.
3. Practically equal thermal expansion of both concrete and steel.

- Universal testing machine - UTM (digital)

  This machine is used to test the strength of concrete, steel.

  This include tensile strength, compressive strength, shear test, bending, bond test.

23/4/21

Grade of Concrete

Group	Grade	Characteristic compressive strength (N/mm² or mpa)	Mix ratio C:FA:CA	
	M_5	5 N/mm²	1:5:10	Nominal mix concrete
	$M_{7.5}$	7.5 N/mm²	1:4:8	Nominal mix concrete
Ordinary concrete	M_{10}	10 N/mm²	1:3:6	Nominal mix concrete
Ordinary concrete	M_{15}	15 N/mm²	1:2:4	Nominal mix concrete
Ordinary concrete	M_{20}	20 N/mm²	1:1.5:3	Nominal mix concrete
Standard concrete	M_{25}	25 N/mm²	1:1:2	Nominal mix concrete
Standard concrete	M_{30}	30 N/mm²		design mix concrete
Standard concrete	M_{35}	35 N/mm²		design mix concrete
Standard concrete	M_{40}	40 N/mm²		design mix concrete
Standard concrete	M_{45}	45 N/mm²		design mix concrete
Standard concrete	M_{50}	50 N/mm²		design mix concrete
Standard concrete	M_{55}	55 N/mm²		design mix concrete
High Strength concrete	M_{60}	60 N/mm²		design mix concrete
High Strength concrete	M_{65}	65 N/mm²		design mix concrete
High Strength concrete	M_{70}	70 N/mm²		design mix concrete
High Strength concrete	M_{75}	75 N/mm²		design mix concrete
High Strength concrete	M_{80}	80 N/mm²		design mix concrete

After M_{80} grade, the next group is known as high performance concrete or special grade concrete.

M_{10} grade is the minimum grade use for PCC.

M_{20} grade is the minimum grade use for RCC.

* Nominal mix :- Mix in a specific ratio

Design mix concrete :- Mixing (with respect to code). So, that structure will be economical.

M_{20} :- M refers to → Mix (C : FA : CA)

20 refers to → characteristics compressive strength (f_{ck})

Characteristics compressive strength :- (f_{ck})

It is the strength below which not more than 5% of the test results are expected to fall.

For example,

If we take 100 cubes of concrete for testing, then if more than 95 cubes have strength of ≥ 20 MPa (assume) or less than 5% of cube have strength of less than 20 Mpa, then this strength is known as f_{ck}

Example

100 cube

No. of cube		Strength (fck)
1	→	22 mpa
2	→	23.5 mpa
3	→	23.5 mpa
4	→	24.5 mpa
5	→	26 mpa
6	→	26.5 mpa
7	→	27.5 mpa
8	→	28 mpa
9	→	28.5 mpa
10	→	28.5 mpa
⋮		
93	→	30 mpa
94	→	30.5 mpa
95	→	30.5 mpa
96	→	30.5 mpa
97	→	34.5 mpa
98	→	34.5 mpa
99	→	34.5 mpa
100	→	35 mpa

Process of calculating fck

100 cube → (20 → Not proper quality control)
(30 → moderate condition)

↓

After 28 days curing

↓

Testing in UTM

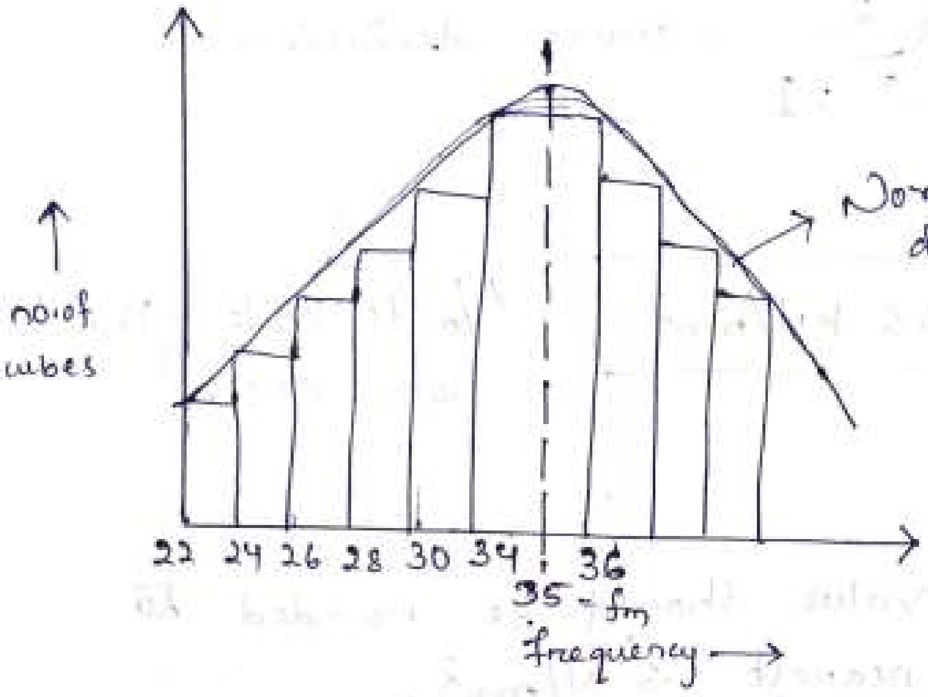
↓

↓

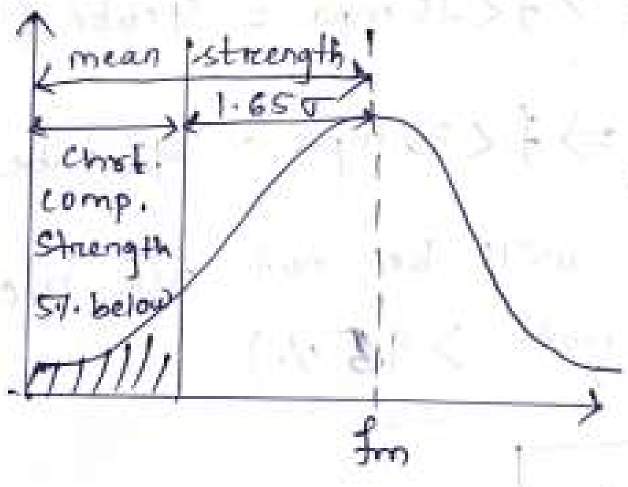
Result (in ascending order)

↓

Draw frequency distribution curve.



35 = f_m (mean Strength)



$f_{ck} = f_m - 1.65\sigma$

$f_m = \frac{\Sigma F}{n}$

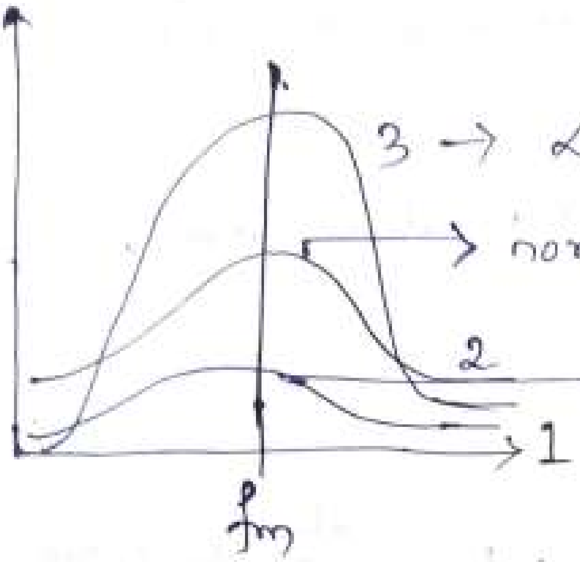
$= \frac{\text{Total strength}}{\text{no. of cubes}}$

σ = standard deviation

$\sigma = \sqrt{\frac{\Sigma (f_s - f_m)^2}{n-1}}$

f_s = Strength of each cube.

frequency → how many cubes are there in same strength.



5% ⇒ f_{ck} = 26 KN/mm² → A/c to code this is not ccs...

N.B ⇒

f_{ck} ⇒ value should be rounded to nearest 5 N/mm².

26 → M_{25} ⇒ f < 25 mpa = 4 cube (4%)

26 → M_{30} ⇒ f < 30 mpa = 92 cube (92%)

{This will be not f_{ck} b/c it is not > 95%.)

So, f_{ck} = 25 mpa

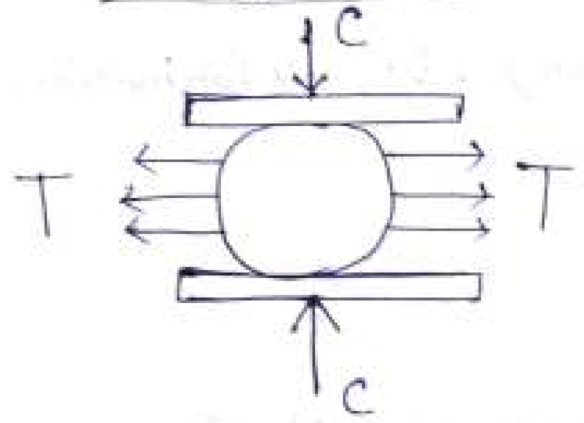
26/4/21

Tensile strength of concrete

→ As concrete weak in tension we cannot perform direct tensile test (X).

SO, we perform indirect tensile test { $f_{cr} = 0.7\sqrt{f_{ck}}$ N/mm^2

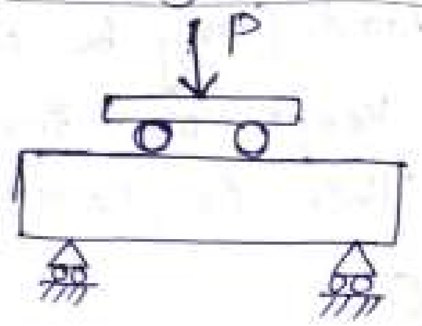
Split tensile strength test



P= failure load

Split strength = $\frac{2P}{\pi L D}$

Flexural tensile strength test.



P= failure load

Flextural strength = $\frac{PL}{6d^2}$

Elasticity :-

It is the tendency of a body, when a body deformation occurs in a body after relaxing it changes to its original shape & size.

$$E_c = 5000\sqrt{f_{ck}}$$

Shrinkage :-

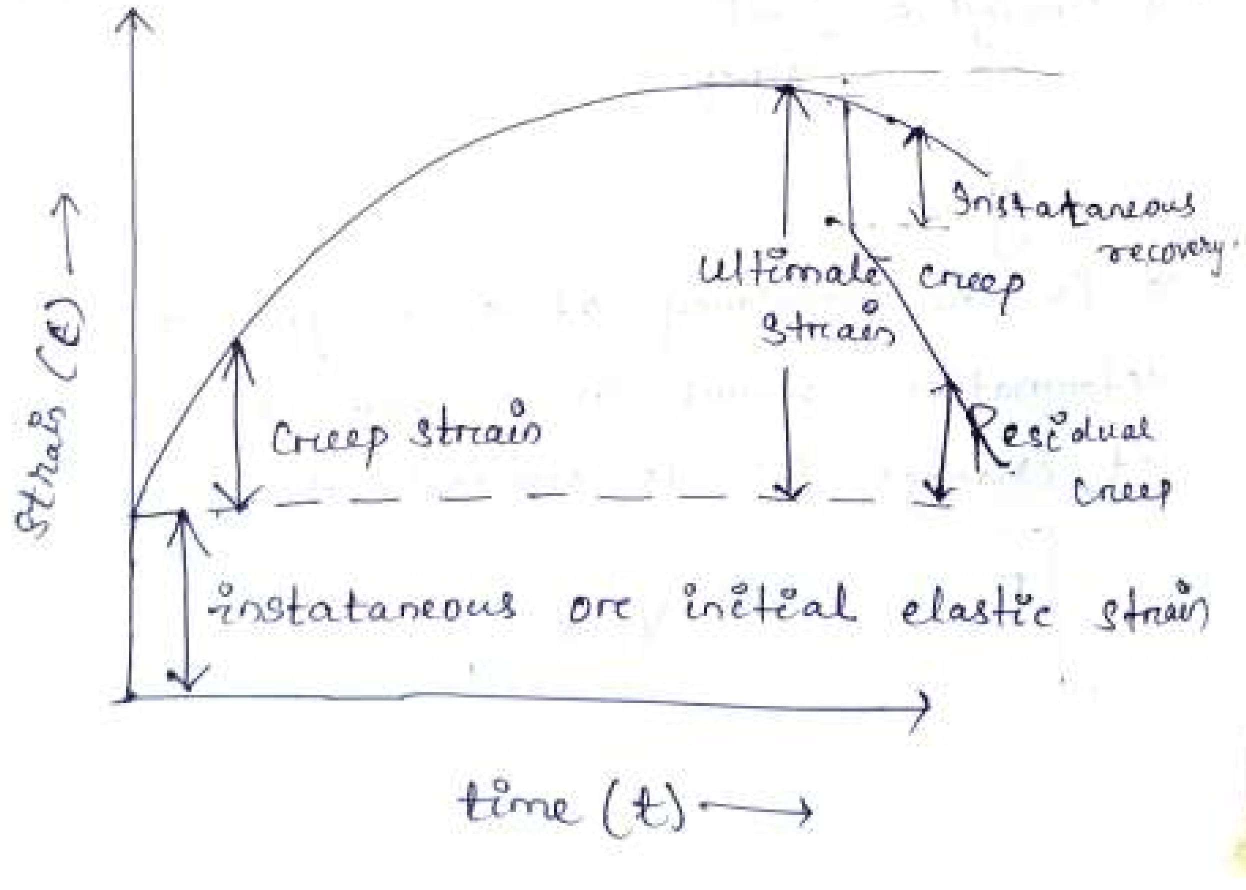
- Reduction of volume create shrinkage strain.
- The approximate value of total shrinkage strain for design may be taken as 3×10^{-4}.

Creep :-

When concrete subjected to sustained or constant load (may be compression or tension), its deformation keeps on increasing.
This is called creep.

OR

It is the time dependant part of strain resulting due to stress.



Creep coefficient (θ) = $\frac{\text{ultimate creep strain}}{\text{elastic strain}}$

Age at loading

Shuttering → formwork which supports the vertical surface is known as shuttering.

Centering → Formwork which supports the horizontal surface such as beam, slab bottoms is known as centering.

After removing formwork, the loading is known as age at loading. (self weight)

Strength $\propto \frac{1}{\theta}$

(Age at loading) Time $\propto \frac{1}{\theta}$

Age at loading	creep coefficient
7 days →	2.2
28 days →	1.6
1 year →	1.1

23/4/21 Reinforcing Material :- Purpose :-

1. To take up all the tensile stresses develop in the ~~structure~~ structure.
2. To increase the strength of concrete sect
3. To prevent the propagation of cracks due to temperature and shrinkage stres

4. To make the sections thinner as compare to plane concrete section.

Explanation :-

If external load of structure > its self weight than extra reinforcement is provided.

In the other way, in case of providing reinforcement its (structure) weight can be increased but by increasing weight its create cracks because of its self weigh so, we preffered reinforcement.

Types of steel reinforcement (clause 5.6) Pg-15

(1) Mild steel or plane bar :-

↦ It has low strength.
↦ It is ductile in nature.
↦ Grade of Mild steel ⟶ Fe 250.

(2) HYSD Bar :- (High Yield Strength deformation)

↦ In steel, grip is provided to increase the strength of the concrete.
↦ It has high strength.
↦ It is brittle in nature.
↦ Grade ⟶ Fe 415, Fe 500, Fe 550.

(3) Hand drawn steel wire fabric

(4) Structural steel.

For WSM

Permissible stress in concrete

Annex - B (Table - 21) (Pg - 81)

Grade	f_{ck}	Permissible stress: Bending (σ_{cbc})	Permissible stress: Direct (σ_{cc})
M_{10}	10	$\frac{10}{3} = 3$	$\frac{10}{4} = 2.5$
M_{20}	20	$\frac{20}{3} = 7$	$\frac{20}{4} = 5$
M_{25}	25	$\frac{25}{3} = 8.5$	$\frac{25}{4} = 6$
M_{50}	50	$\frac{50}{3} = 16$	$\frac{50}{4} = 12$

$$\text{Permissible stress } (\sigma_p) = \frac{\text{Strength of material } (f_{ck})}{\text{Factor of safety (FOS)}}$$

Permissible = Stress allow to structure.

I.S. code (For concrete)

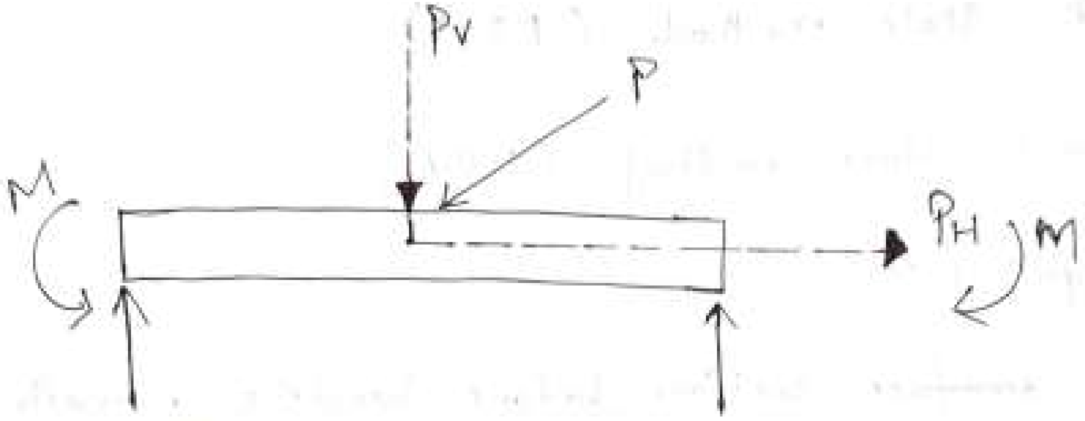
FOS in bending compression (σ_{cbc}) = 3
FOS in direct compression = 4

σ_{cbc} = Permissible stress

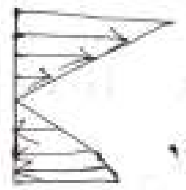
- c → compression
- b → bending
- c → in concrete

σ_{cc} = permissible stress in direct / constant compression.

FOS for steel = 1.8



[σ_{cc}, b/c stress is constant]



(Stress diagram of direct stress PH)

(Bending stress distribution)

P_H ⟶ induced direct stress.

P_V ⟶ induced bending or flexural stress.

Permissible stress in steel (Table-22)

Type	Mild steel (Fe 250)	HYSD bar (Fe 415)
(1) Tension		
(a) upto & include 20mm ϕ bar	$\frac{250}{1.8} \simeq$ 140 mPa	$\frac{415}{1.8} \simeq$ 230 mpa
(b) Over 20 mm ϕ bar	130 mpa	230 mpa
(2) Compression in column	130 mpa	190 mpa

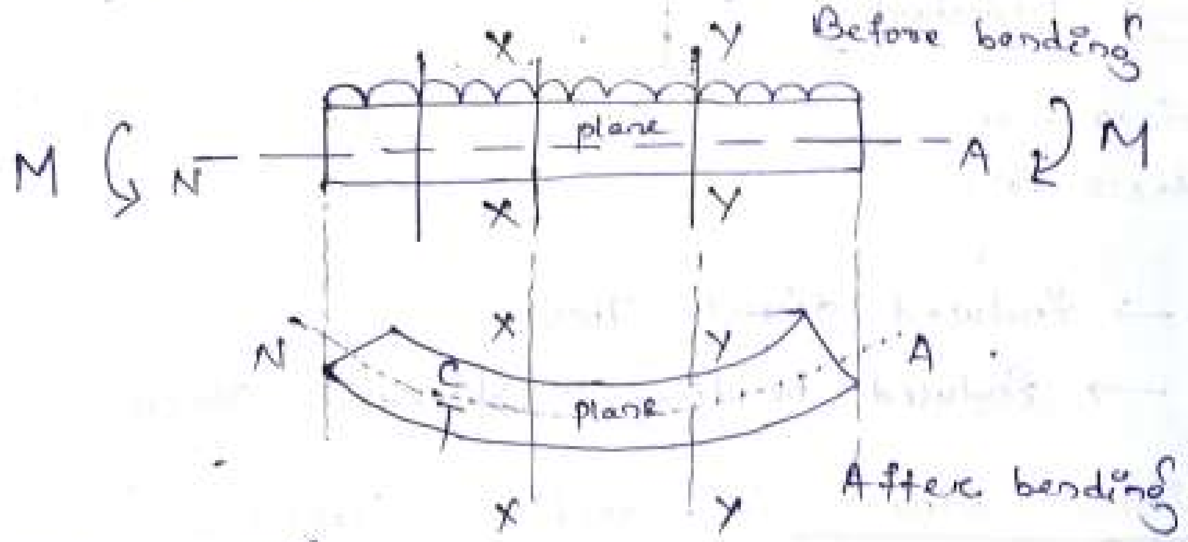
Method of design of RCC structure :-

(1) Working stress method (WSM)
(2) Ultimate load method (ULM)
(3) Limit State method (LSM).

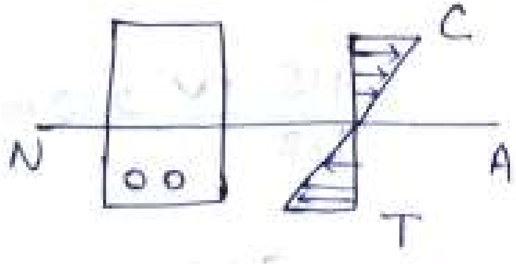
(1) Working stress method (WSM) (Pg-80)

Assumptions :-

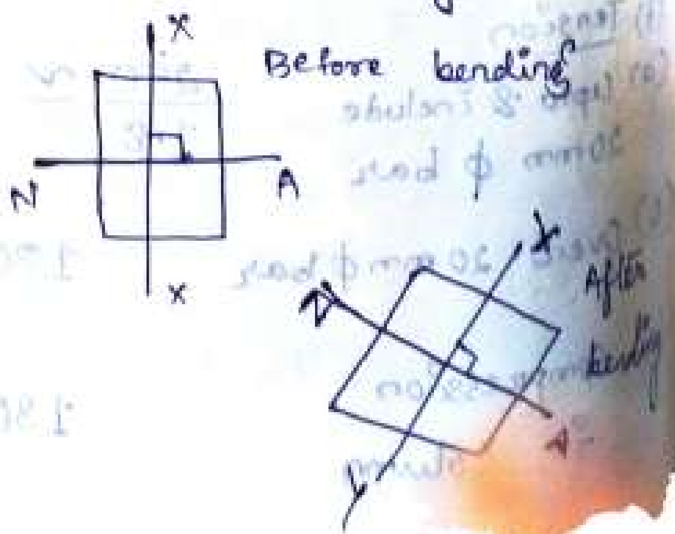
1. Plane ~~surface~~ section before bending remain plane after bending.
(i.e. Strain is linear)



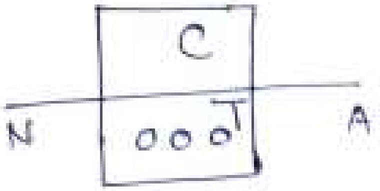
Strain diagram



In another way



2. All the tensile stress are taken by steel not by concrete.

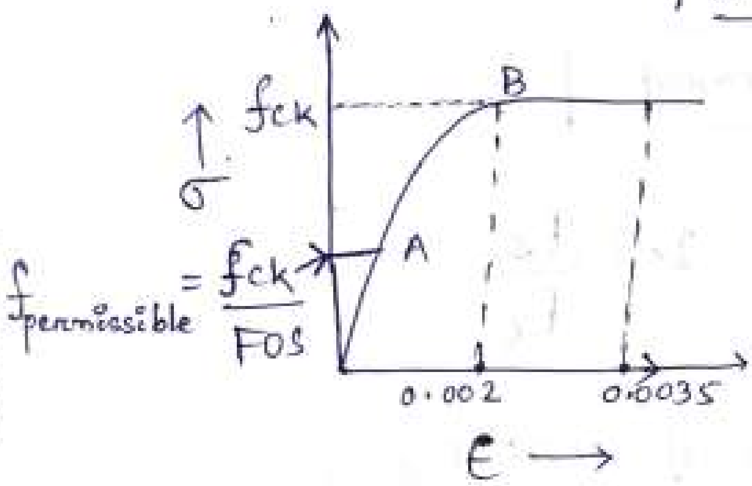


∴ Steel is provided in tension phase.

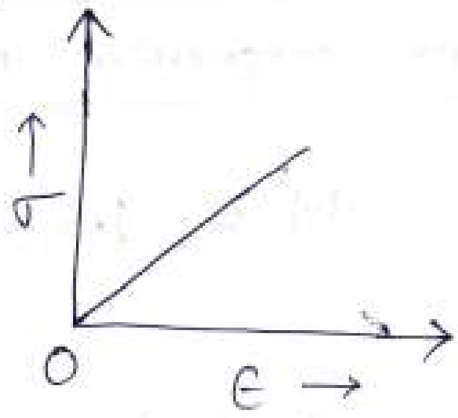
As concrete week in tension, Steel is provided to prevent it.

3. The stress - strain relationship of steel and concrete is linear or straight line.

For concrete

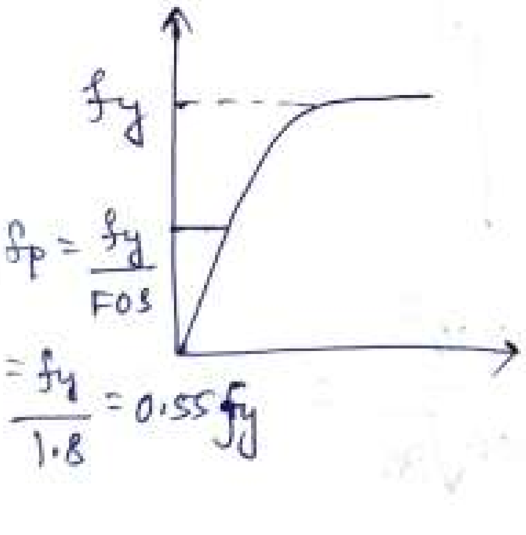


≃

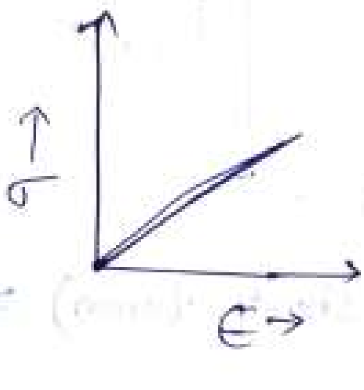


0 — 0.002 → Parabola

0.002 — 0.0035 → straight line.



≃



4. The modular ratio m has the value $\frac{280}{3\sigma_{cbc}}$.

For tensile reinforcement

$$\text{Modular ratio } (m) = \frac{280}{3\sigma_{cbc}}$$

$$M = \frac{\text{Modulus of elasticity of steel}}{\text{Modulus of elasticity of concrete}}$$

$$= \frac{E_s}{E_c}$$

For compressive reinforcement

$$M' = 1.5M = 1.5\frac{E_s}{E_c}$$

E_c = Short term modulus of elasticity.

E_{cr} = Reduced modulus of elasticity

or

E_{ce} = effective modulus of elasticity

$$E_{cr} = \frac{5000\sqrt{f_{ck}}}{1+\theta}$$

Eg:- M30.

$$M \text{ (for short term)} = \frac{E_s}{E_c} = \frac{2\times10^5}{5000\sqrt{30}} = 7.3$$

(Long term) $M = \frac{E_s}{E_{cr}} = \frac{2\times10^5}{\left(\frac{5000\sqrt{30}}{1+1.6}\right)} = 18.9$

↳ for 28 days.

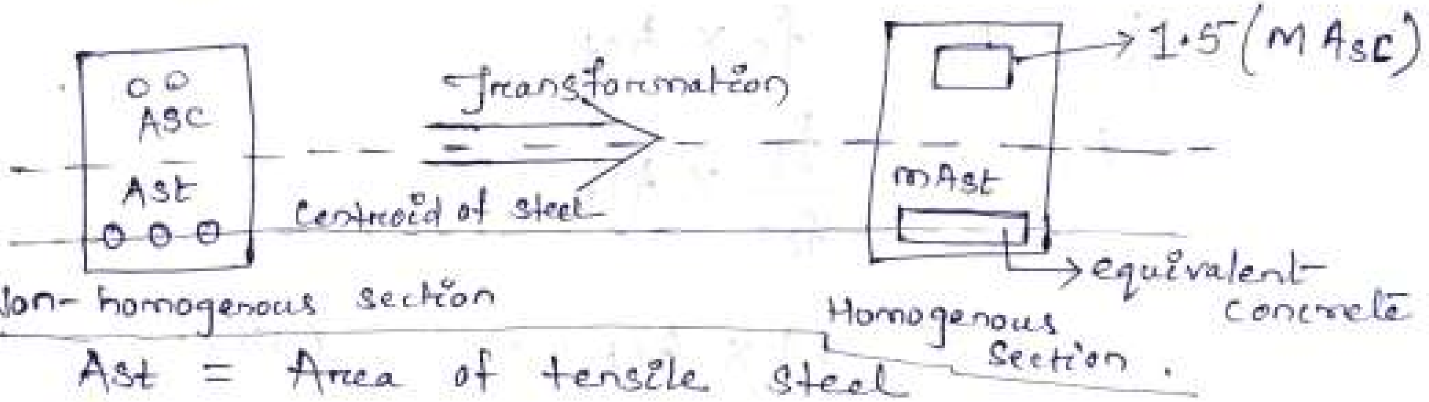
$$M = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times\left(\frac{f_{ck}}{3}\right)} = \frac{280}{3\times\left(\frac{30}{3}\right)} = 9.33$$

↦ IS 456 : 2000 considere partial effect of creep and shrinkage for calculating modulare ratio.

*. Transformation :-

Changes of non-homogeneous section to homogenous section is known as transformation.

steel —convert→ concrete



Ast = Area of tensile steel

Asc = Area of steel in compression.

Modulare ratio is the transformation factor.

*. Strength ⟶ less

(i) $\epsilon_{concrete} = \epsilon_{steel}$

$$\frac{f_c}{E_c} = \frac{f_s}{E_s} \qquad \left(\because \sigma = E \cdot \epsilon \quad f = E \cdot \epsilon \quad \frac{f}{E} = \epsilon \right)$$

$$\Rightarrow f_c = f_s \cdot \frac{E_c}{E_s}$$

$$= \frac{f_s}{m} \qquad \left(\because m = \frac{E_s}{E_c} \right) \text{——— (i)}$$

$$\Rightarrow \frac{f_s}{f_c} = M \text{——— (ii)}$$

(ii) Force in concrete = Force in steel.

(Stress × Area = Force)

$$f_c \times A_c = f_s \times A_{st}$$

$$A_c = \frac{f_s}{f_c} \times A_{st}$$

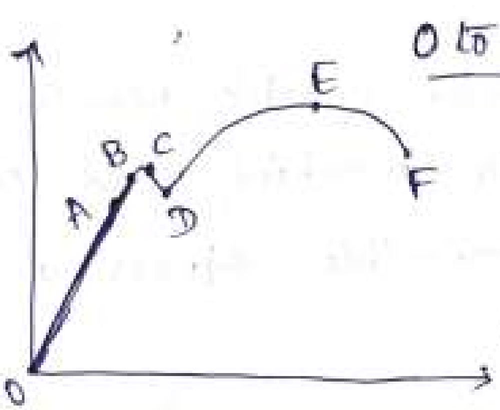
$$= M \times A_{st} \text{——— (iii)}$$

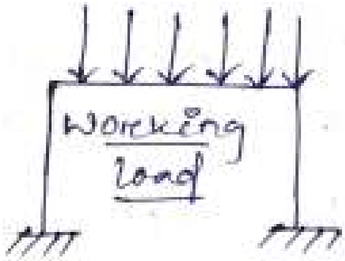
$$M = \frac{280}{3\sigma_{cbc}}$$

Working Stress Method (WSM) :-

↦ It is the traditional method of structural design.

↦ It assumes structural material behaves in a linear elastic manner and adequate safety is ensured by restricting the stress in the material induced by the expected working load (service load) on the structure.





It is the actual load acting on a structure. (internally as well as externally).

↦ As specific permissible (allowable) stress are kept well below the material strength, so the assumption of linear elastic behaviour is justified.

$$\text{Permissible stress} = \frac{\text{Strength of Material } (f_{ck})}{FOS}$$

* Working stress $\leq$ Permissible stress (σ_{cbc})

Advantage

↦ As we use larger FOS the section is larger area ($\sigma = \frac{F}{A}$). So, it provide better servicability performance (i.e. less deviation less crack, less vibration).

Disadvantages

↦ As the section has larger area, so the self weight of the structure is increases.

↦ The main assumption of linear elastic behaviour, such that the stress under working load can be kept within the permissible stress are not justified because :-

It does not consider, the long term effect of creep & shrinkage, the effect of stress concentration etc. So all this effect results in increasing stress into the inelastic range..

↦ Working stress method does not provide realistic measure of FOS for design.

↦ It is fails to descriminate betwn different types of load acting on a structure simultaneously.

(It ~~appli~~ carry only one type of load (either dL, lL etc)

Working stress method used in → bridges, water-tank, chimney etc.

WSM
It works on only for the strength of material not load.

strength of material → FOS.
Load → does not determine actual load.

Ultimate Load Method (ULM) / Load factor method / ultimate strength method / plastic design method.

↦ In this method, stress condition at the state of impeding failure of section (i.e. at ultimate strength) is analyzed.

↦ So, non-linear stress – strain curve of concrete and steel is used.

↦ The concept of modular ratio is avoided.

↦ Safety in this design is introduced by an appropriate choice of load factor.

$$\text{Load factor} = \frac{\text{ultimate load}}{\text{working load}}$$

↦ In this method, we can assign different types of load to different load factor i.e. Combined loading condition satisfied.

Advantage

↦ It results more slender (thin) section and more economical design of structure.

Disadvantage

↦ It doesnot guarantee about serviceability performance.

↦ So, it results excessive deflection and crack under service load.

↦ Use of non-linear stress-strain behaviour is meaningful is appropriate non-linear analysis is performed, but such type of analysis is generally not performed on reinforced concrete structure. So, it is difficult to predict behaviour of plastic analysis.

Reinforcement ⟶ elastic limit.

Due to high load → Plastic hinge doesnot predict.

Loading Standard (IS 875)

(i) Dead load (IS 875 - part I):→

⇒ It is the self weight of the structure.

⇒ It includes the weight of all permanent construction (i.e. wt. of roof, wall, floor, coloumn, footing etc.)

(ii) Live load / imposed load (IS 875 part II)

⇒ Loads are keep on changing from time to time.

Eg. weight of person, movable partition, furniture etc.

(iii) Wind load (WL) (IS 875 – part III)

⇒ The force exerted by wind on a structure.

(iv) Snow load (IS 875 – part IV)

(v) Special load and load combination (IS 875 – part V)

Special load ⟶ Accidental load

- Impact and collision.
- Explosion
- Fire

Load combination ⟶ Load are acted in combination.

LSM (limit state Method) (Probabilistic approach)

- Limit state is a state of impeding failure, beyond which a structure ceases to perform its function satisfactorily interms of either safety or servicability.

- LSM aims by providing safety at ultimate load and servicability at working (service) load.

- So, it use partial safety factor (PSF) format which provide adequate safety at ultimate load and servicability at service load by considering all possible limit state.

(Partial factor of safety → Multiple factor of safety)
⇒ OR it is also known as balanced → By probability)
⇒ Partial effect of creep and shrinkage.)

Method	Material FOS	Load FOS
WSM	✓	×
ULM	×	✓
LSM	✓	✓

It is of two types :-

(i) limit state of collapse (ultimate limit state)

⇒ collapse ⟶ completely failure.

(i)⇒ limit state of collapse in flexure (bending)
(In beam)

(ii)⇒ Limit state of collapse in compression. (In column)

(iii)⇒ Limit state of collapse in shear. (Beam & Slab)

(iv)⇒ limit state of collapse in torsion. (Beam)

⟶ It deals with the maximum load carrying capacity i.e. the safety requirement of structure.
load < ultimate moment of structure or max. limit
Resistance of structure > total limiting moment.

⟶ Strength of material is sufficient to carry ultimate load.

(ii) Limit State of Servicability

⟶ A structure is of no use if it is not servicable.

↦ Thus limit state is introduce to prevent excessive deflection and cracking.

So, it ensure satisfactory performance of structure at working load.

Working load ⟶ deform : like vibration.
⟶ it doesnot fail.

→ It includes :-

(i) Limit state of deflection.

(ii) Limit state of cracking

It also includes vibration,

In code book
Pg - 67

Characteristic strength of Material (f_{ck}). (Pg - 67)

↦ It's the strength of the material below which not more than 5% of the test result are expected to fall.

Concrete → f_{ck}

Steel → f_y

Characteristics load

↦ It's the value of the load which has a 95% probability of not being exceeded during life time of structure.

Design Value :-

(i) Design strength of Material (f_d)

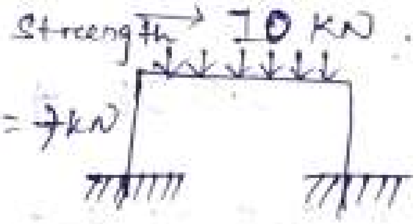
$$f_d = \frac{f}{\gamma_m} = \frac{\text{Characteristic strength of material}}{\text{Partial safety factor to the material.}}$$

For concrete $\gamma_m = 1.5$
For steel $\gamma_M = 1.15$

* Steel has high partial safety factor as compared to concrete.

Because in steel proper quality control is measured. (∵ It manufacture in machine).

But in concrete ~~pro~~ quality control is not properly measured. (i.e. quarrying is not done properly etc.)



$f_d = \frac{10}{1.5} = 7 kN$

$F_d = 10 \times 1.5 = 15 kN$

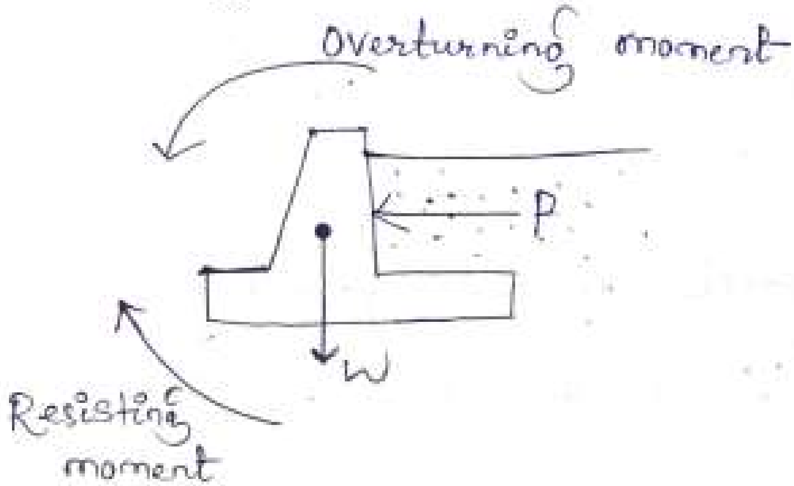
Design load.

$$F_d = F \times \gamma_f$$

F = characteristic load
γ_f = PSF ~~to~~ load

Load combination	LSC (limit state of collapse) DL	LL	WL	LSS (limit state of serviceability) DL	LL	WL
DL + LL	1.5	1.5	—	1	1	—
DL + WL (or EL)	1.5 or 0.9	—	1.5	1	—	1
DL + LL + WL or (EL)	1.2	1.2	1.2	1	0.8	0.8

WL and EL both the load could not act at same time in a structure because its Probability of occurance is very very less.



E.g. DL = 10 kN·m , LL = 20 kN·m , WL = 30 kN·m
EL = 50 kN·m .

(I) 1·5 DL + 1·5 LL = 15 + 30 = 45 kN·m

(II) 1·5 DL + 1·5 WL = (1·5 × 10) + (1·5 × 30)
= 15 + 45 = 60 kN·m.

or

1·5 DL + 1·5 EL = 15 + 75 = 90 kN·m

(III) (1·2 × 10) + (1·2 × 20) + (1·2 × 30) = 12 + 24 + 36
= 72 kN·m .

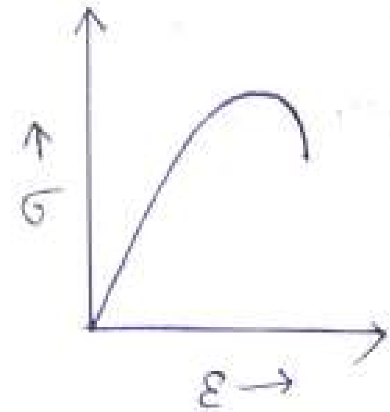
or

(1·2 × 10) + (1·2 × 20) + (1·2 × 50) = 12 + 24 + 60
= 96 kN·m

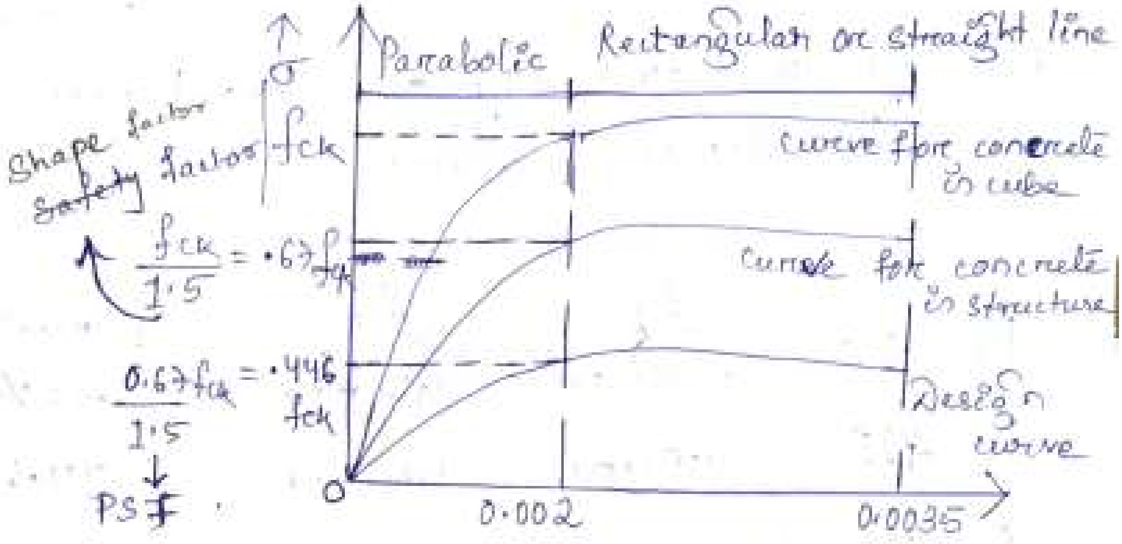
In code book
Pg - 68

7/5/21

Stress - strain curve of concrete :-



(Actual)



(Design curve)

Compressive strength of cube = fck.

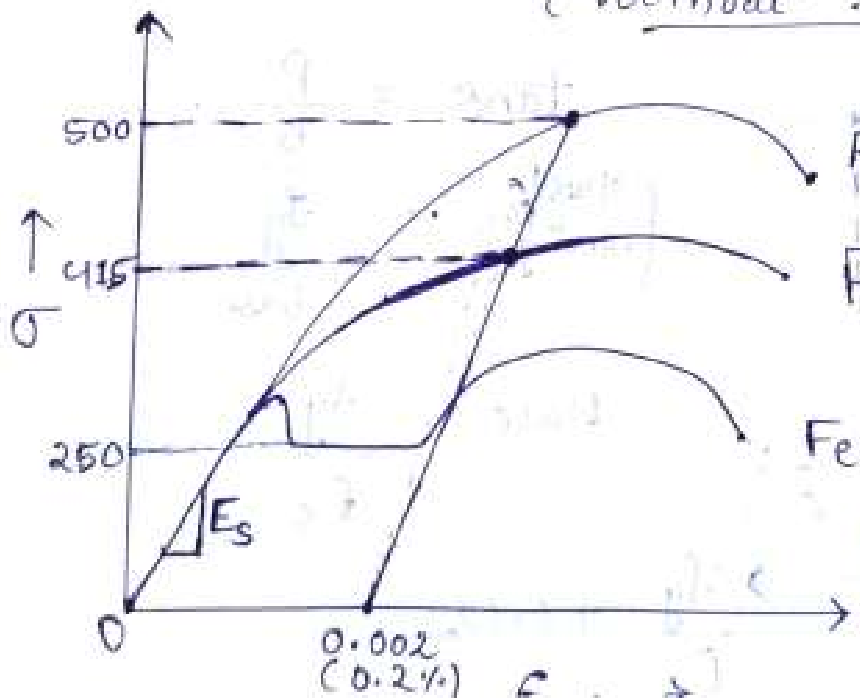
Compressive strength of actual contrete member =

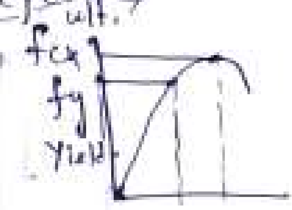
$$\frac{f_{ck}}{1.5} = 0.67 f_{ck}$$

Design compressive strength of actual concrete member $= \frac{0.67 f_{ck}}{1.5} = 0.45 f_{ck}$.

Stress - Strain curve Steel :-

(Without factor of safety)





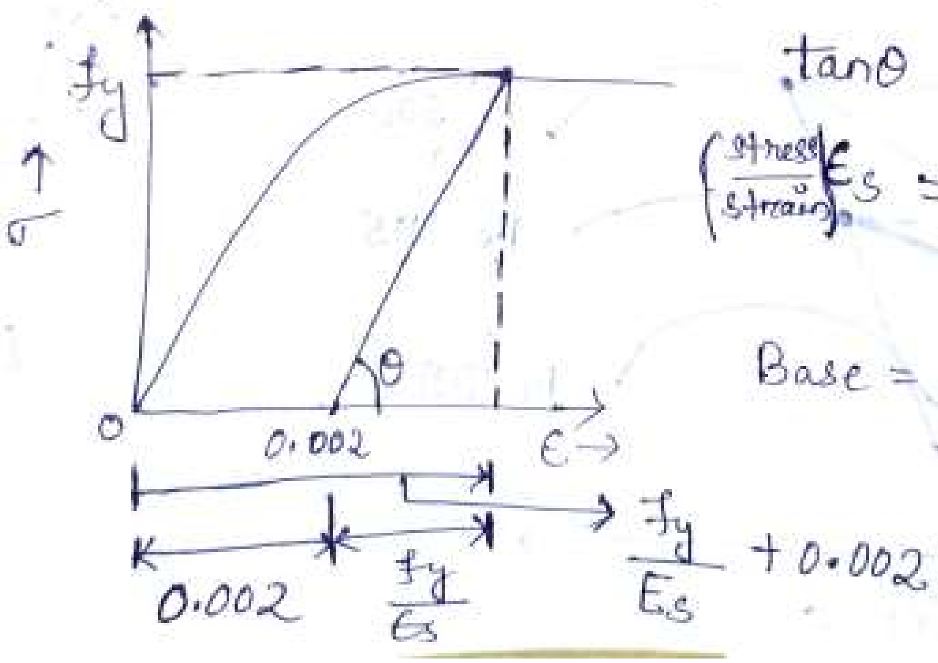
$E = \frac{\sigma}{\epsilon}$

By adding carbon its grade increases and yield strength and ultimate strength also increases but its ~~ofet~~ ductility ~~and~~ ~~brittleness~~ decreases and brittleness increases.

	f_y	f_u	Elongation
Fe 250	250 mpa	412 mpa	23%
Fe 415	415 mpa	485 mpa	14%
Fe 500	500 mpa	545 mpa	10%

↦ Mild steel has clearly and well defined yield point.

↦ But in HYSD steel yield point is not clearly visible.
SO choose a proof stress of 0.2% which is equivalent to Yield stress.
Proof stress is inelastic.



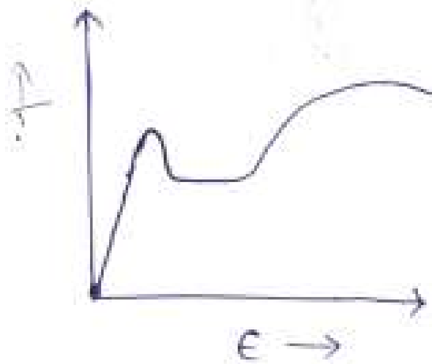
$\tan\theta = \frac{P}{b}$

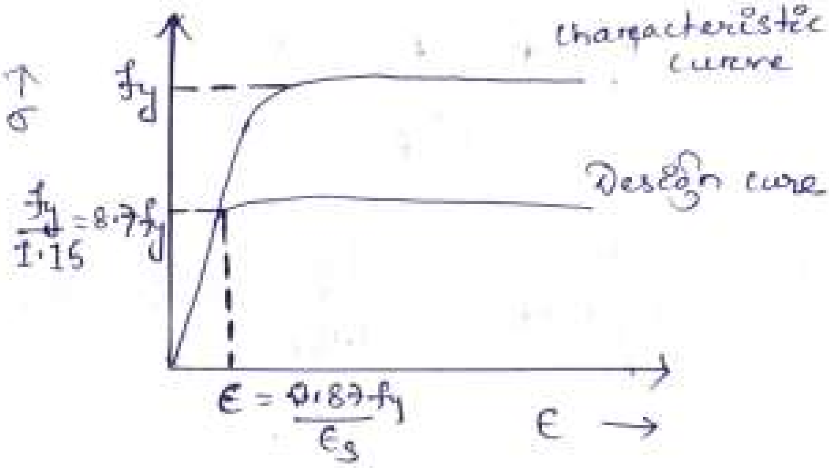
$\left(\frac{\text{stress}}{\text{strain}}\right) E_s = \frac{f_y}{\text{base}}$

$\text{Base} = \frac{f_y}{E_s}$

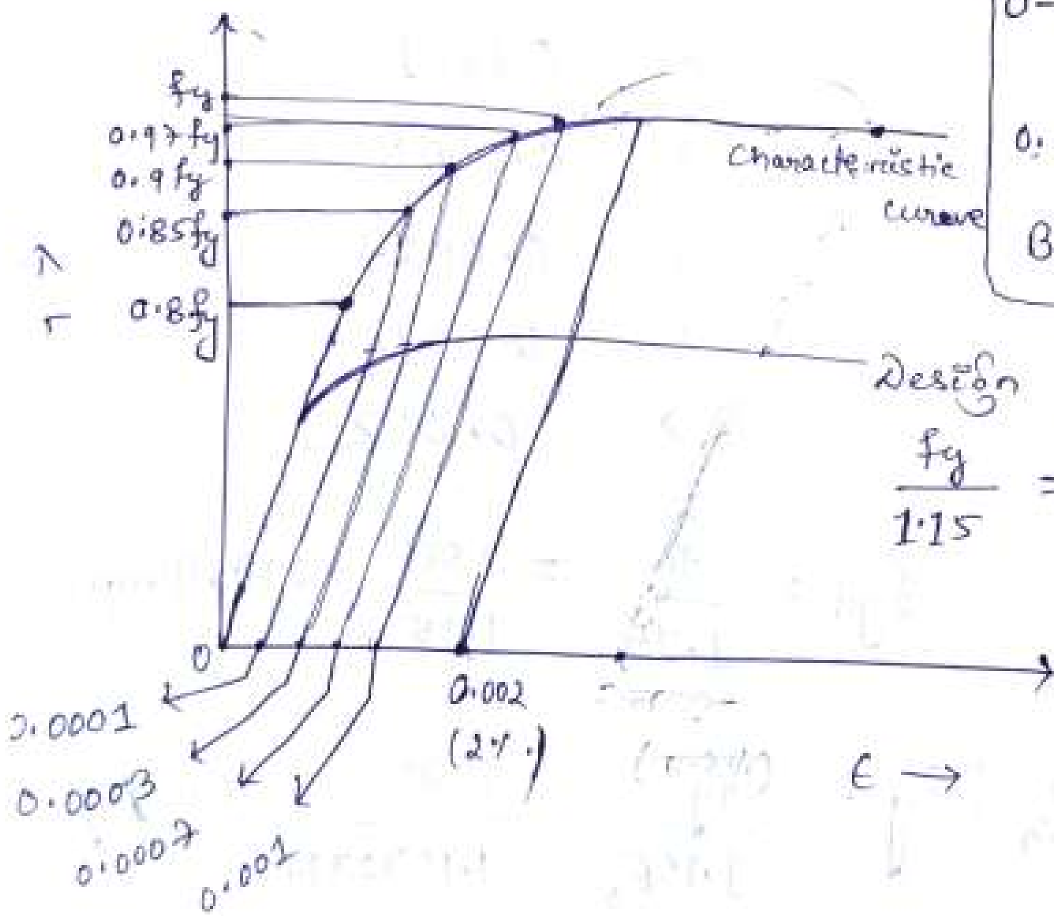
In HYSD Bar ⇒ { Strain at yield point $= \frac{f_y}{E_s} + 0.002$
Stress at yield $= f_y$.

(i) Design stress-strain curve for mild steel (Fe 250)





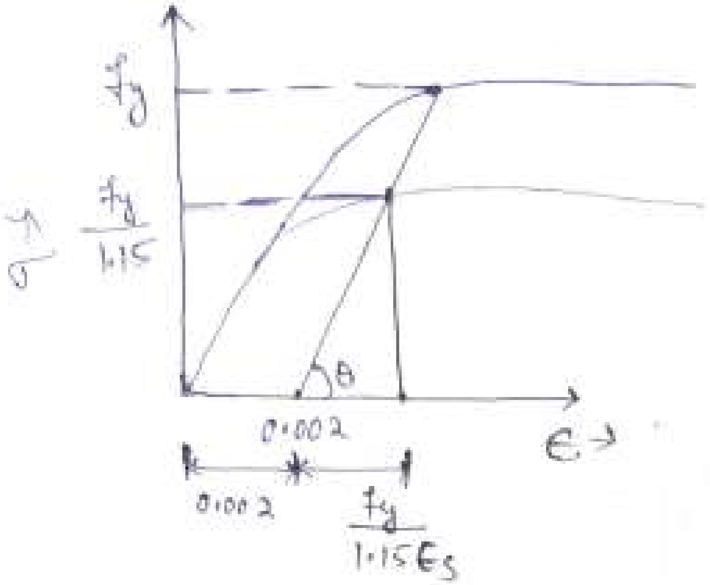
(ii) Design stress-strain curve for HYSD ~~steel~~ Bar:-



0 − 0.8fy → Straight line or Linear
0.8fy − fy → Non-linear
Beyond fy → Straight

Design curve

$\frac{f_y}{1.15} = 0.87 f_y$



$\tan\theta = \frac{P}{b}$

$E_s = \frac{\frac{f_y}{1.15}}{base}$

$\Rightarrow Base = \frac{f_y}{1.15 E_s}$

(Strain in steel) $\boxed{\epsilon_{st} = \frac{f_y}{1.15 E_s} + 0.002}$

Design stress	Inelastic or Proof strain
0.8 fyd →	0
0.85 fyd →	0.0001
0.9 fyd →	0.0003
0.95 fyd →	0.0007
0.975 fyd →	0.001
1 fyd →	0.002

For, Fe 250, $f_{yd} = \frac{f_y}{1.15} = \frac{250}{1.15} = 217.4$ mpa

(Yield strain) $\epsilon_y = \frac{f_y}{1.15 E_s} = \frac{250}{1.15 \times 2 \times 10^5} = 0.00109$

Stress level	Fe 415 ϵ_y	Fe 415 σ (mpa)	Fe 500 ϵ_y	Fe 500 σ (mpa)
$0.8 f_{yd}$	$= \frac{.8 f_y}{1.15 E_s} + 0 = \frac{.8 \times 415}{1.15 \times 2 \times 10^5} = 0.00144$ →	$\frac{0.8 \times 415}{1.15} = 288.7$	0.00174 →	347.8
$0.85 f_{yd}$	$= \frac{0.85 \times 415}{1.15 \times 2 \times 10^5} + .0001 = 0.00163$	$\frac{0.85 \times 415}{1.15} = 306.7$	0.00195 →	369.6
$0.90 f_{yd}$	= 0.00192	324.8	0.00226 →	391.3
$0.95 f_{yd}$	= 0.00241	342.8	0.00277 →	413
$0.975 f_{yd}$	= 0.00276	351.8	0.00312 →	423.9
$1 f_{yd}$	$= \epsilon = \frac{f_y}{1.15 E_s} + 0.002 = \frac{415}{1.15 \times 2 \times 10^5} + 0.002 = 0.0038$	$\frac{f_y}{1.15} = \frac{415}{1.15} = 360.9$	$\epsilon = \frac{500}{1.15 \times 2 \times 10^5} + 0.002 = 0.00417$ →	$434.8 = \frac{500}{1.15}$

Fe 250 : $\epsilon_y = 0.00109$.

Fe 415 : $\epsilon_y = 0.0038$.

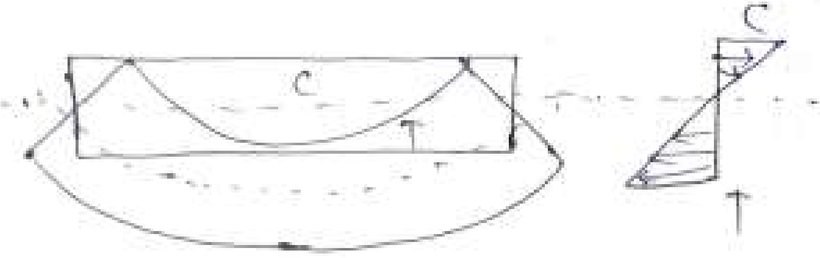
Fe 500 : $\epsilon_y = 0.0042$.

In code book
Pg - 69
Pg - 70,

Assumptions of Limit State of collapse : Flexure.

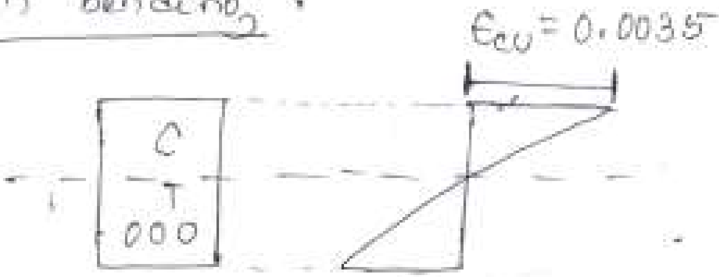
(1) Plane sections normal to the axis remain plane after bending.

(i.e. strain is linear).



(shear diagram)

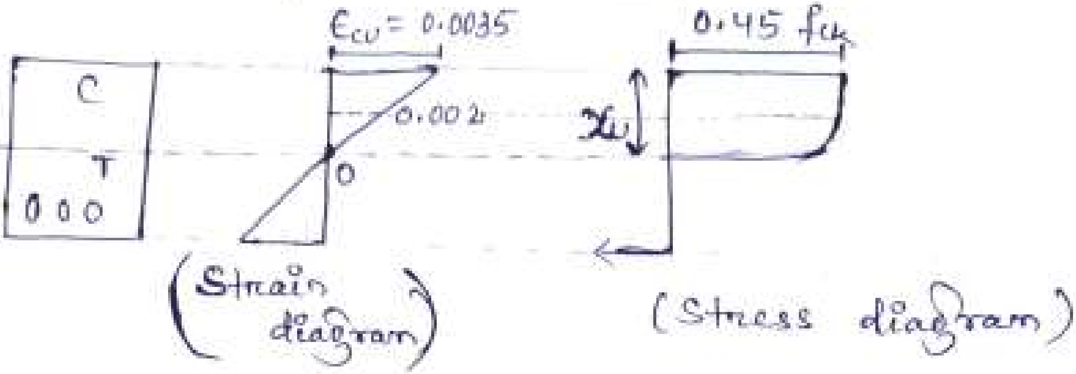
(2) The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.



ϵ_{cu} = strain of outermost compression fibre

(3) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, Parabola or any other shape which results in Prediction of strength in substantial ~~agreem~~ agreement with the result of test. An acceptable stress - strain curve is given in fig. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic

Strength. The partial safety factor $\gamma_m = 1.5$ shall be applied in addition to this.



x_u = depth of N-A from outermost compression fibre

(4) The tensile strength of the concrete is ignored. (i.e. Tensile strength of concrete = 0)

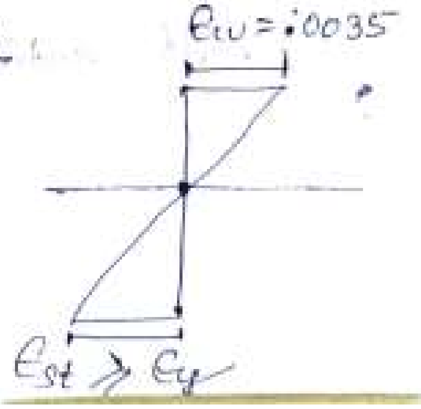
(5) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. For design purposes the partial safety factor γ_m equal to 1.15 shall be applied.

$$f_d = \frac{f_y}{1.15} = 0.87 f_y$$

(6) The maximum strain in the tensile reinforcement in the section at failure shall not be less than :

$$\epsilon_y = \frac{f_y}{1.15 E_s} + 0.002$$

$$\epsilon_{st} \nless \epsilon_y .$$



*. Failure must be ductile but should not be brittle.

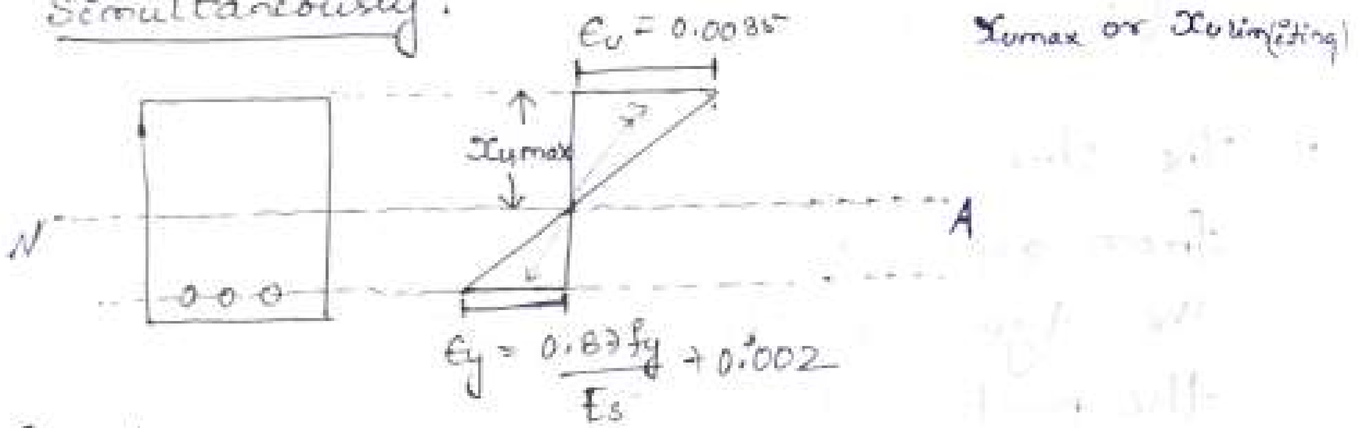
11/5/21

Types of section

(i) Balanced section (BS)
(ii) Under reinforced section
(iii) Over reinforced section

(i) Balanced section (BS):-

→ In BS the ~~compression~~ compressive strain in extreme fibre of concrete reach the ultimate strain (ϵ_{cu}) and tensile strain at the level of centroid of steel reaches the yield strain (ϵ_y) simultaneously.



Condition

(i) In BS, $x_u = x_{u,max}$ or $x_{u,lim}$

(ii) Compressive strain at top fibre = 0.0035

(iii) Tensile strain at level of centroid of steel

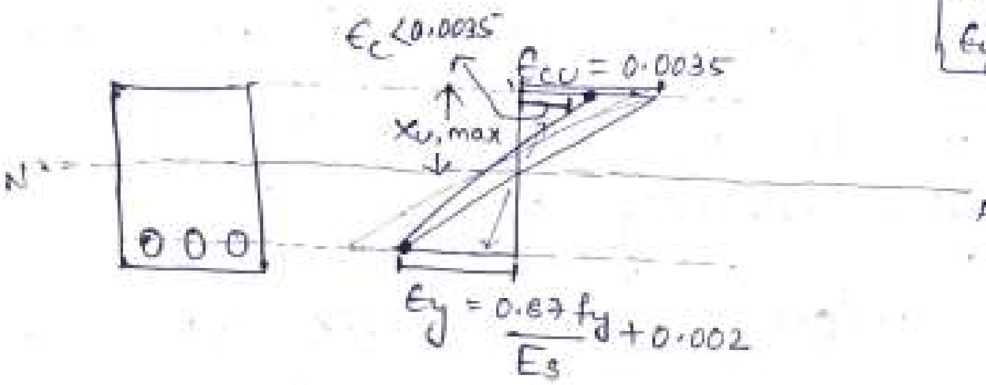
$$= \frac{0.87 f_y}{E_s} + 0.002$$

∴ It satisfy assumption (2) & (6).

(2) Under reinforced section (URS)

→ In under reinforced section the tensile strain at level of centroid of steel reaches yield strain before the compressive strain in extreem fibre of concrete reach the ultimate strain.

ε_{cu} → before
ε_y → reaches.



* Case - 1

$\varepsilon_{cu} < 0.0035$

$\varepsilon_{st} = \varepsilon_y = \frac{0.87 f_y}{E_s} + 0.002$

∴ It is satisfy assumption (2) & (6).

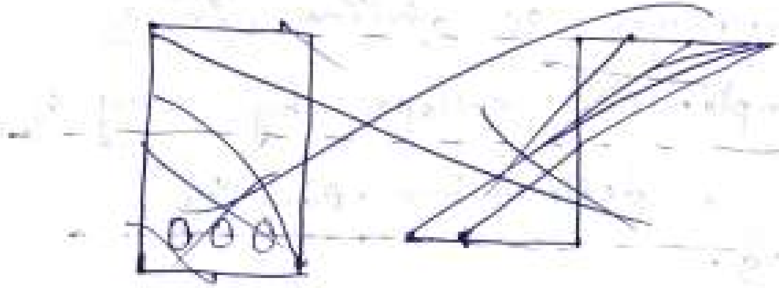
* Case - 2

$\varepsilon_c = 0.0035 = \varepsilon_{cu}$

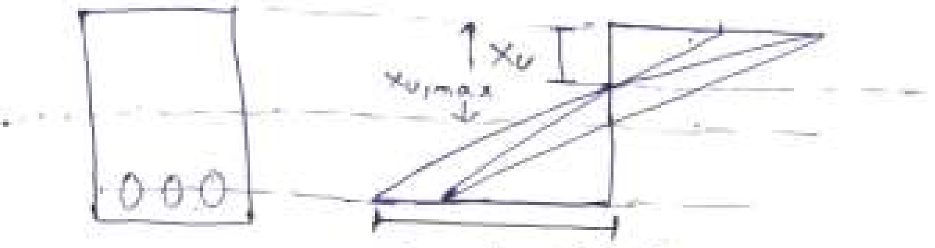
$\varepsilon_{st} > \varepsilon_y$

$\varepsilon_{st} > \frac{.87 f_y}{E_s} + 0.002$

∴ Assumption (2) & (6) satisfy.



- ultimate strain of steel is greater than its ~~point~~ yield point.
- ultimate strain of steel > ultimate strain of concrete.



$$\epsilon_{st} > \frac{0.87 f_y}{E_s} + 0.002$$

*→ Under reinforced section should be provided.

*→ Steel is provide in small quantity.

*→ Load carrying capacity high.

*→ (steel) It give priore warning after failure.

*→ If the concrete first reaches the ultimate strain then sudden failure will occur.

$$x_u < x_{u,max}$$

→ The failure of URS is called tension failure, as primary cause of failure is yielding in tension of steel.

→ This failure is gradual, so ~~giving~~ giving priore warning of implending collapse by way of increasing curvature, ~~deflection~~ causing deflection and cracking.

→ So such mode of failure is highly prefered in design.

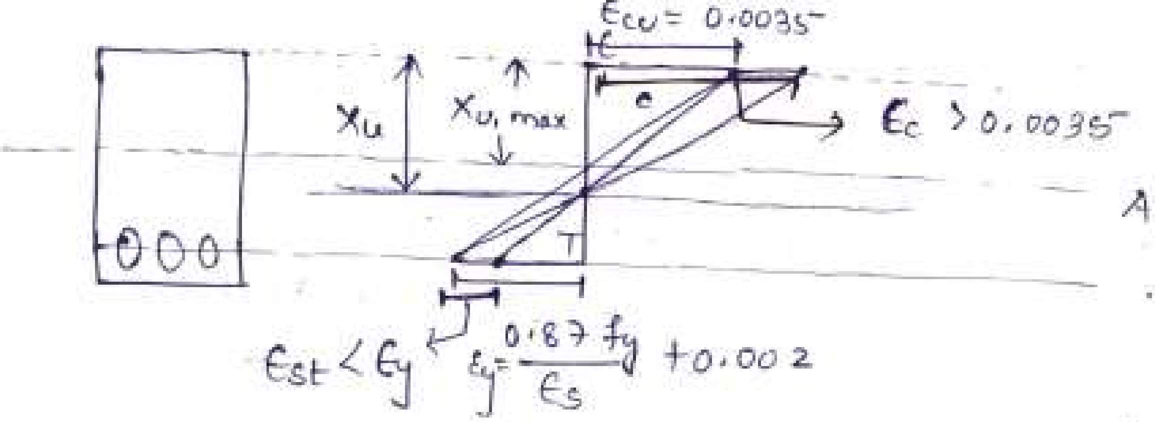
→ The actual collapse although due to yielding of Steel, occurs by means of crushing of concrete, in compression.

→ So it is called <u>secondary compression failure</u>.

→ It is also called <u>ductile failure</u>.

(3) Over reinforced section (ORS)

→ In ORS, strain in extreem fibre of concrete reach the ultimate strain <u>earlier</u> than the tensile strain at level of centroid of steel reach yield strain.



* **Case - 1**

$$\epsilon_{cu} = 0.0035$$

$$\epsilon_{st} < \frac{0.87 f_y}{E_s} + 0.002$$

∴ Assumption (2) is Satisfied.
Assumption (6) is not Satisfied.

* **Case - 2**

$$\epsilon_{cu} > 0.0035$$

$$\epsilon_{st} = \frac{.87 f_y}{E_s} + 0.002$$

∴ Assumption (2) is not Satisfied.
Assumption (6) is Satisfied.

$$\Rightarrow \boxed{X_u > X_{u,max}}$$

⇒ The concrete fail in compression before the steel reaches its yield point.

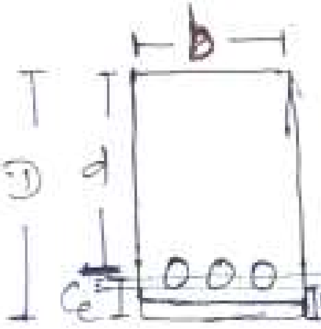
⇒ So, this type of failure is called primary compression failure (∵ here concrete fails)

⇒ As this type of failure occurs suddenly without any prior warning, ORS are not permitted.

⇒ It is also called brittle failure.

12/5/21

Cover to reinforcement :- (Pg-18)



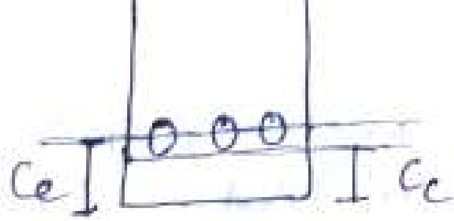
C_e = effective cover.

D = over depth of the beam

d = effective depth of beam

$$d = D - C_e$$

$$d = D - C_c - \frac{\text{diameter of bar}}{2}$$



b = width of beam

→ Cover is the shortest distance between the surface of a concrete member to the nearest surface of reinforcement.

→ It protect steel against corrosion, provide fire resistance and develop bond between concrete and steel.

Clear cover (cc)

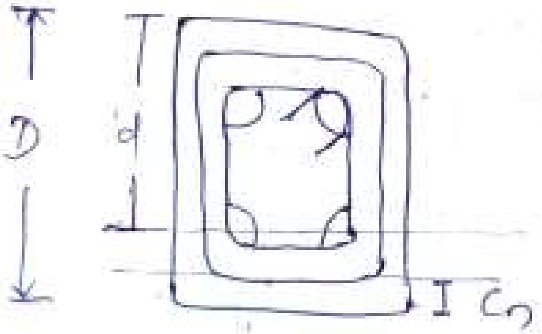
↦ Distance measured from the exposed surface of concrete to the nearest surface of reinforcement.

Effective cover (C_e):-

↦ Distance between exposed concrete surface to centroid of main reinforcement.

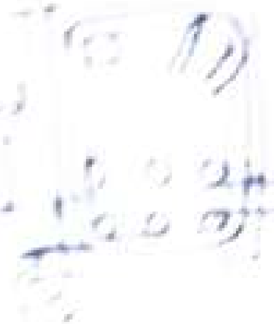
Nominal cover (C_n) :- (cl. 26.4.1) (Pg - 46)

Design depth of de concrete cover to all steel reinforcement including links.



$$d = D - C_n - \text{dia of stirrups} - \frac{\text{dia of main reinforcement}}{2}$$

(Pg 18)



[illegible] (horizontal spacing between two bars)

[illegible] dia of aggregate – dia of main [illegible]

Minimum tensile reinforcement in beams :-
(Cl 26.5.1.1)

[illegible]

(Ast)min [illegible] minimum tensile reinforcement

Maximum tensile reinforcement (Pg-47)

[illegible] (compaction of concrete)

→ use of large amount of steel [illegible],
[illegible] concrete cannot be done properly [illegible] compaction of concrete.

Spacing of reinforcement in beams :- (Pg-46)

(i) Spacing of horizontal reinforcement :- (S_h)
(Cl 26.3.2)

[illegible]

16, 12, 8

[illegible]

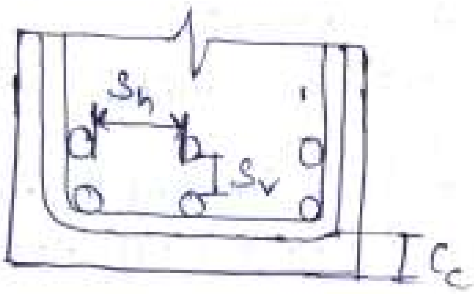
$S_h \not< max.$ { dia. of bar (if dia are equal)
dia of larger bar (if dia are unequal)
5mm more than nominal maximum size of coarse aggregate.

Spacing of vertical reinforcement :-

Vertical spacing between two parallel reinforcement :- (S_v)

$S_v \not< max.$ { 15mm
dia of larger bar
$\frac{2}{3}$rd of nominal maximum size of coarse aggregate

15mm
(16 mm)
$20 = \frac{2}{3} \times 20 = 13mm$



In code book

Pg - 18	(Cl - 26.5.1.1.)
Pg - 45	(Cl - 26.3.2)
Pg - 47	(Cl - 8.2.2.1 and 35.3.2)
Pg - 46	(Cl - 26.3.3)
	(Cl - 26.4.2)

Table 3. Environmental Exposure Conditions.
(Clauses 8.2.2.1 and 35.3.2)

Environment	Exposure Condition
Mild →	Concrete surfaces protected against weather or aggressive conditions, except those situated in coastal area.
Moderate →	Concrete surfaces sheltered from severe rain or freezing whilst wet. Concrete exposed to condensation and rain concrete continuously under water. Concrete in contact or buried under non-aggressive soil/ground water. Concrete surfaces sheltered from saturated salt air in coastal area.
Severe →	Concrete surfaces exposed to severe rain, alternate wetting and drying or occasional freezing whilst wet or severe condensation. Concrete completely immersed in sea water. Concrete exposed to coastal environment.
Very severe →	Concrete surfaces exposed to sea water spray, corrosive fumes or severe freezing conditions whilst wet. Concrete in contact with or buried under aggressive sub-soil/ground water.
Extreme →	Surface of members in tidal zone. Members in direct contact with liquid/solid aggressive chemicals.

Table 16. Nominal cover to Meet Durability Requirements
(Clause 26.4.2)

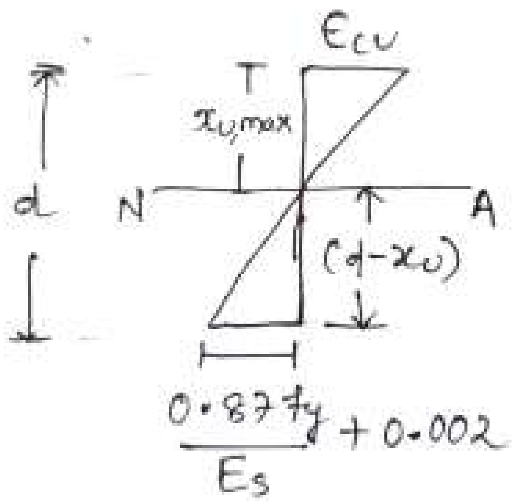
Exposure	Nominal Concrete cover in mm not less than
Mild →	20
Moderate →	30
Severe →	45
Very severe →	50
Extreme →	75

17/5/21

Pg-70

Calculation of limiting value of NA :-

f_y		$x_{u,max}/d$
250	$\frac{700}{0.87(f_y)+1100}$	0.53
415		0.48
500		0.46



$$\frac{\epsilon_{cu}}{x_{u,max}} = \frac{\frac{0.87f_y}{E_s}+0.002}{d-x_{u,max}}$$

$$\Rightarrow \frac{d-x_{u,max}}{x_{u,max}} = \frac{\frac{0.87f_y}{E_s}+0.002}{0.0035}$$

$$\Rightarrow \frac{x_{u,max}}{d-x_{u,max}} = \frac{0.0035}{\frac{0.87f_y}{E_s}+0.002}$$

$$\Rightarrow x_{u,max}\left(\frac{0.87 f_y}{E_s} + 0.002\right) = 0.0035d - 0.0035x_{u,max}$$

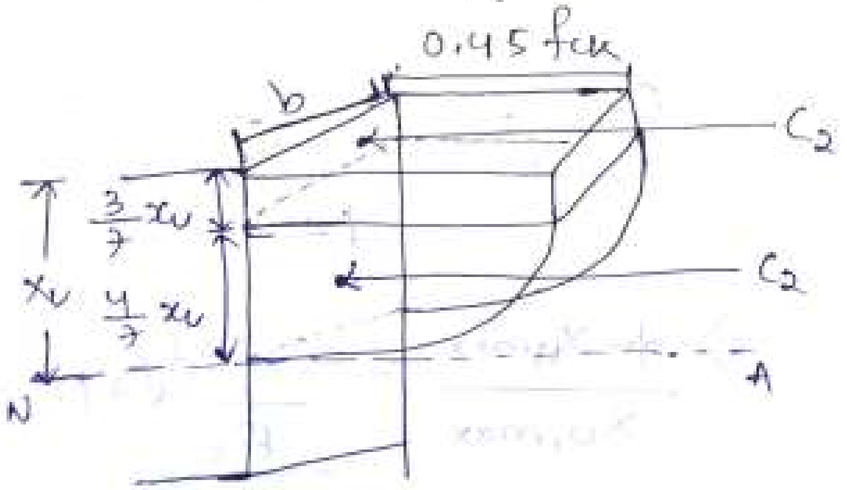
$$\Rightarrow x_{u,max}\left(\frac{0.87 f_y}{E_s} + 0.002 + 0.0035\right) = 0.0035d$$

$$\Rightarrow \boxed{\frac{x_{u,max}}{d} \leq \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}}$$

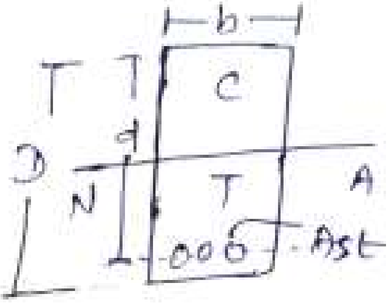
$$\Rightarrow \frac{x_{u,max}}{d} \leq \frac{0.0035}{\frac{0.87 f_y + 0.0055 E_s}{E_s}}$$

$$\Rightarrow \boxed{\frac{x_{u,max}}{d} \leq \frac{700}{0.87 f_y + 1100}}$$

$$\left(\frac{x_{u,max}}{d}\right) \propto \frac{1}{f_y}$$



Analysis & Design of Singly Reinforced Beam :-



Singly reinforced beam (SRB) :-

The beam reinforcement is provided only in tension zone is called singly reinforced beam.

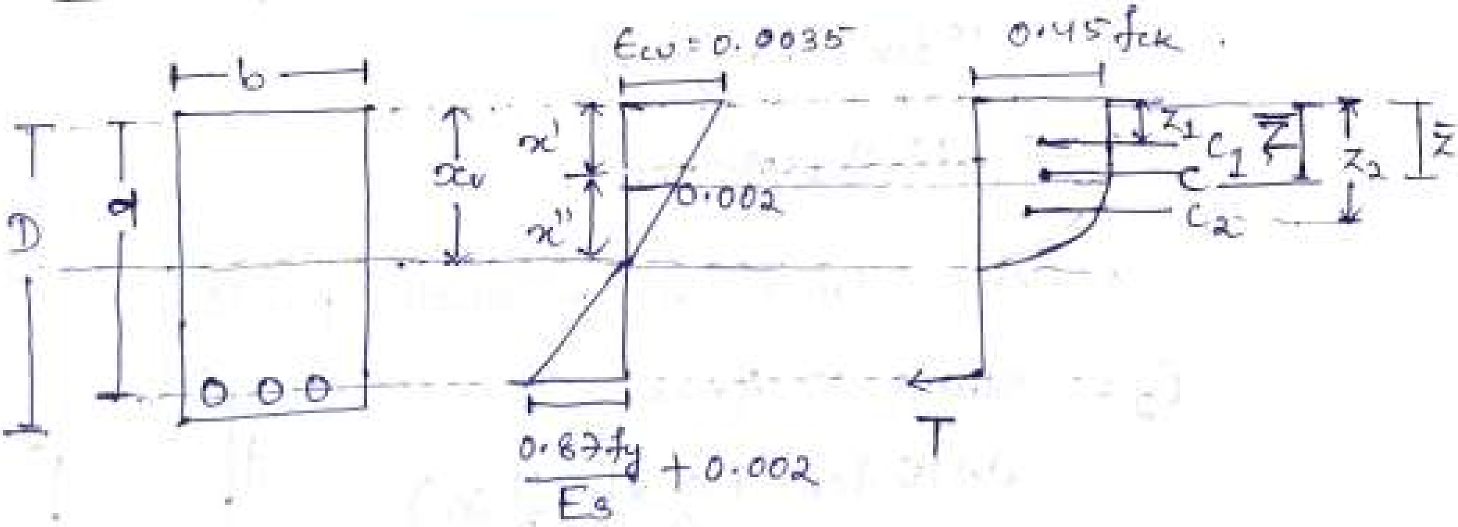
Analysis :- It is also known as review problem.. In analysis, Ast, d, b are given (i.e. dimensions are given.) We have to find moment, resistance of the moment.

Design:-

In design moment is given,. We have to find Ast, b, d.

fck is assumed.

Singly reinforced beam section :-



From strain diagram,

$$\frac{0.0035}{x_u} = \frac{0.002}{x''}$$

$$\Rightarrow x'' = \frac{0.002\, x_u}{0.0035} = \boxed{\frac{4}{7} x_u}$$

$$\boxed{x' + x'' = x_u}$$

$$\Rightarrow x' = x_u - \frac{4}{7} x_u$$

$$\Rightarrow \boxed{x' = \frac{3}{7} x_u}$$

C_1, C_2 is the compressive stress

18/5/21

$C_1 \rightarrow$ compressive force on the rectangular portion of the stress block

$$C_1 = \text{Stress} \times \text{Area}$$

$$= 0.45 f_{ck} \left(b \cdot \frac{3}{7} x_u \right)$$

$$= 0.193\, f_{ck} \cdot b \cdot x_u$$

$\sigma = \frac{F}{A}$

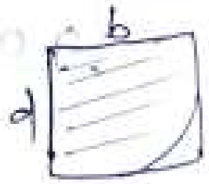
$\Rightarrow F = \sigma \times A$

$C_2 \rightarrow$ compressive force on parabolic portion

$$C_2 = \text{Stress} \times \text{Area}$$

$$= 0.45 f_{ck} \left(\frac{2}{3} \times b \times \frac{4}{7} x_u \right)$$

$$= 0.171\, f_{ck} \cdot b \cdot x_u$$



$\frac{2}{3} \times b \times d$

C = Total compressive force

$= C_1 + C_2 = 0.193 f_{ck} \cdot b \cdot x_u + 0.171 f_{ck} \cdot b \cdot x_u$

$\Rightarrow \boxed{C = 0.364\, f_{ck} \cdot b \cdot x_u}$ (Eq - 69)

Z_1 = Location of C_1 force from top fibre

$Z_1 = \frac{1}{2}\left(\frac{3}{7} x_u\right) = \boxed{\frac{3}{14} x_u}$

Z_2 = Location of C_2 force from top fibre.

$Z_2 = \frac{3}{7} x_u + \frac{3}{8}\left(\frac{4}{7} x_u\right)$ [CG of parabola $= \frac{3}{8} d$]

$= \frac{3}{7} x_u + \frac{12}{56} x_u$

$= \frac{9}{14} x_u$

$\bar{Z}$ = location of compressive force C from top fibre.

$= \frac{C_1 Z_1 + C_2 Z_2}{C_1 + C_2}$

$= \frac{\left(0.193 f_{ck} \cdot b \cdot x_u \cdot \frac{3}{14} x_u\right) + \left(0.171 f_{ck} \cdot b \cdot x_u \cdot \frac{9}{14} x_u\right)}{0.36 f_{ck} \cdot b \cdot x_u}$

$= 0.416\, x_u$

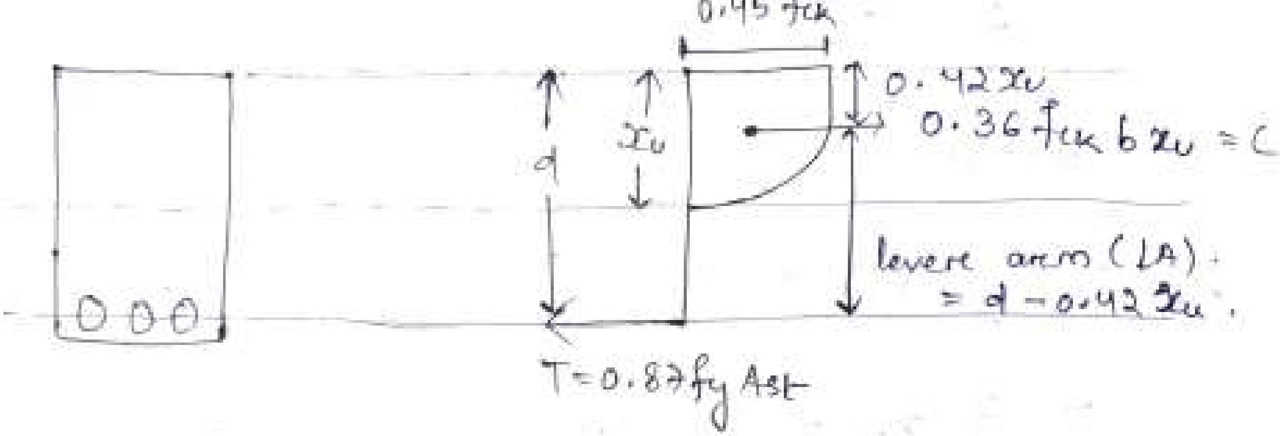
$\Rightarrow \boxed{\bar{Z} = 0.42\, x_u}$

T = Tensile force acting at the level of centroid of steel.

T = Design Stress in steel × Area of steel.

$= f_{st} \times A_{st}$

$= 0.87 f_y \cdot A_{st}$



Lever Arm :-

Distance between the compressive and tensile force.

$LA = d - 0.42\,x_u$

Compressive causes clockwise direction and Tension causes anti-clockwise direction. It stable the structure.

Depth of Neutral axis of a given beam

Total tensile force = Total compressive force

T = C

$0.87 f_y A_{st} = 0.36 f_{ck} \cdot b \cdot x_u$.

$$\Rightarrow \boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b}}$$

$$\Rightarrow \boxed{\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot bd}}$$ (dividing 'd')

Expression for moment of resistance :- (M_u)

M_u = ultimate moment of resistance (MOR)

$= C \times LA$ or

$M_u = T \times LA$ (lever arm)

(i) Expression of MOR interms of steel strength.

FOR:- URS

$M_u = T \times LA$ (lever arm)

$= 0.87 f_y A_{st} \cdot (d - 0.42 x_u)$ ——(1)

$$\frac{x_u}{d} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot bd}$$

$LA = d - 0.42 x_u$.

$$= d - 0.42\left(\frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b}\right)$$

$$= d - \frac{f_y \cdot A_{st}}{f_{ck} \cdot b} \longrightarrow (II)$$

Putting eqn (II) in eqn (I).

$$M_u = 0.87 f_y A_{st}\left(d - \frac{f_y A_{st}}{f_{ck} \cdot b}\right)$$

$$\boxed{M_u = 0.87 f_y A_{st} \cdot d\left(1 - \frac{f_y A_{st}}{f_{ck} \cdot bd}\right)} \longrightarrow (III)$$

A_{st} = Area of tension reinforcement.

P = percentage of tensile reinforcement.

$$\boxed{P = \frac{100 A_{st}}{bd}} \longrightarrow (IV)$$

$$\boxed{A_{st} = \left(\frac{P}{100}\right) bd} \longrightarrow (V)$$

If percentage is given,

Putting eqn (V) in eqn (III)

$$M_u = 0.87 f_y \left(\frac{P}{100}\right) bd \cdot d\left(1 - \frac{f_y}{f_{ck}\, bd} \cdot \frac{P}{100} \cdot bd\right)$$

$$\boxed{M_u = 0.87 f_y \left(\frac{P}{100}\right)\left(1 - \frac{f_y}{f_{ck}} \cdot \frac{P}{100}\right) \cdot bd^2} \longrightarrow (VI)$$

Expression of moment of resistance in terms of concrete strength :-
(For BS)

$$M_u = C \times LA$$
$$= 0.36 f_{ck} \cdot b \cdot x_u \cdot (d - 0.42 x_u)$$
$$= 0.36 f_{ck} \cdot b \cdot x_u \cdot d \left(1 - 0.42 \frac{x_u}{d}\right)$$
$$= 0.36 f_{ck} \cdot b \left(\frac{x_u}{d}\right) d \cdot d \left(1 - 0.42 \frac{x_u}{d}\right)$$

$$\boxed{M_u = 0.36 f_{ck} \cdot \left(\frac{x_u}{d}\right)\left(1 - 0.42 \frac{x_u}{d}\right) \cdot bd^2} \quad -(vii)$$

$$= \underbrace{0.36 \left(\frac{x_u}{d}\right)\left(1 - 0.42 \frac{x_u}{d}\right)}_{k} \cdot f_{ck} \, bd^2$$

$$\Rightarrow \boxed{M_u = k \cdot f_{ck} \, bd^2}$$

$\frac{x_u}{d}$ is constt.

Fe 250 → $\frac{x_u}{d} = 0.53$

$\Rightarrow K = 0.148 = 0.36 \times 0.53 \times (1 - 0.42 \times 0.53)$

Fe 415 → $\frac{x_u}{d} = 0.48$

$\Rightarrow K = 0.138$

Fe 500 → $\frac{x_u}{d} = 0.46$

$\Rightarrow K = 0.133$

For Fe 250 = 0.149
Fe 500 = 0.133

OR

$M_{u,lim} = 0.138 f_{ck} \cdot bd^2 \rightarrow$ (Fe 415)

Limiting value of MOR :-

Put $x_u \rightarrow x_{u,max}$ or $x_{u,lim}$.

$$M_{u,lim} = 0.36 f_{ck} \left(\frac{x_{u,max}}{d}\right)\left(1 - 0.42\frac{x_{u,max}}{d}\right) bd^2 \quad \text{—(VIII)}$$

$M_{u,lim} = 0.148 f_{ck} bd^2 \rightarrow$ Fe 250.

$= 0.138 f_{ck} bd^2 \rightarrow$ Fe 415

$= 0.133 f_{ck} bd^2 \rightarrow$ Fe 500

Calculate percentage of steel (P) :-

$T = C$

$0.87 f_y A_{st} = 0.36 f_{ck} \cdot b \cdot x_u$

$$A_{st} = \frac{0.36 f_{ck} \cdot b \cdot x_u}{0.87 f_y} \quad \text{—(IX)}$$

$$P = 100 \cdot \frac{A_{st}}{bd}$$

$$= 100\left(\frac{0.36 f_{ck} \cdot b \cdot x_u}{0.87 f_y \cdot bd}\right)$$

$$= \left(\frac{0.36}{0.87}\right) \times 100 \cdot \frac{f_{ck}}{f_y} \cdot \frac{x_u}{d}$$

$$\Rightarrow p = 41.3 \left(\frac{f_{ck}}{f_y}\right) \cdot \left(\frac{x_u}{d}\right) \quad \text{—— (x)}$$

If $x_u \rightarrow x_{u,max}$.

$$p_{lim} = 41.3 \left(\frac{f_{ck}}{f_y}\right) \cdot \left(\frac{x_{u,max}}{d}\right) \quad \text{—— (xi)}$$

Calculation of Ast.

$$A_{st} = \frac{0.5 f_{ck}}{f_y} \left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} \cdot bd^2}}\right) \cdot bd \quad \text{—— (xii)}$$

$$A_{st,lim} = \frac{p_{lim} \times bd}{100} \quad \text{—— (xiii)}$$

(1) Balanced Section

(i) $\frac{x_u}{d} = \frac{x_{u,max}}{d}$

(ii) $p = p_{lim}$

(iii) $M_u = M_{u\,lim}$

(2) URS

(i) $\frac{x_u}{d} < \frac{x_{u,max}}{d}$

(ii) $p < p_{lim}$

(iii) $M_u < M_{u,lim}$

BS

$$M_{u,lim} = 0.36 \left(\frac{x_{u,max}}{d}\right)\left(1 - 0.42 \frac{x_{u,max}}{d}\right) \cdot bd^2 f_{ck}$$

$$= k\, f_{ck}\, bd^2$$

For URS,

$M_U = 0.87 f_y A_{st}$

(iii) ORS :-

(i) $x_u > x_{u,max}$ or $\frac{x_u}{d} > \frac{x_{u,max}}{d}$

(ii) $P > P_{lim}$

(iii) $M_u > M_{u,lim}$

(iv) ORS is redesigned

or/ designed as balanced section by $x_u = x_{u,max}$

Analysis

18/5/21

Q1. Determine the depth of neutral axis of a beam 250mm x 400 mm, reinforced with 3 bars of 20mm diameter. Also check for the type of Section. Use M20 concrete and Fe 415 steel.

Ans:- Given data,

$b = 250mm$

$D = 400\ mm$

Reinforcement details = 3 bar @ 20mm dia

$A_{st} = \frac{\pi}{4} (20)^2 \times 3$

$= 942.5\ mm^2$

$M_{20} \Rightarrow f_{ck} = 20\ MPa$

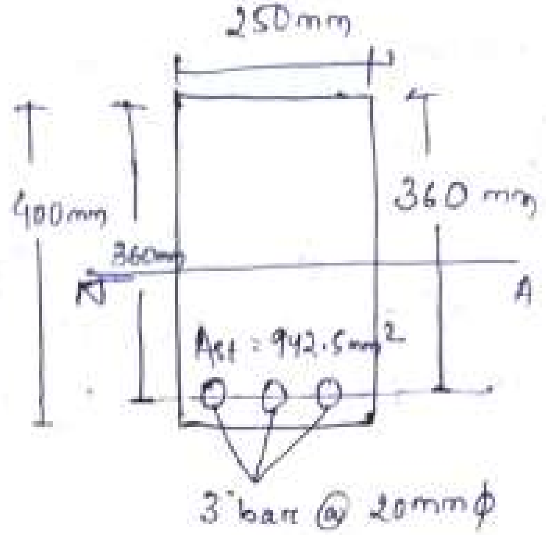
For Fe 415 → $f_y = 415\ MPa$

Let's Assume,

Clear cover = 30 mm

$d = 400 - 30 - \frac{20}{2}$

$= 360$ mm



Data required,

$x_u = ?$

$x_{u,max} = ?$

Calculate depth of NA :- $x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$

$= \frac{0.87 \times 415 \times 942.5}{0.36 \times 20 \times 250}$

$= 189.$ mm

Calculate limiting depth of NA :- For Fe 415

$\frac{x_{u,max}}{d} = 0.48$

$\Rightarrow x_{u,max} = 0.48d$

$= 0.48 \times 360$

$= 173$ mm

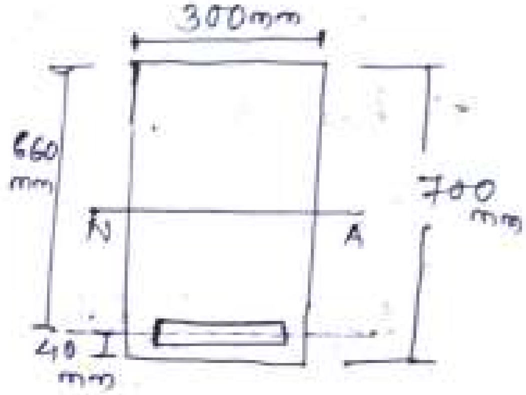
$x_u > X_{u,max}$.

Section is ORS.

21·5·21

Design

Q1. A reinforced concrete beam is 300mm × 700mm is subjected to a bending moment of 150 KNm. Determine the area of reinforcement if M_{20} concrete and Fe 415 steel is used. Take effective cover as 40mm.



Ans:- Given data,

$b = 300 \text{ mm}$

$D = 700 \text{ mm}$

Effective cover = 40mm

$d = 700 - 40 = 660 \text{ mm}$

$f_{ck} = 20 \text{ MPa}$, $f_y = 415 \text{ MPa}$

$M = 150 \text{ kN·m}$

Calculation of design moment :-

$M = 150 \text{ KN·m}$

$M_U = M \times \gamma_{load} = 150 \times 1.5 = 225 \text{ kN·m}$

Calculation of limiting moment of resistance :-

For, Fe 415,

$M_{u,lim} = 0.138 f_{ck} b d^2$

$= 0.138 \times 20 \times 300 \times 660^2$

$= 360.6 \text{ kN·m}$

OR

$$M_{u,lim} = 0.36 \frac{x_{u,max}}{d} \left(1 - \frac{0.42\, x_{u,max}}{d}\right) f_{ck} b d^2$$

$M_u < M_{u,lim}$

So, the section is URS.

Calculation of area of steel :-

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$\Rightarrow A_{st} \Rightarrow$

OR

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} bd^2}}\right) bd$$

$$= \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 225 \times 10^6}{20 \times 300 \times 660^2}}\right) \times 300 \times 660$$

$$= 1063 \text{ mm}^2.$$

Q2. A singly reinforced R.C.C. beam is subjected to a moment of 80 kNm. The width of the beam is 200mm. Calculate the depth of beam and area of steel reinforcement required for balanced design. Use M_{20} concrete and Fe 415 steel.

Doubly Reinforced Beams

Double reinforced beam :-

The R.C. beam in which steel reinforcement placed in tension as well as compression zone is called ~~BBB~~ double reinforced beam section.

$$M_u > M_{u,lim}$$

(DRS)

Necessity of doubly reinforced section

1. When the dimensions (bxd) of the beam are restricted due to any constraints like availability of head room, architectural or space consideration and the moment of resistance of singly reinforced section is less than the external moment.

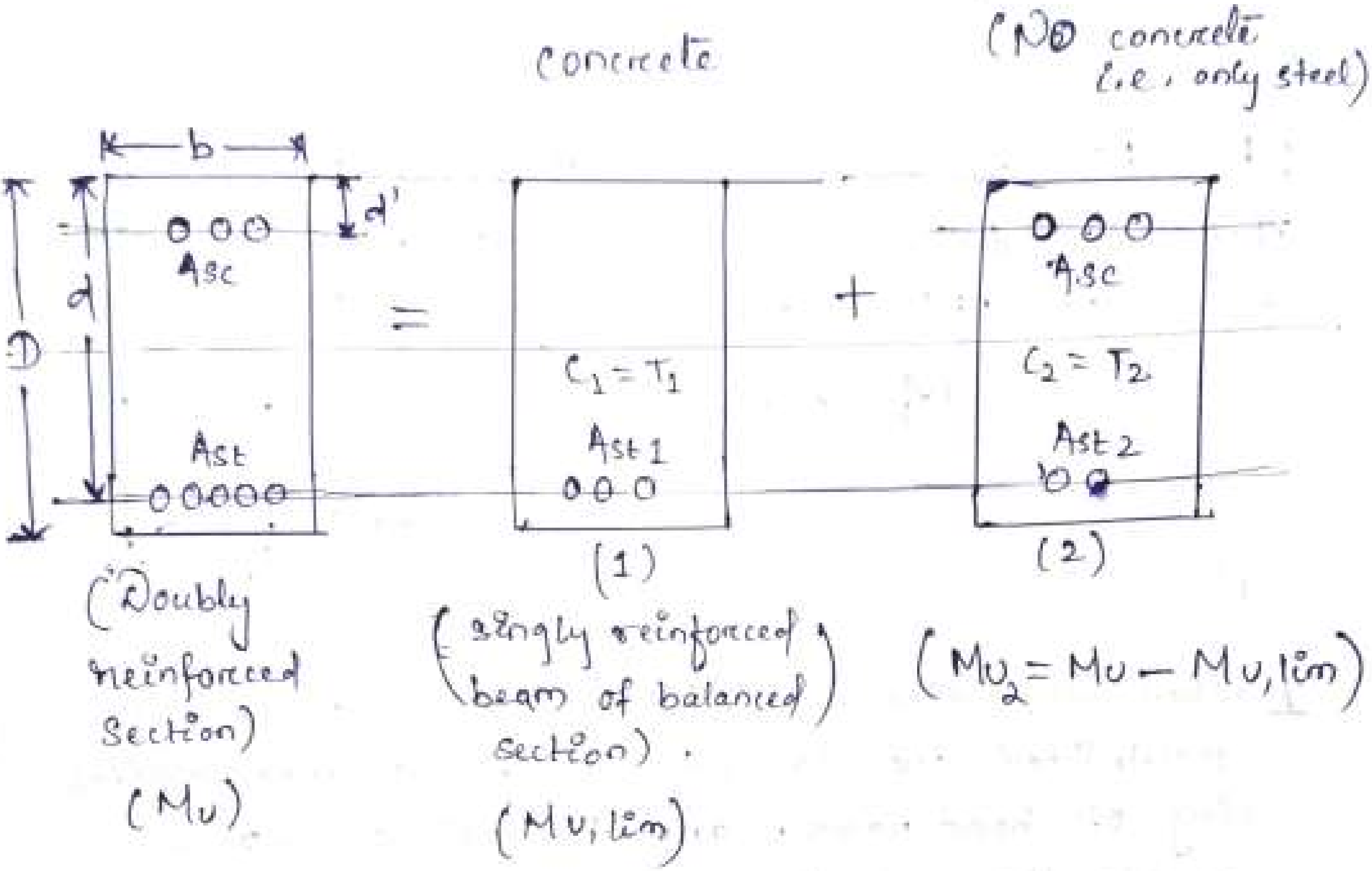
2. When the external loads may occur on either face of the member i.e. the ... are alternating or reversing and ma... on both faces of the member

3. When the loads are eccentric

4. When the beam is subjected ... or sudden lateral loads.

5. In the case of continuous beams ... the sections at supports are gen... designed as doubly reinforced section

Analysis of Double reinforced beam

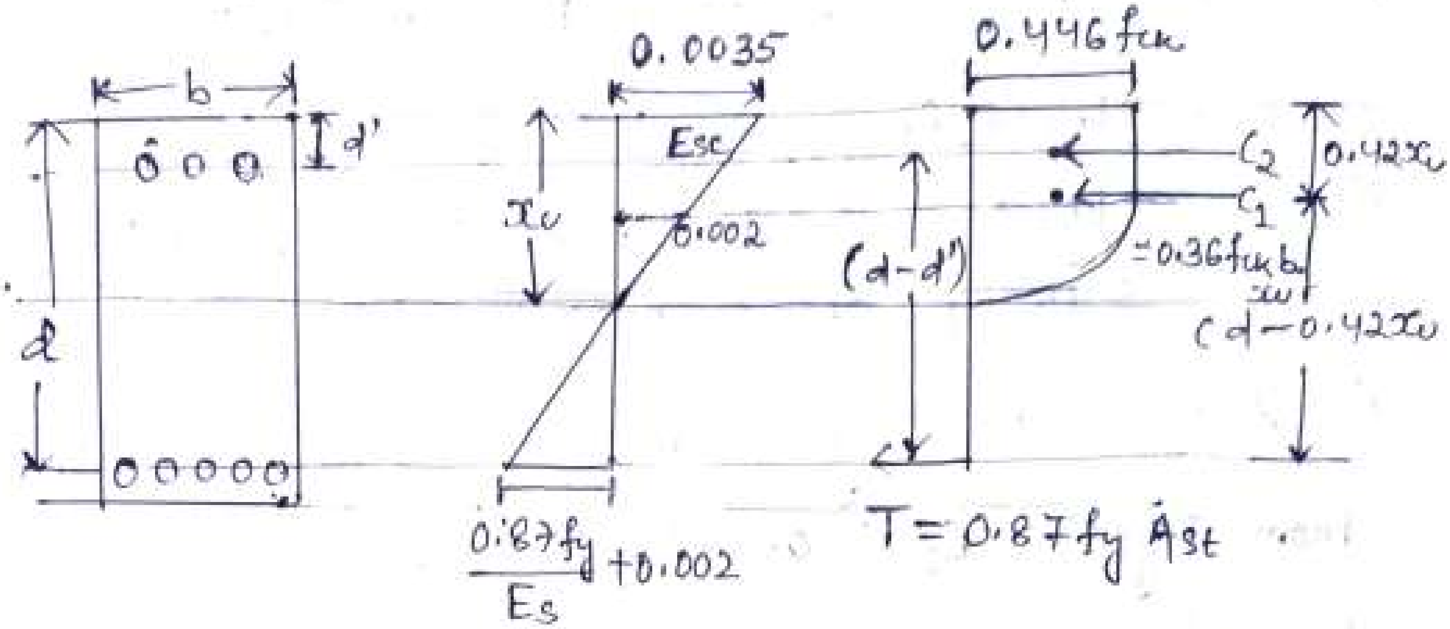


<u>Section 1</u> :- It consists of singly reinforced balanced section having area of steel Ast1 and moment Mu.

<u>section 2</u> :- It consists of compression steel Asc and additional tensile steel Ast2 correspond to Asc and moment of resistance is M_{u_2}. i.e. $(M_u - M_{u,lim})$.

$$M_u = M_{u,lim} + M_{u_2}$$

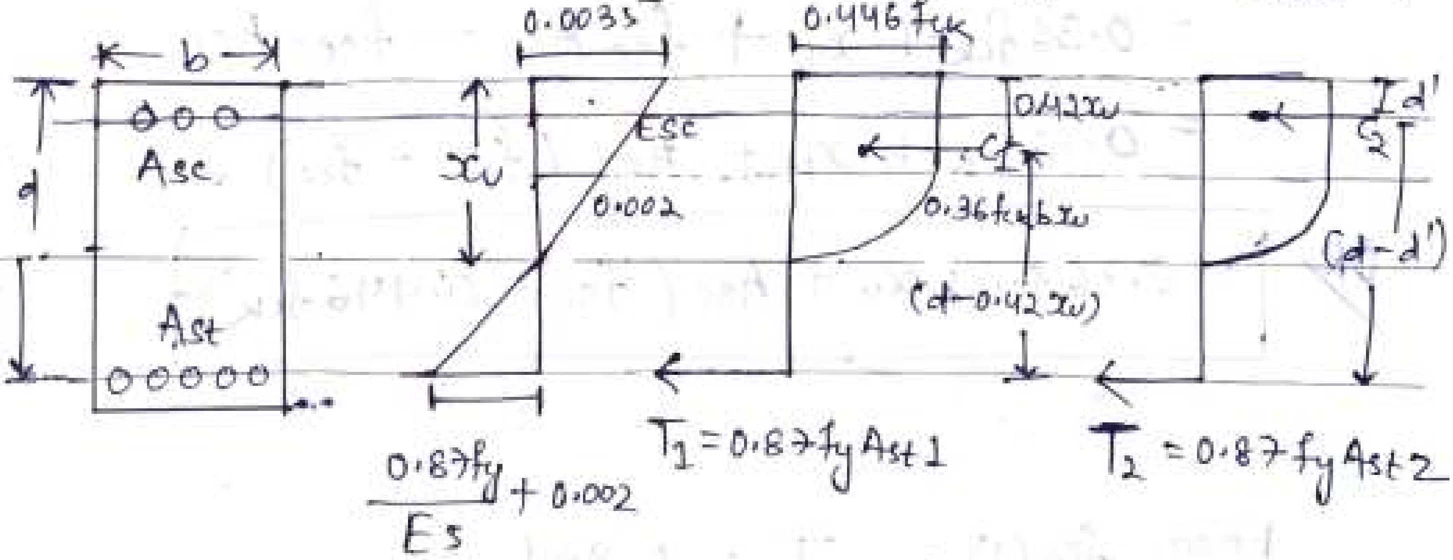
$$A_{st} = A_{st1} + A_{st2}$$



(strain diagram) (stress diagram)

$C_1 \rightarrow$ compressive force carried by concrete area

$C_2 \rightarrow$ Compressive force carried by compressive steel.



b → Width of beam

D → Overall depth of beam

d → Effective depth of beam

d' → Effective cover to compression steel.

Ast → Area of tension steel.

Asc → Area of compression steel.

ε_{sc} → Strain in concrete at level of compression steel.

f_{sc} → Stress in compression steel

f_{cc} → stress in concrete at the level of centroid of compression steel.

(1) Determination of compressive force

From fig(1) $C_1 = 0.36 f_{ck} \cdot b \cdot x_u$.

From fig(2) $C_2 = f_{sc} \cdot A_{sc} - f_{cc} \cdot A_{sc}$

$(f_{cc} = 0.446 f_{ck})$

$\sigma = \frac{F}{A}$

$\Rightarrow F = \sigma \cdot A$

$C = C_1 + C_2$

$= 0.36 f_{ck} \cdot b \cdot x_u + f_{sc} A_{sc} - f_{cc} \cdot A_{sc}$

$= 0.36 f_{ck} \cdot b \cdot x_u + A_{sc} (f_{sc} - f_{cc})$ ——— (I)

OR $= 0.36 f_{ck} b x_u + A_{sc} (f_{sc} - 0.446 f_{ck})$ ——— (II)

2) Determination of tensile force

From fig(1), $T_1 = 0.87 f_y A_{st1}$.

fig(2), $T_2 = 0.87 f_y A_{st2}$

$T = T_1 + T_2 = 0.87 f_y A_{st1} + 0.87 f_y A_{st2}$

$T = 0.87 f_y A_{st}$ ——— (III)

3) Calculation of depth of Neutral axis (x_u):-

$$C = T$$

$$0.36 f_{ck} b x_u + A_{sc}(f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st} - A_{sc}(f_{sc} - f_{cc})}{0.36 f_{ck} b} \quad \text{—— (IV)}$$

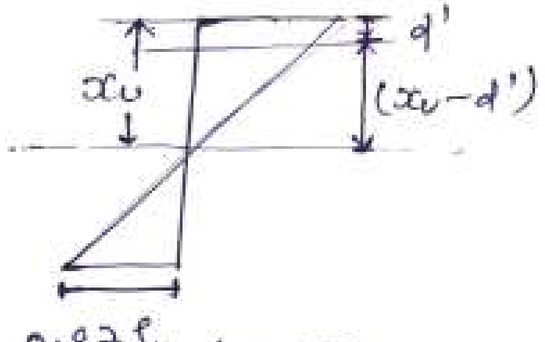
* The value of f_{cc} is very small with respect to f_{sc}.

So, f_{cc} is neglected.

$$x_u = \frac{0.87 f_y A_{st} - A_{sc} f_{sc}}{0.36 f_{ck} b} \quad \text{—— (V)}$$

4) Calculation of ε_{sc} :-

From strain diagram,



(Strain diagram)

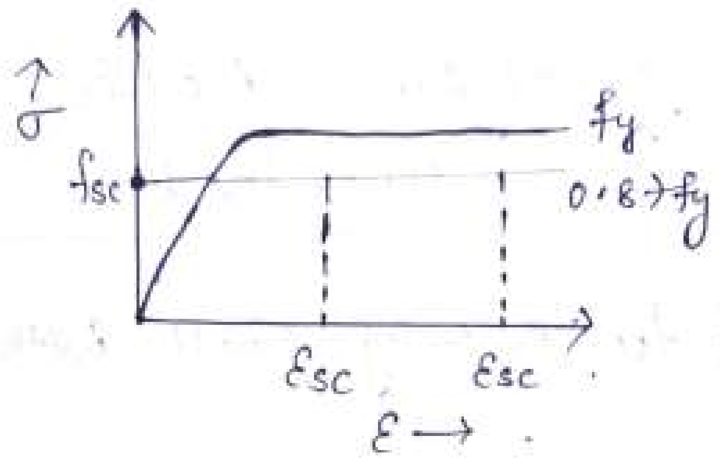
$$\frac{0.0035}{x_u} = \frac{\varepsilon_{sc}}{x_u - d'}$$

$$\Rightarrow \varepsilon_{sc} = \frac{0.0035(x_u - d')}{x_u}.$$

$$\Rightarrow \varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_u}\right) \quad \text{—— (VI)}$$

(5) Calculation of f_{sc} :-

1) For mild steel :-

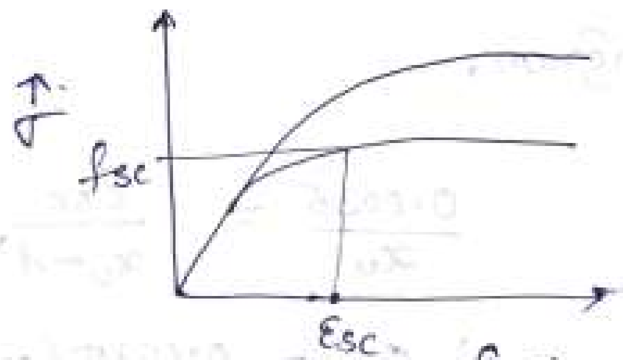


For mild steel $f_{sc} = 0.87 f_y$

$= 0.87 \times 250$

$= 217 \; N/mm^2$ ——— (7)

(II) For HYSD Bar :-



Grade of steel	d'/d			
	0.05	0.1	0.15	0.2
Fe 415	355	353	342	329
Fe 500	424	412	395	370

Calculation of moment of resistance (MOR):-

$$M_u = M_{u,lim} + M_{u_2} \quad \text{———} (8)$$

$$M_{u,lim} = C_1 \times \text{lever arm}$$
$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) \quad \text{———} (9)$$

$$M_{u_2} = C_2 \times \text{lever arm}$$
$$= C_2 (d - d')$$
$$= (f_{sc} A_{sc} - f_{cc} \cdot A_{sc})(d - d')$$
$$= A_{sc}(f_{sc} - f_{cc})(d - d') \quad \text{———} (10)$$

$$M_u = M_{u,lim} + M_{u_2}$$
$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc}(f_{sc} - f_{cc})(d - d') \quad \text{———} (11)$$

MOR in terms of tensile steel reinforcement :-

$$M_u = T \times \text{Lever arm}$$
$$= 0.87 f_y A_{st1}(d - 0.42 x_u) + 0.87 f_y A_{st2}(d - d') \quad \text{———} (12)$$

Calculation of Ast :-

(I) Calculation of Ast 1 :-

Ast1 resist $M_{u,lim}$

$$M_{u,lim} = 0.87 f_y A_{st1} (d - 0.42 x_{u,max})$$

$$\Rightarrow \boxed{A_{st1} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}}$$

(II) Calculation of Ast2 :-

$$M_{u2} = 0.87 f_y A_{st2} (d - d')$$

$$M_u - M_{u,lim} = 0.87 f_y A_{st2} (d - d')$$

$$\boxed{A_{st2} = \frac{M_u - M_{u,lim}}{0.87 f_y (d - d')}} \longrightarrow (14)$$

$$A_{st} = A_{st1} + A_{st2}$$

$$= \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})} + \frac{M_u - M_{u,lim}}{0.87 f_y (d - d')}$$

$$\longrightarrow (15)$$

Calculation of Asc :-

We provide Asc to resist $(M_u - M_{u,lim})$

From fig (2)

$$C_2 = T_2$$

$$A_{sc}(f_{sc} - f_{cc}) = 0.87 f_y A_{st2}.$$

$$\Rightarrow A_{sc}(f_{sc} - 0.446 f_{ck}) = 0.87 f_y A_{st2}.$$

$$\Rightarrow \boxed{A_{sc} = \frac{0.87 f_y A_{st2}}{f_{sc} - 0.446 f_{ck}}} \quad \text{——— (16)}$$

OR

$$M_{u_2} = C_2 \times (d - d')$$

$$= (f_{sc} A_{sc} - f_{cc} A_{sc})(d - d')$$

$$= A_{sc}(f_{sc} - f_{cc})(d - d')$$

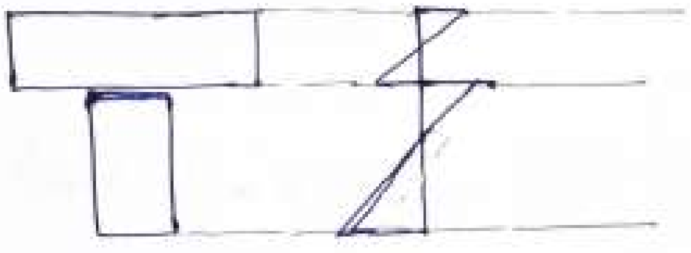
$$\Rightarrow A_{sc} = \frac{M_{u_2}}{(f_{sc} - f_{cc})(d - d')}$$

$$\boxed{= \frac{M_u - M_{u,lim}}{(f_{sc} - f_{cc})(d - d')}} \quad \text{——— (17)}$$

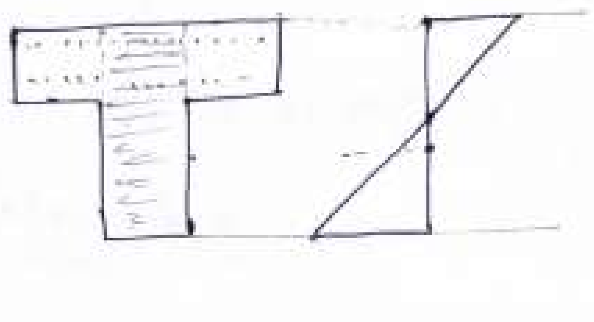
Flanged beam / T- beam.

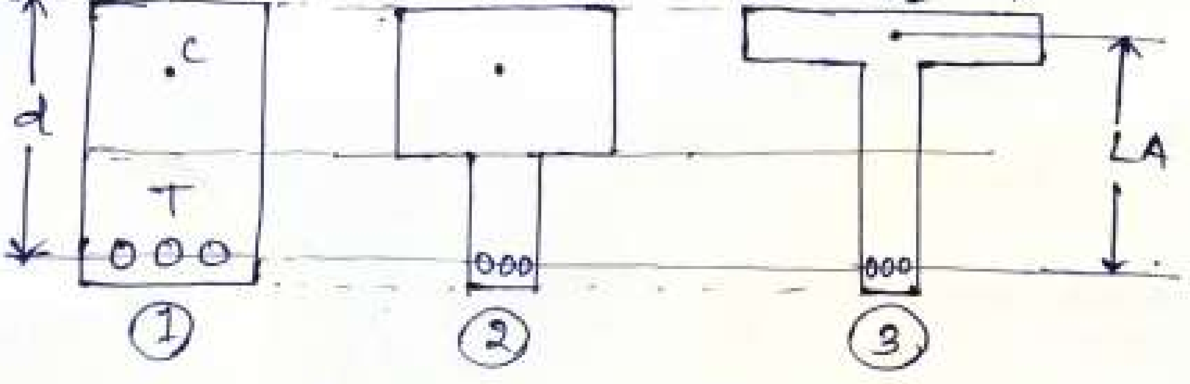
- In RCC Structure, slabs and beams are cast monolithically (at a time).
- The intermediate beams supporting the slab are called as T- beams and end beams are called L- beams. (Slab present on only one side)

Not cast - monolithically



casted monolithically



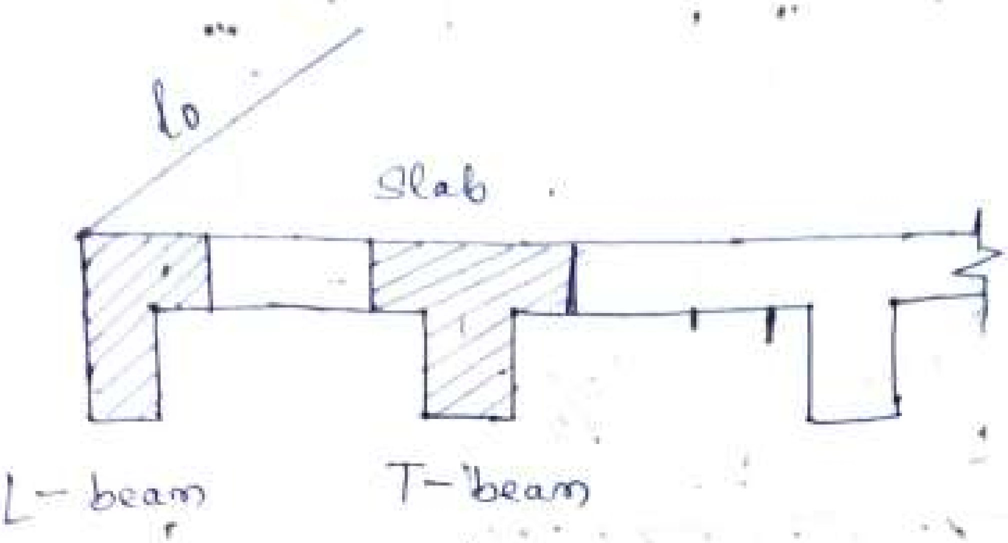


$A_1 > A_2 = A_3$.

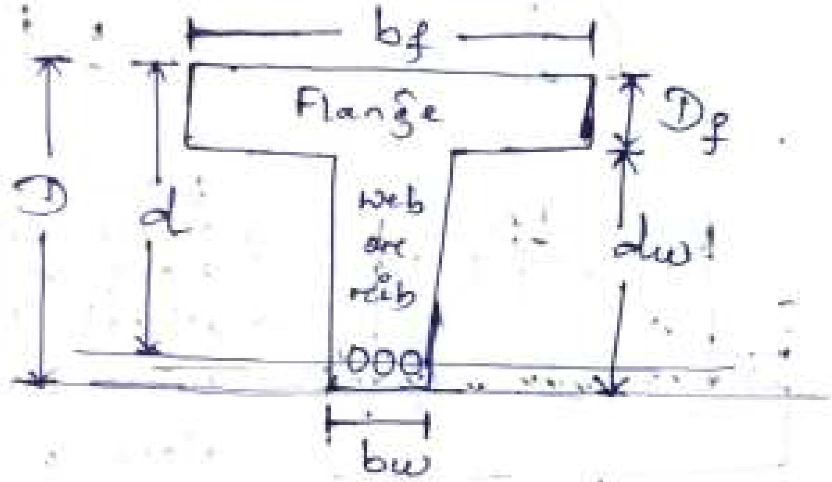
$LA_1 = LA_2 < LA_3$

$MOR_1 = MOR_2 < MOR_3$

$$\begin{pmatrix} MOR = C \times LA \\ MOR \propto LA \end{pmatrix}$$



<u>Terms</u> :-



b_f = Effective width of flange.

D_f = Depth or thickness of flange.

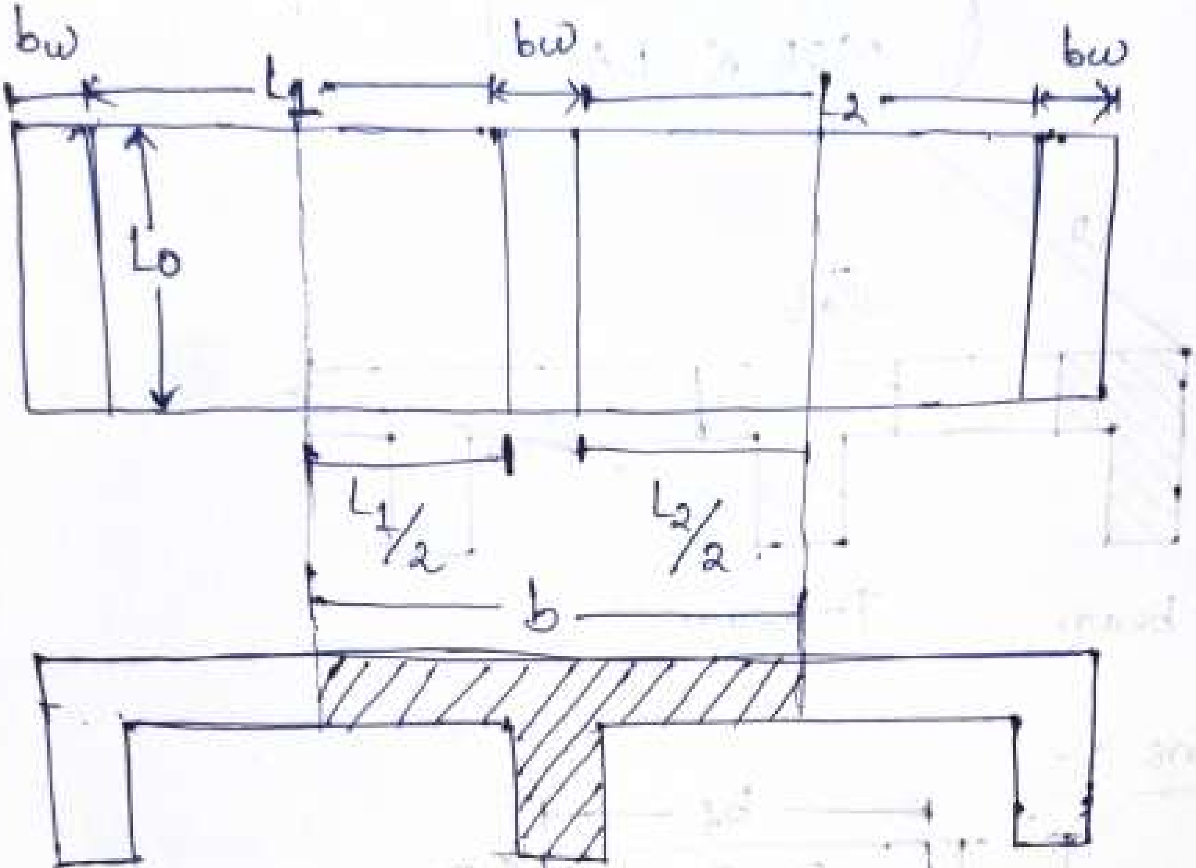
d_w = Depth of web.

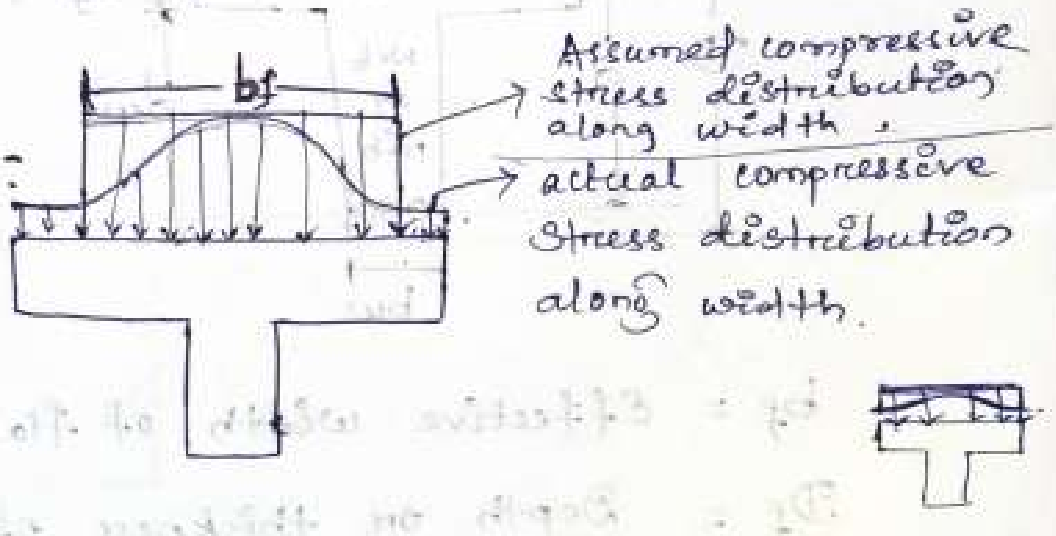
D = Overall depth of T beam

d = effective depth of T-beam

b_w = width of beam

L_1, L_2 = Clear distance between beams.





(Actual compressive stress = assumed compressive stress.)

b = actual width of T-beam

$= \frac{L_1}{2} + b_w + \frac{L_2}{2}$

b_f = effective width of the flange

$b_f \not> b$

i.e. $b_f = b$

Effective width of Flange (b_f)

1) Monolithic T-beam

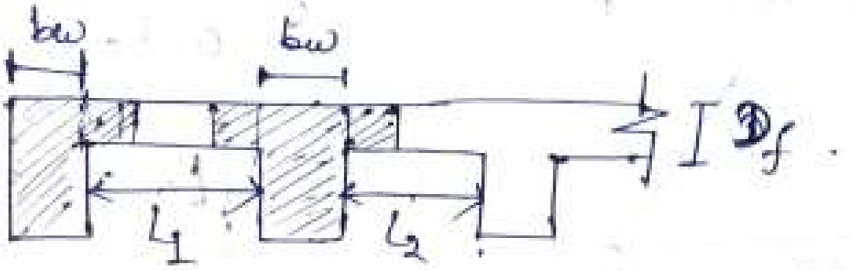
$$b_f = b_w + 6D_f + \frac{l_o}{6}$$

$$b_f \not> \frac{L_1 + L_2}{2} + b_w$$

2) Monolithic L-beam

$$b_f = b_w + 3D_f + \frac{l_o}{12}$$

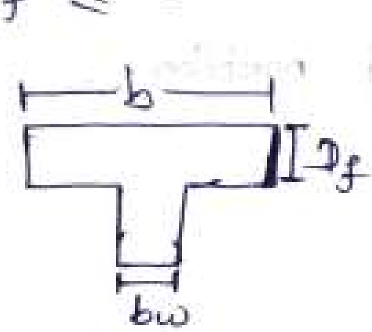
$$b_f \not> b_w + \frac{L_1}{2}$$



3) Isolated T-beam

$$b_f = b_w + \frac{l_o}{\left(\frac{l_o}{b} + 4\right)}$$

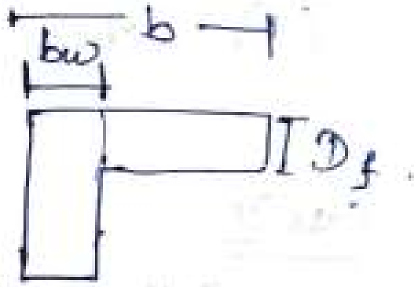
$$b_f \leq b$$



4) Isolated L-beam

$$b_f = b_w + \frac{0.5 l_o}{\left(\frac{l_o}{b} + 4\right)}$$

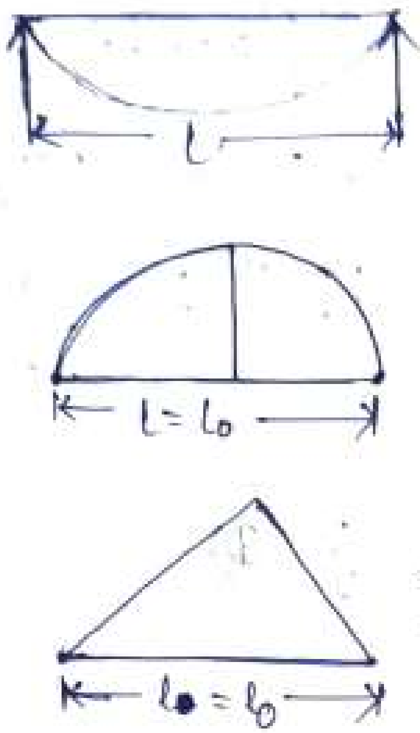
$$b_f \leq b$$



l_o → effective span (distance between point of contraflexure or zero moment).

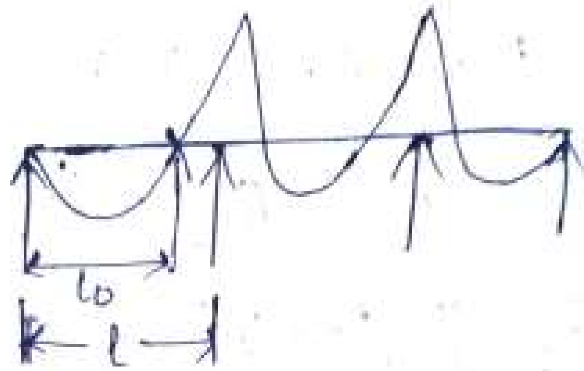
Pg-37

1) In simply supported beam



$\boxed{l_0 = l}$ in S.S.B.

11) In ~~combined~~ continuous beam



In continuous beam

$\boxed{l_0 = 0.7l}$

Analysis of T-beam / flanged beam (Annex – G)

Case – I

(1) when NA is in flanged portion

$(x_u < D_f)$

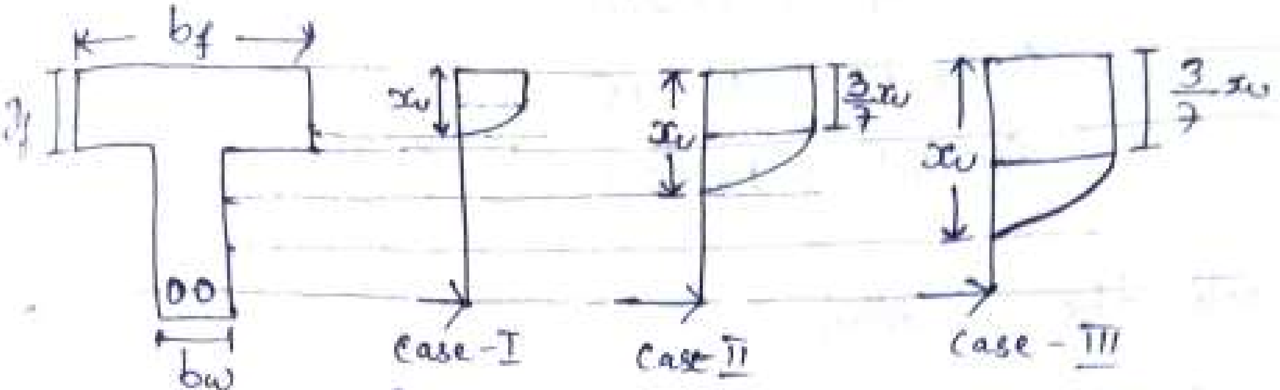
Case II

when NA is in web.

$(x_u > D_f)$

when $\frac{3}{7}x_u < D_f$

when $\frac{3}{7}x_u \geq D_f$

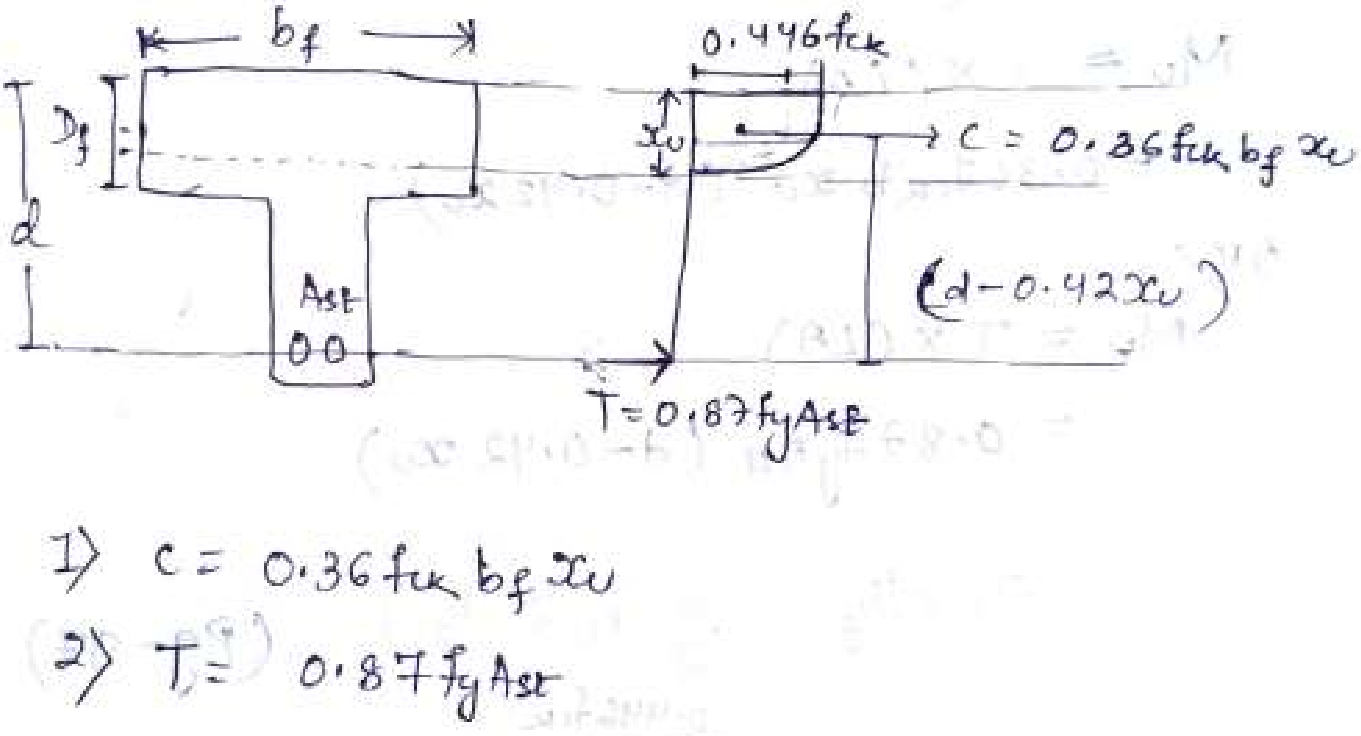


Case-I: $x_u < D_f$

Case-II: $x_u > D_f$, $\frac{3}{7}x_u < D_f$

Case-III: $x_u > D_f$, $\frac{3}{7}x_u \geq D_f$.

Case-I (when N-A is in flange) i.e. $x_u < D_f$



1) $C = 0.36 f_{ck} b_f x_u$

2) $T = 0.87 f_y A_{st}$

3) $C = T$

$$0.36 f_{ck} \cdot b_f \cdot x_u = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b_f \cdot}$$

4) $x_{u,max} = 0.53d \Rightarrow Fe\,250$

$= 0.48d \Rightarrow Fe\,415$

$= 0.46d \Rightarrow Fe\,500$

5) Check the section

$x_u < x_{u,max} \Rightarrow$ URS (Take x_u)

$x_u = x_{u,max} \Rightarrow$ BS (Take $x_{u,max}$)

$x_u > x_{u,max} \Rightarrow$ ORS (Take $x_{u,max}$)

6) Find MOR

$M_u = C \times (LA)$

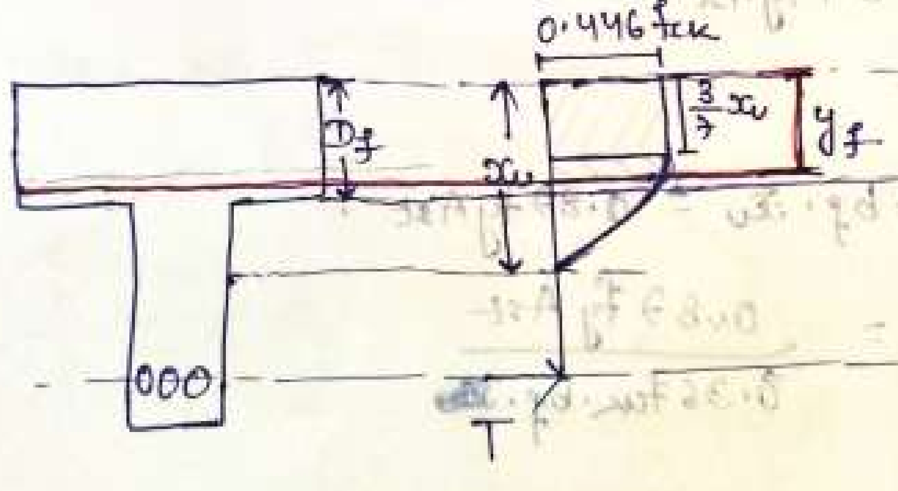
$= 0.36 f_{ck} b\, x_u\, (d - 0.42 x_u)$

or/

$M_u = T \times (LA)$

$= 0.87 f_y A_{st}\, (d - 0.42 x_u)$

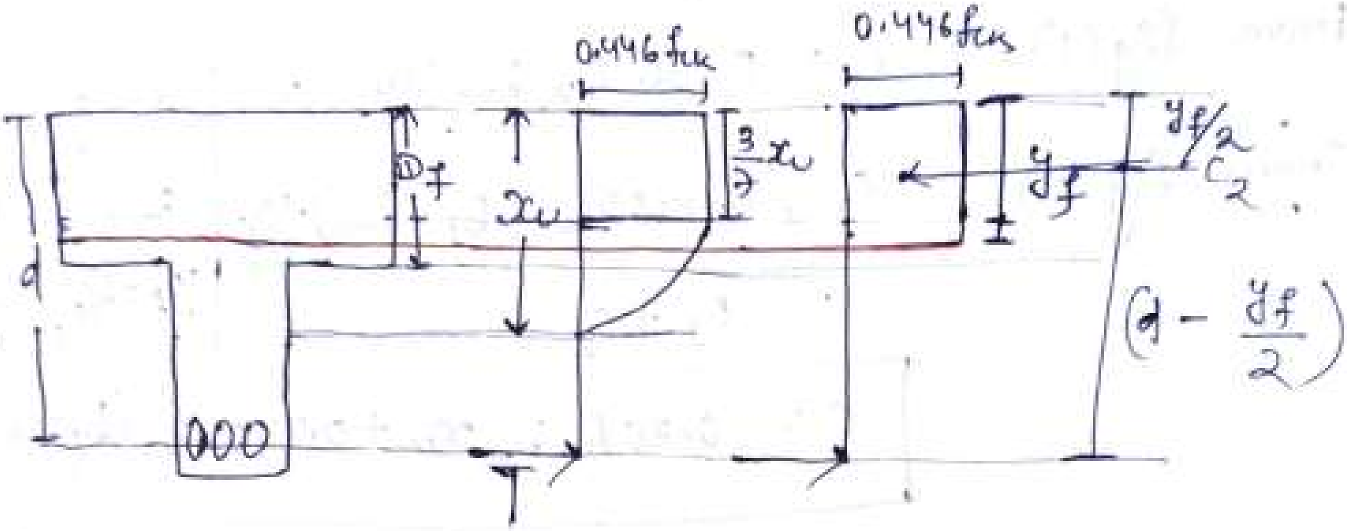
Case - II ($x_u > D_f$ & $\frac{3}{7} x_u < D_f$) (Pg-97)



As it is a trunkated parabola, it can't be analysis.
so, uniformly stress block is taken.

Why $y_f < D_f$?

If we subtract AB from BC
Then, $y_f < D_f$.

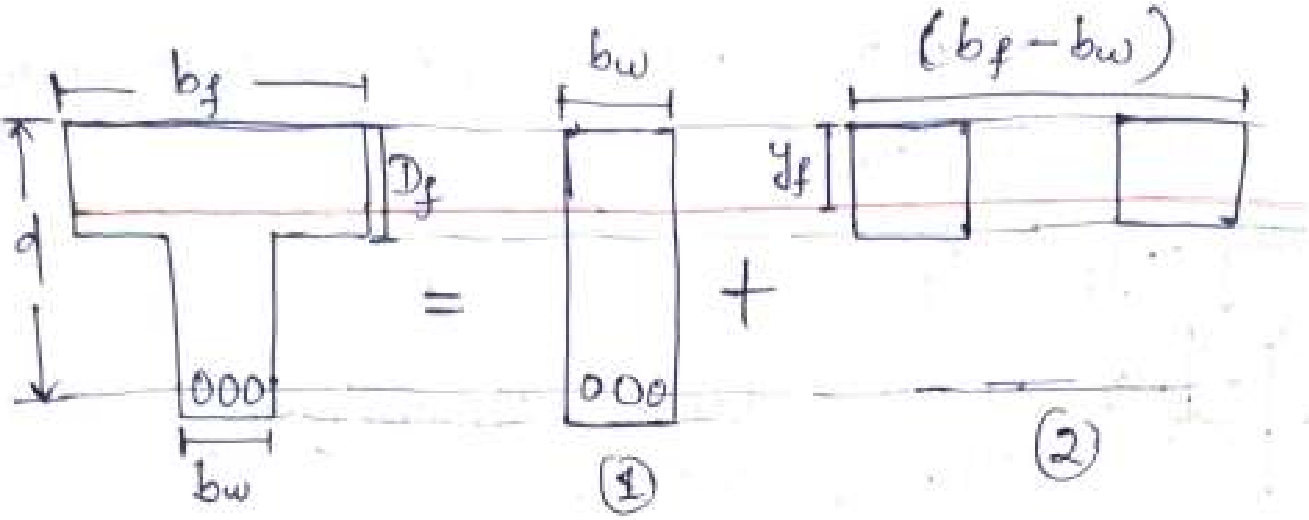


$$y_f = 0.15x_u + 0.65D_f$$

$(y_f \ngtr D_f)$

y_f = depth of equivalent rectangulare stress block..

i.e. y_f is not greater than D_f.



From fig (1) = $C_1 = 0.36 f_{ck} b_w x_u$

from fig (2) $C_2 = 0.446 f_{ck} (b_f - b_w) \cdot y_f$

$\sigma = \frac{F}{A}$

$F = \sigma \times A$

y_f = b/c C lies on y_f.

$C = C_1 + C_2$

$$C = 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f \quad \text{—(I)}$$

$$T = 0.87 f_y A_{st} \quad \text{—(II)}$$

$C = T$

$$0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w)(0.15 x_u + 0.65 D_f) = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 f_{ck} b_w x_u + 0.067 f_{ck} b_f x_u + 0.29 f_{ck} b_f D_f - 0.67 f_{ck} b_w x_u - 0.29 f_{ck} b_w D_f = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 f_{ck} b_w x_u + 0.067 f_{ck} x_u (b_f - b_w) + 0.29 f_{ck} D_f (b_f - b_w) = 0.87 f_y A_{st}.$$

$$x_u = \frac{0.87 f_y A_{st} - 0.29 f_{ck} D_f (b_f - b_w)}{0.36 f_{ck} b_w + 0.067 f_{ck} (b_f - b_w)} \quad \text{—(iii)}$$

$$M_u = C \times (LA)$$

$$= [C_1 \times (LA)_1] + [C_2 \times (LA)_2]$$

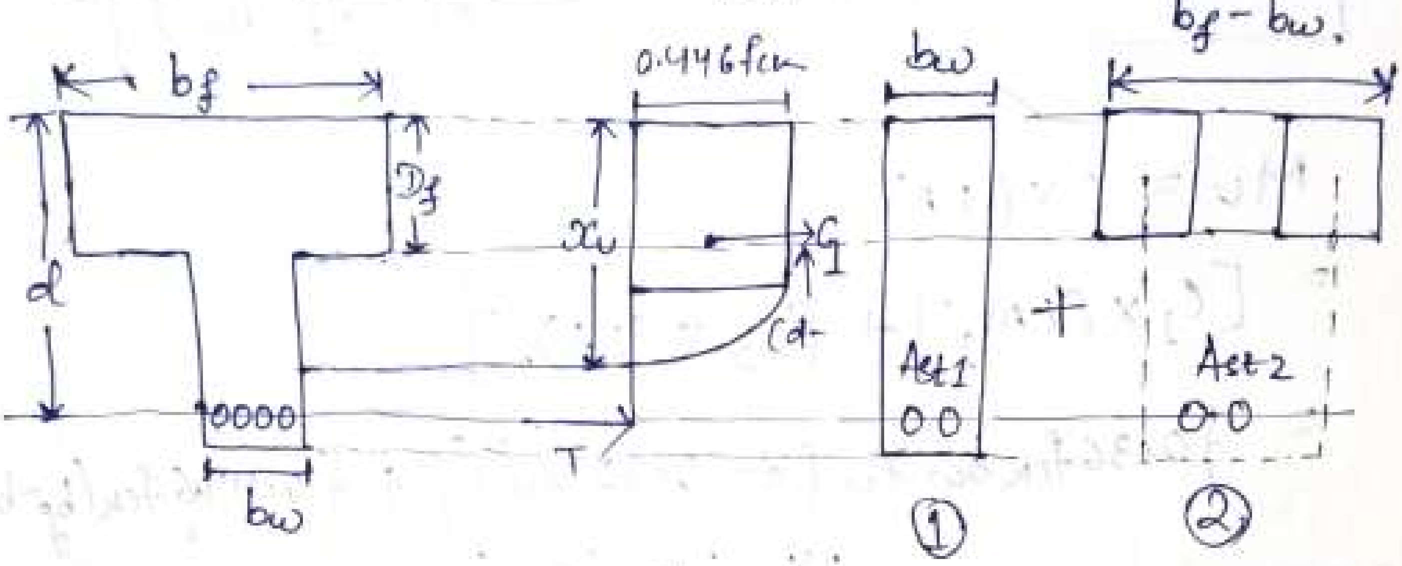
$$= \{0.36 f_{ck} b_w x_u (d - 0.42 x_u)\} + \{0.446 f_{ck} (b_f - b_w) \cdot y_f (d - \frac{y_f}{2})\}$$

$$x_i \rightarrow x_{u,max}$$

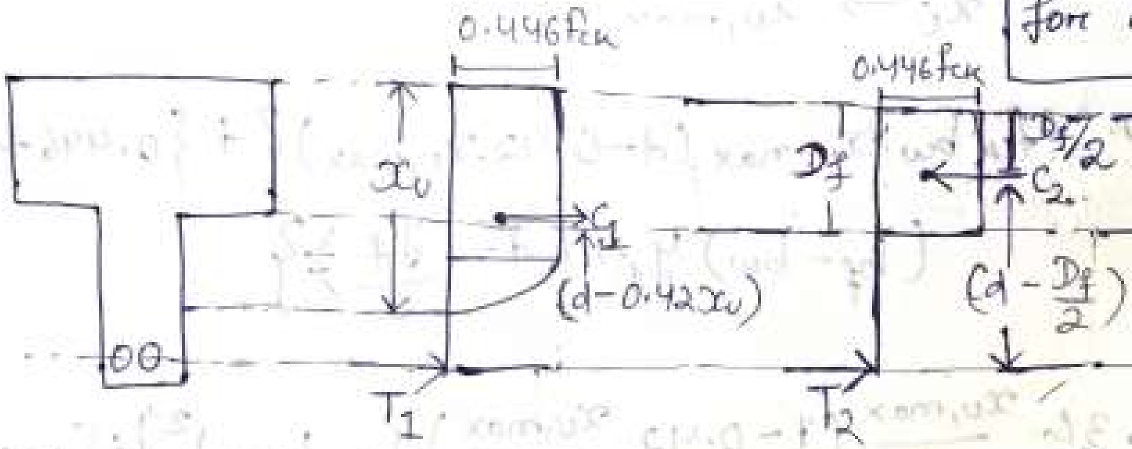
$$M_u = \{0.36 f_{ck} b_w x_{u,max} (d - 0.42 x_{u,max})\} + \{0.446 f_{ck} (b_f - b_w) y_f (d - \frac{y_f}{2})\}$$

$$M_u = \left[0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d}\right) f_{ck} b_w d^2\right] + \left[0.446 f_{ck} (b_f - b_w) y_f \cdot (d - \frac{y_f}{2})\right] \quad \text{—(v)}$$

Case - III ($x_u > D_f$ and $\frac{3}{7} x_u \geq D_f$)



dotted line for no concrete



From fig (1) $C_1 = 0.36 f_{ck} b_w x_u$.

(Pg-96)

From fig (2). $C_2 = \sigma \cdot A$

$= 0.446 f_{ck} (b_f - b_w) \cdot D_f$.

$$C = C_1 + C_2$$
$$= 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) \cdot D_f \quad \text{—— (1)}$$

$$T_1 = 0.87 f_y A_{st1}$$

$$T_2 = 0.87 f_y A_{st2}$$

$$T = T_1 + T_2$$

$$= 0.87 f_y (A_{st1} + A_{st2})$$

$$\boxed{= 0.87 f_y A_{st}} \quad \text{——— (ii)}$$

$$C = T.$$

$$\Rightarrow 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) \cdot D_f = 0.87 f_y A_{st}$$

$$\Rightarrow \boxed{x_u = \frac{0.87 f_y A_{st} - 0.446 f_{ck} (b_f - b_w) \cdot D_f}{0.36 f_{ck} b_w}} \quad \text{——— (iii)}$$

$$M_u = C \times LA$$

$$= \{C_1 \times (LA)_1\} + \{C_2 \times (LA)_2\}.$$

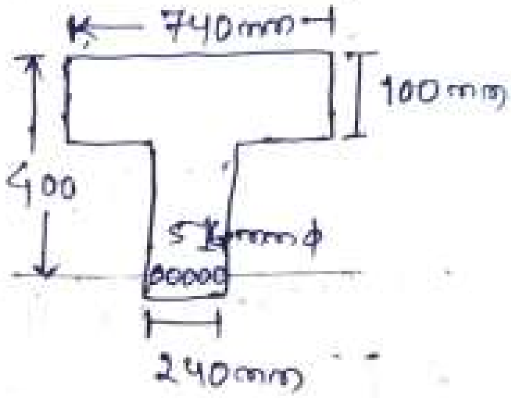
$$= [0.36 f_{ck} b_w x_u (d - 0.42 x_u)] + [0.446 f_{ck} D_f (b_f - b_w) \cdot (d - \frac{D_f}{2})]$$

$$x_u \longrightarrow x_{u,max}$$

$$\boxed{= 0.36 \frac{x_{u,max}}{d} \left(1 - 0.42 \frac{x_{u,max}}{d}\right) f_{ck} \cdot b_w d^2 + 0.446 f_{ck} D_f (b_f - b_w)\left(d - \frac{D_f}{2}\right)} \quad \text{—— (iv)}$$

Example 1 :- Find the moment of resistance of a T-beam having a web width of 240 mm, effective depth of 400 mm, effective flange width of 740 mm and flange thickness equal to 100 mm. The beam is reinforced with 5-16 mm diameter, Fe 415 bars. Use M20 concrete.

Ans:- Given data,



$b_w = 240$ mm

$d = 400$ mm

$b_f = 740$ mm

$D_f = 100$ mm

5-16mm ϕ bar

$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3 \text{ mm}^2$

$f_{ck} = 20 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$

Lets assume the N.A fall in the flange

(i.e. $x_u < D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 1005.3}{0.36 \times 20 \times 740}$$

$$= 68.1 \text{ mm} < 100$$

$\therefore x_u < D_f$

So, our assumption is correct

⇒ NA lies in the flange.

3) Check the section.

For Fe 415, $x_{u,max} = 0.48d$

$= 0.48 \times 400$

$= 192$ mm

$x_u < x_{u,max}$ (URS)

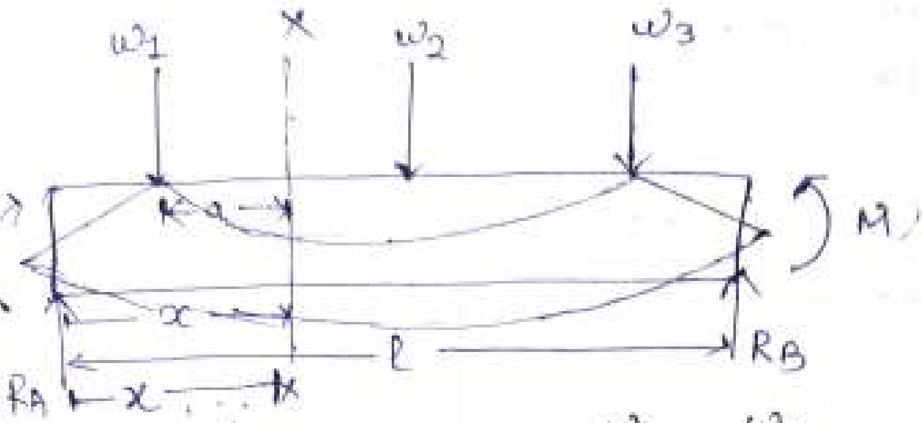
4) Cal. of Moment of resistance :-

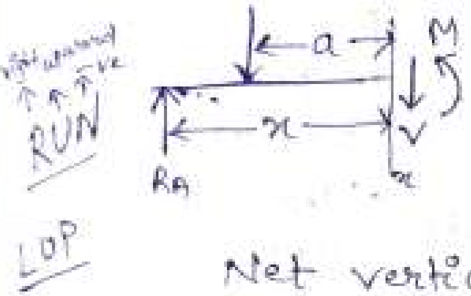
$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b_f d}\right)$$

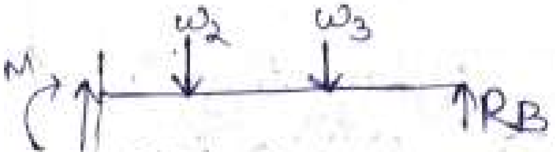
$$= 0.87 \times 415 \times 1005.3 \times 400 \left(1 - \frac{1005.3 \times 415}{20 \times 740 \times 400}\right)$$

$= 134.95$ kN·m

Sheare, bond and development length (LSM)







$\Sigma F_V \big|_{RHS} = (+w_2 + w_3) - R_B$ (↑|↓ | ↓|↑)

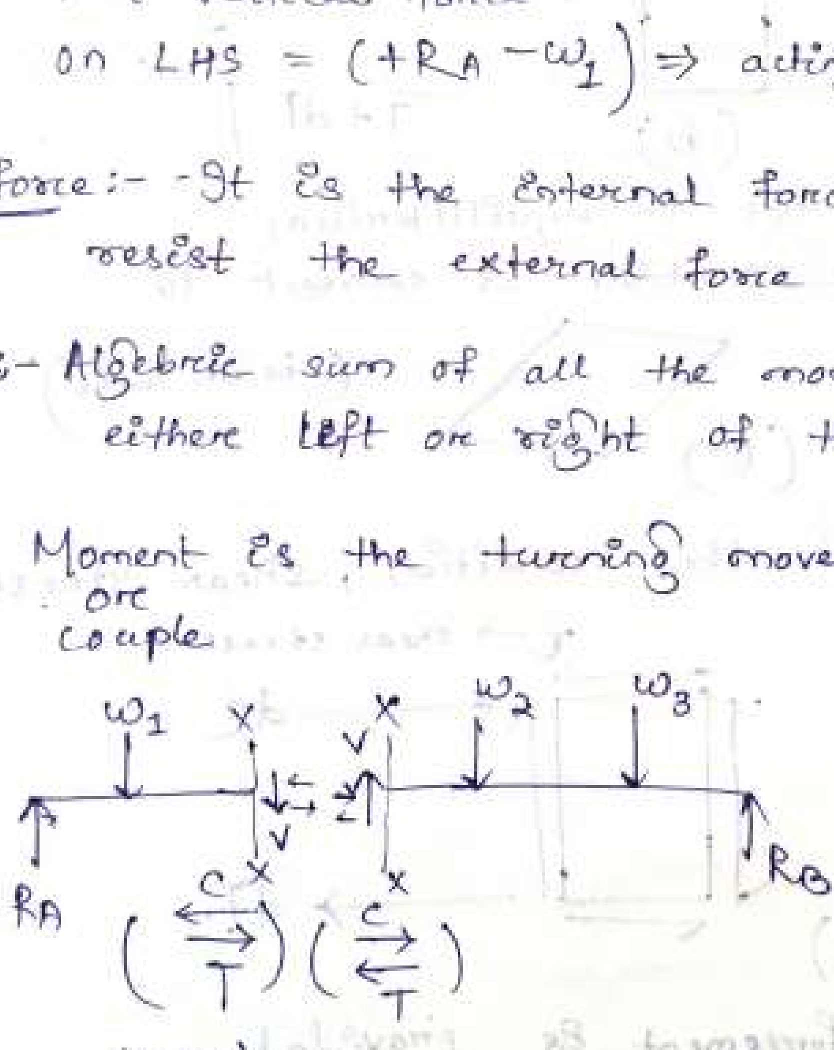
⇒ net force acting downward.

Net vertical force on LHS $= (+R_A - w_1)$ ⇒ acting upward direction

Shear force :- It is the external force which resist the external force.

Bending :- Algebric sum of all the moment acting either left or right of the section.

*. Moment is the turning movement of a body.
or
couple



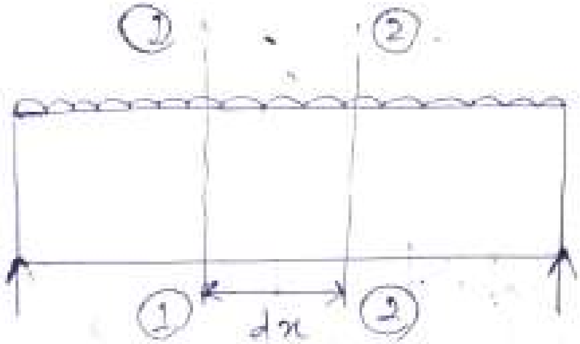
Moment

Relation betw' shear force & bending moment.

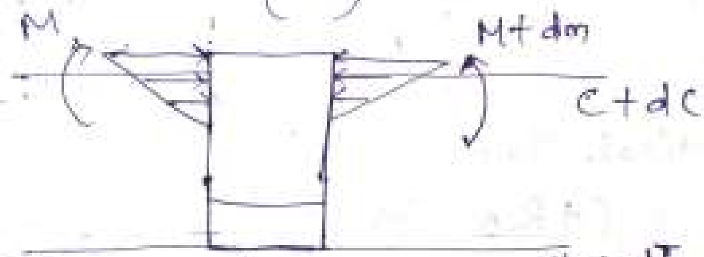
Rate of change of bending moment = Shear force

$$\frac{dm}{dx} = V$$

Sheare force vary with change in bending moment.



(a)



(b)

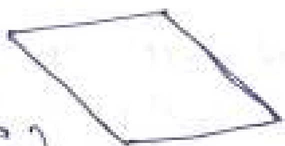
∵ Bending moment is not same in every point.

~~→ value incre.~~

→ value increase upto peak.

dC & dT extra force.

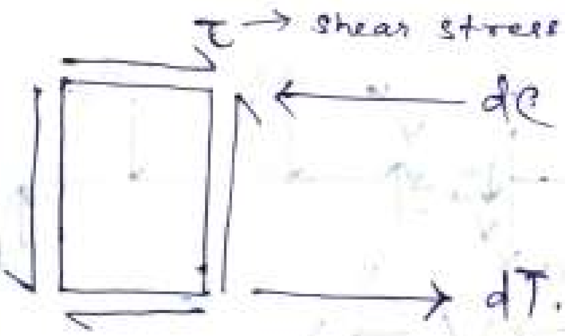
As it is not in equillibrium then, the section is convert to



(beam fail)

(c)

To countaract this condition, shear stress is develop.



(d)

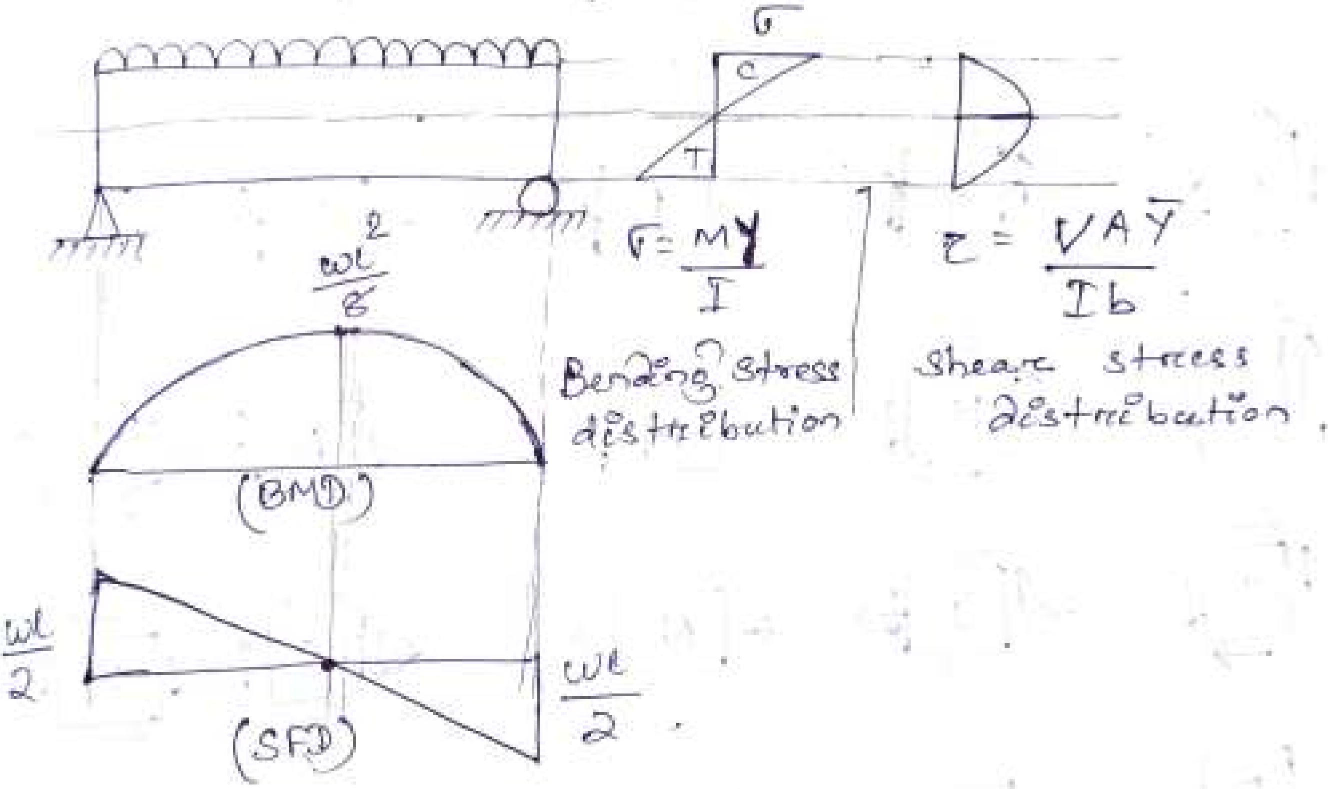
Sheare reinforcement is provided

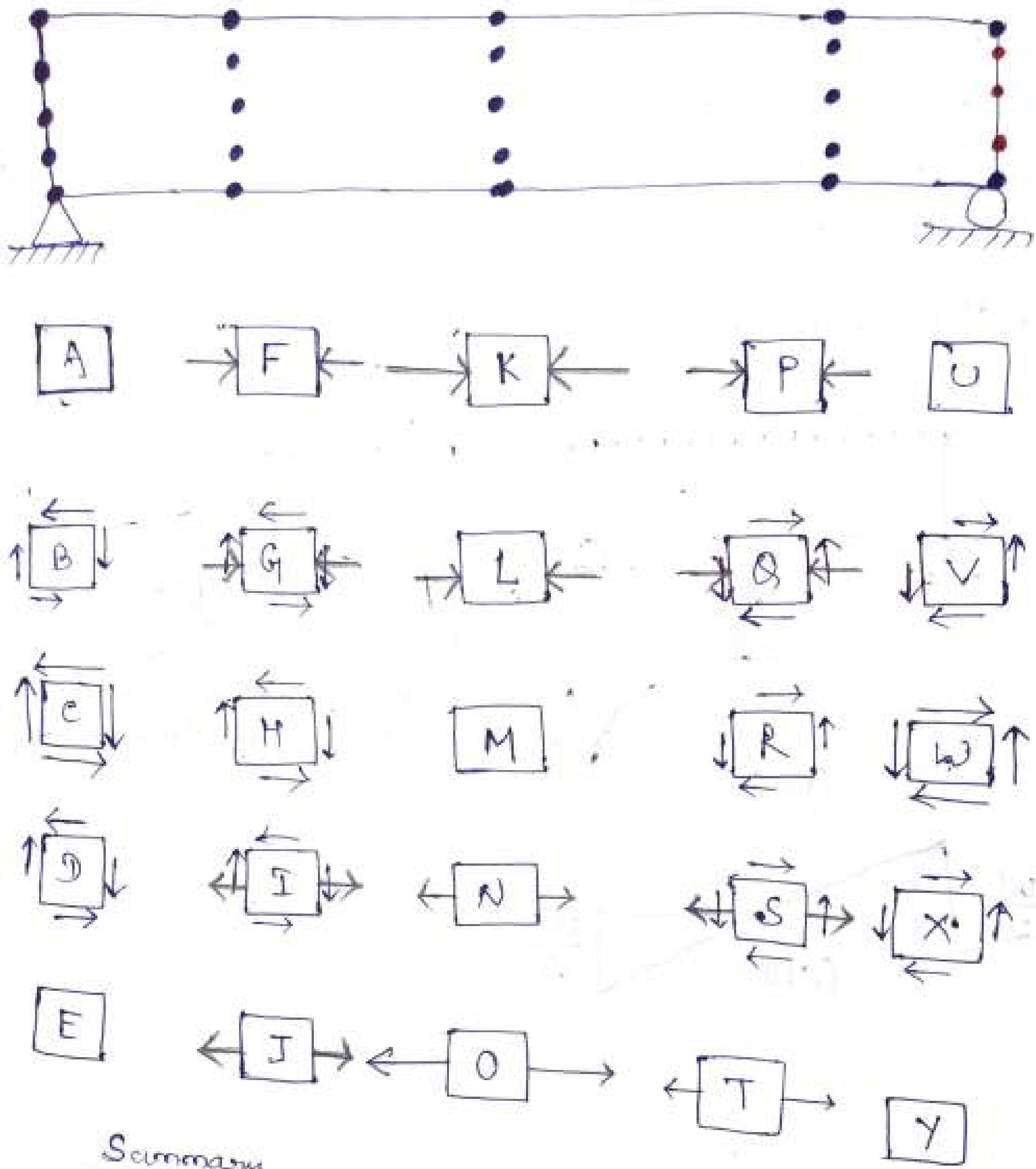
- Shear is maximum in support but zero in centre
- Bending is maximum is centre and zero in support.
(length wise) ↗

top ↙ ↘ bottom

depth wise
Centre → bending stress 'zero'.

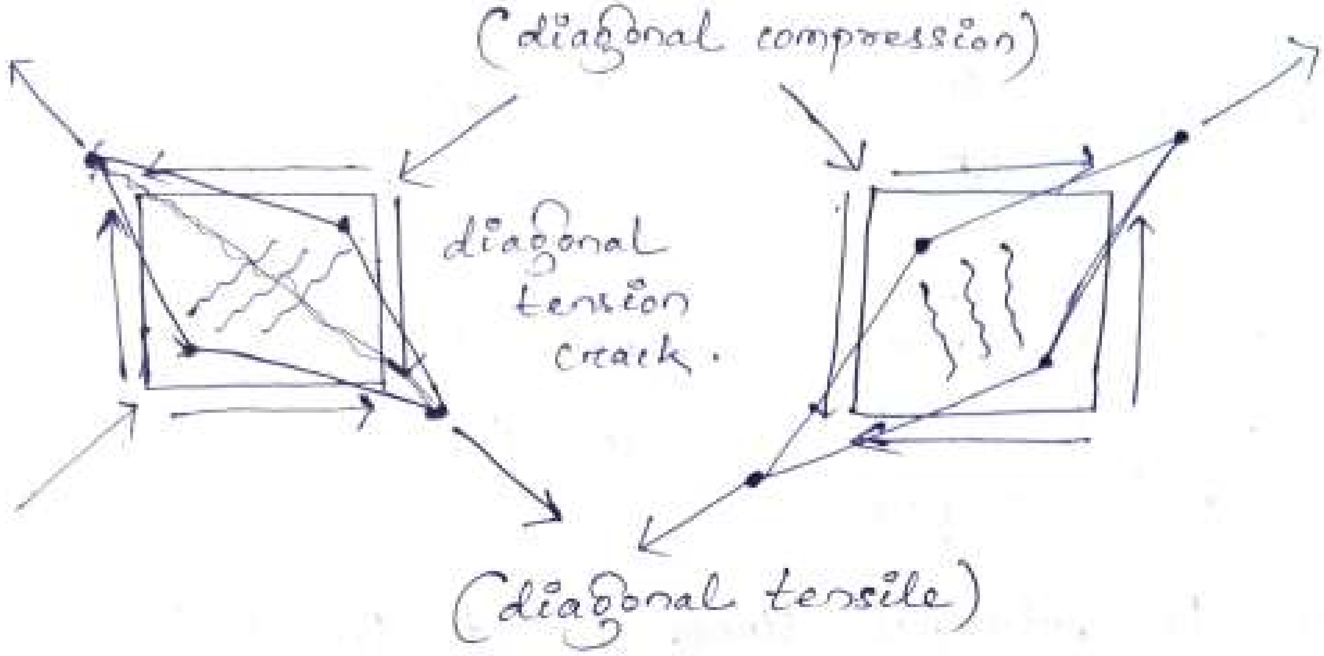
* The crack is always $\perp^{r}$ to principal stress |.



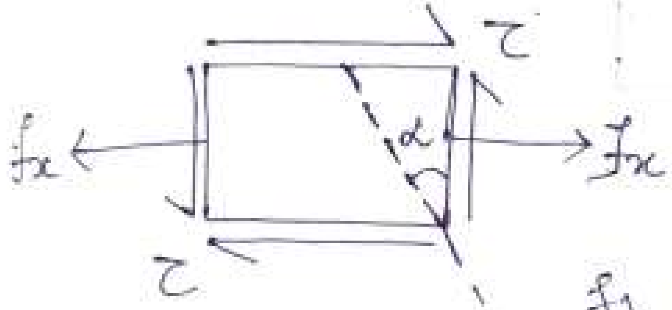


Summary

1) Points of no stress = A, E, M, U, Y.
2) Points of maximum shear stress (τ_{max}) = C, W
3) Points of maximum bending tensile stress = O.
4) Points of maximum bending compressive stress = K.

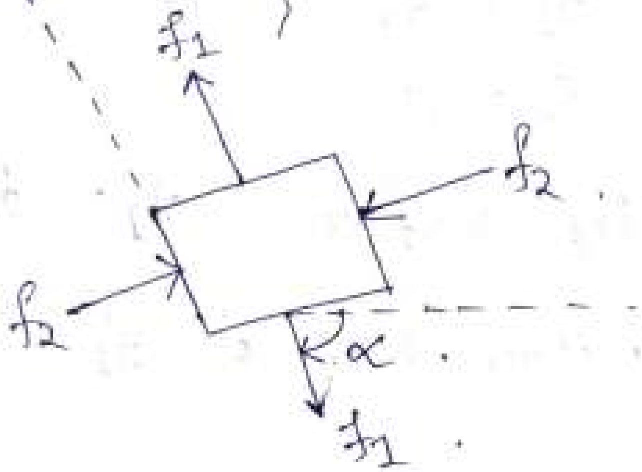


Failure Mechanism



$$f_{1,2} = \frac{f_x}{2} \pm \sqrt{\left(\frac{f_x}{2}\right)^2 + \tau^2}$$

$$\tan 2\alpha = \frac{2\tau}{f_x}$$



1) At top and bottom fibre :-

$$\tau = 0$$

$$\tan 2\alpha = \frac{0}{f_x} \Rightarrow \tan 2\alpha = 0.$$

$$= \tan 0^\circ$$

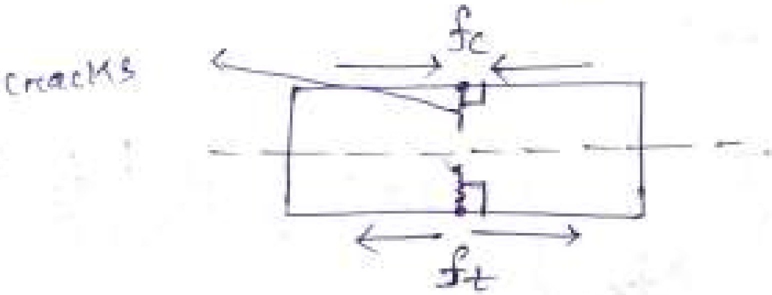
$$\Rightarrow \boxed{\alpha = 0}$$

$\tan 2\alpha = \frac{0}{f_x} = 0 \Rightarrow \tan 2\alpha = \tan 180^\circ$

$\Rightarrow \boxed{\alpha = 90^\circ}$

$\Rightarrow$ i.e. one principal stress is in a direction parallel to the surface & other is perpendicular to the surface.

* The principal stress $\perp^{rl}$ to the surface $= 0$.



So. $f_1 = f_x$.

$f_2 = 0$.

At top surface $f_1 = f_c$.

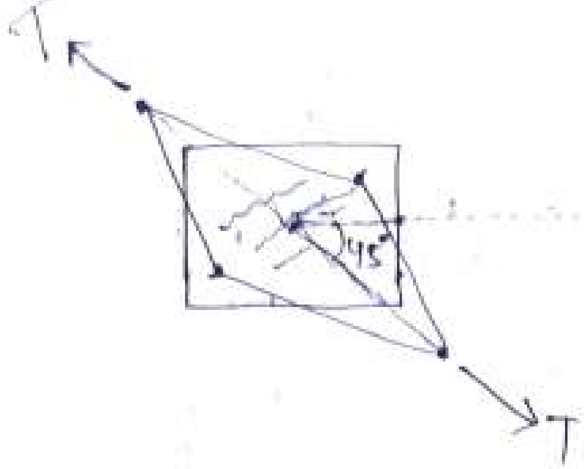
bottom surface $f_1 = f_t$.

<u>At middle fible or N.A</u>

($f_x = 0$, $\tau = max.$

$\tan 2\alpha = \frac{2\tau}{f_x}$

$= \frac{2\tau}{0} \Rightarrow \alpha = 45^\circ$.



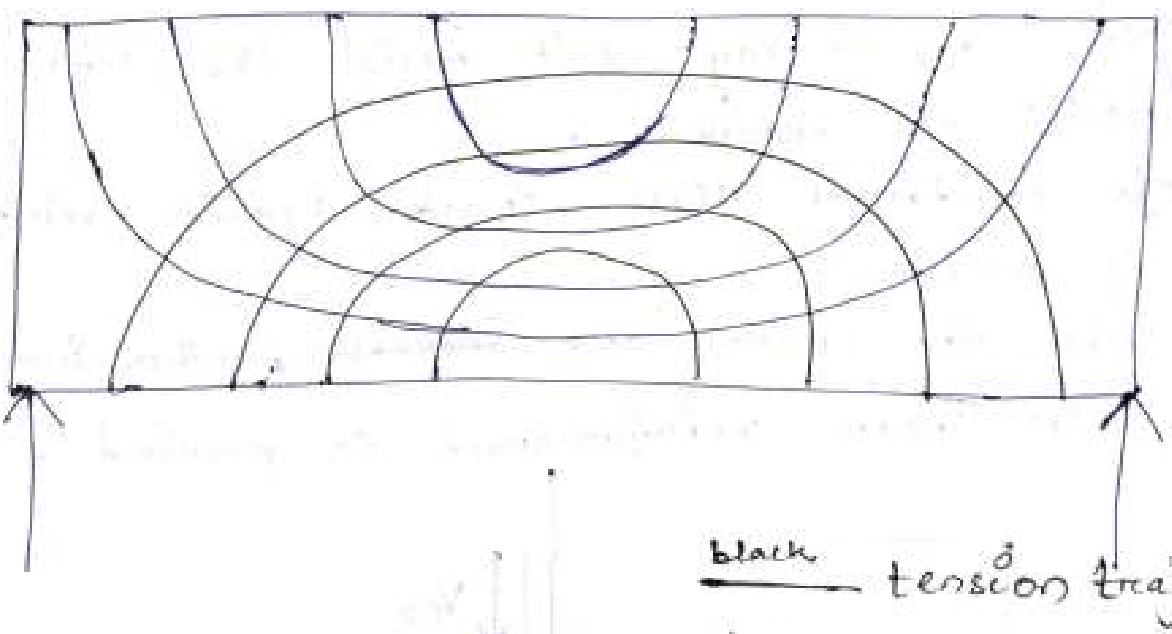
Principal stress trajectory :-

Middle - 45° → shear reinforcement.
(Either vertical, inclined, bent of bar (∠))

Top, bottom → 90° → Vertical.

Top → both doubly & singly reinforcement.

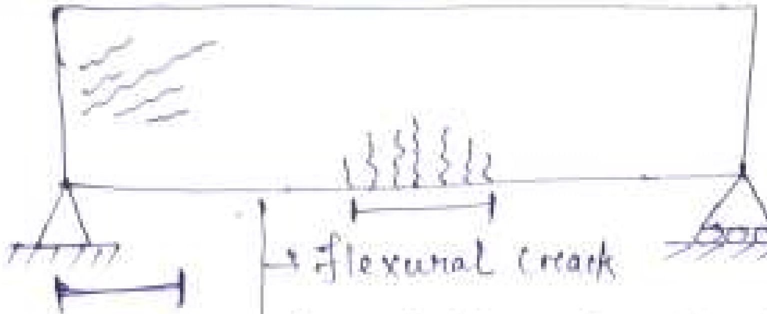
bottom → singly reinforcement.

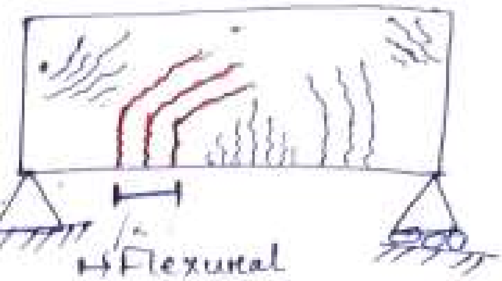


black — tension trajectories

blue — compression trajectories.

Crack Pattern :-





↦ Web shear crack.
→ 45° to horizontal
→ SF ↑ max.
B·M = 0.

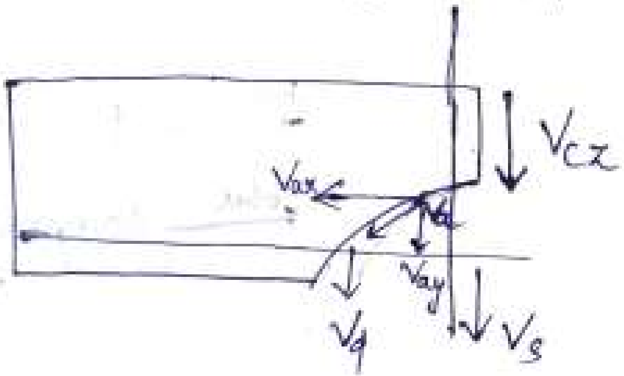
↦ flexural crack
→ 90° from horizontal
→ B·M ↑ maxi
~~S·F ↓ min~~.
S.F = 0 .

↦ Flexural shear crack.
↦ Both BM & SF Present.

Shear transfer mechanism :-

↦ When load act at the beam, ~~the~~ first the load carried by uncracked part.
(V_{cz})

↦ Aggregate resists the interlocking force. shear force is vertical.
'Vay' component resist the shear stress.
When, V_{cz} & Vay can't resist the stress, cracks are formed.
To countaract these cracks tensile reinforcement is provide.
When the cracks are ~~increases~~, further increases then shear reinforcement is provided.



$$V = V_{cz} + V_{ay} + V_d + V_s$$

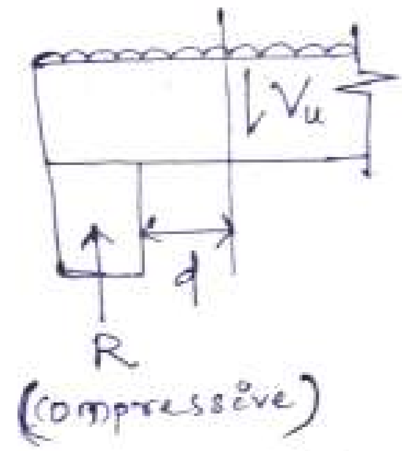
V_{cz} = Shear taken by uncracked portion of concrete.

V_{ay} = Shear taken by aggregate interlocking in cracked portion.

V_d = dowel action in longitudinal reinforcement.

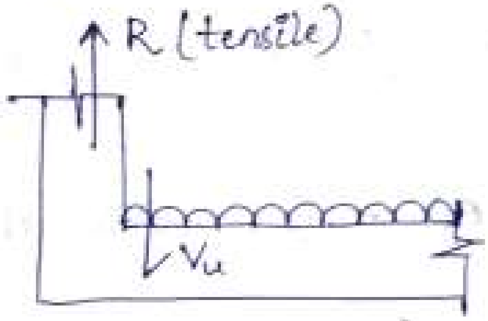
V_s = Shear taken by shear r/f or stirrups.

<u>Critical section in shear</u> :- (Cl 22.6.2 & 22.6.2.1)

① 

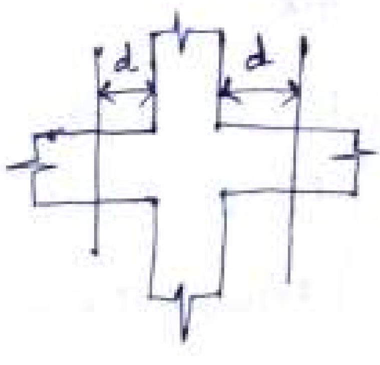
Critical section = (Face of support + d)

↳ if the support under compression.

② 

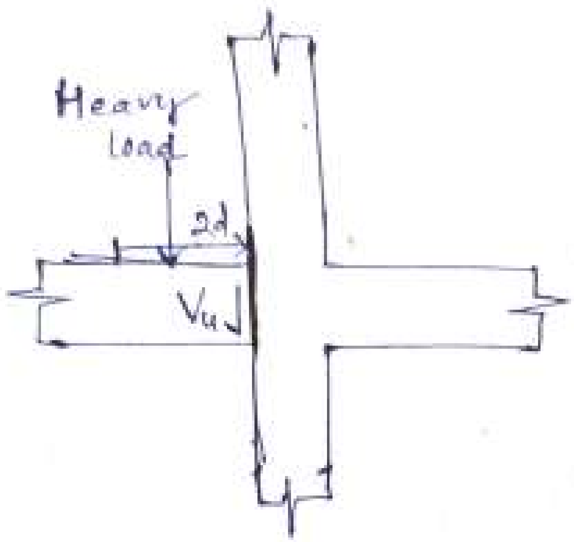
Critical section at face of support.

→ If support is under tension.

③ 

(Beam - column joint)

4)



* If a heavy load is acted within a distance 2d from the face of support.

→ The critical section is at the face of support.

1) Design shear strength of concrete (τ_c) :- (Table - 19)

→ Shear strength of concrete without reinforcement.

→ It depend on grade of concrete and % of tensile steel (P_t).

$$P_t = 100 \frac{A_{st}}{bd}$$

2) Maximum shear strength of concrete ($\tau_{c,max}$): (Table 20)

→ Shear strength of concrete with shear reinforcement.

f_{ck}	$\tau_{c,max}$
M_{15} →	2.5
M_{20} →	2.8
M_{25} →	3.1
M_{30} →	3.5
M_{35} →	3.7
M_{40} & above →	4

It should be redesign if $\tau_{c,max}$ is above 2.5 or ......

Nominal Shear Stress (τ_v) :- Cl: 40

→ Shear stress occur due to external load.

$$\tau_v = \frac{V_u}{bd}$$

V_u → factor S.F. at critical section ($V \times 1.5$)

Conditions :-

(i) $\tau_v < \tau_{c,max}$. (Then the structure safe in shear).

(ii) If $\tau_v > \tau_{c,max}$
→ redesign the section. (i.e. increase b & d
→ or increase $\tau_{c,max}$ (i.e. increase grade of concrete.)

(iii) If $\tau_v < \tau_c$ ⇒ (minimum shear reinforcement provided.) (Pg-48)

Clause 26.5.1.6

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y}$$

$$\Rightarrow 0.87 f_y A_{sv} \geq 0.4 bS_v$$

S_v = Spacing of stirrup

f_y → Should not be more than $415 N/mm^2$

A_{sv} = cross-sectional area of stirrup legs.

(iv) If $\tau_v > \tau_c$ ⇒ Shear reinforcement provided in the form of :-

(a) Vertical stirrups.

(b) Inclined stirrups.

(c) Bent up bar along with stirrups.

stirrup provide to counteract extra shear stress = $(\tau_v - \tau_c)$

SO, design shear force $\boxed{(V_{us}) = (\tau_v - \tau_c) \cdot bd}$ F = S.A

↓

Shear resisted by shear reinforcement (i.e. stirrup).

A_{sv} = Cross sectional area of stirrup legs or bent up bar within a distance S_v.

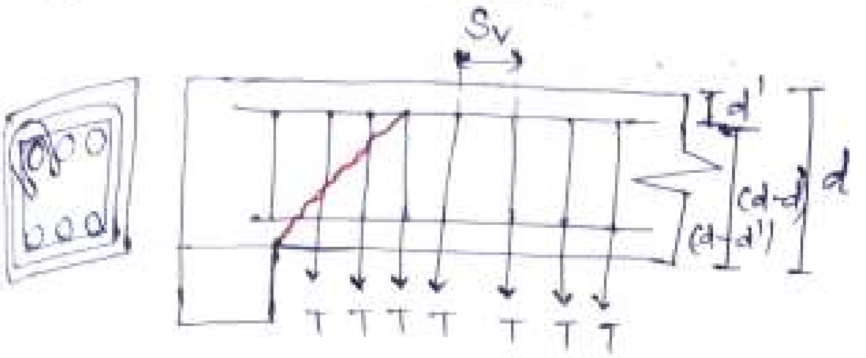
S_v = Spacing of stirrup or bent up bar along the length of member.

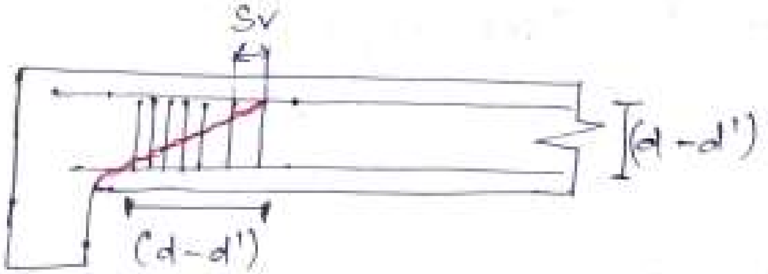
f_y → Characteristic strength of stirrups ($f_y \not> 415$ MPa)

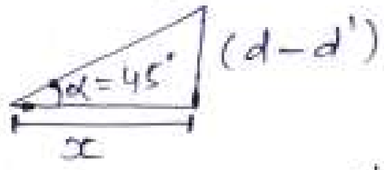
d → effective depth

(1) Vertical stirrup

Area of stirrup = A_{sv}.







$\tan 45^\circ = \frac{d-d'}{x}$

$x = \frac{d-d'}{\tan 45^\circ}$

$x = \frac{d-d'}{1}$

$\boxed{x = d - d'}$

No. of stirrups crossing one crack

$= \left(\frac{d-d'}{S_v}\right) \simeq \frac{d}{S_v}$

(neglect d')

Shear force taken by one stirrup $= \sigma \times A$

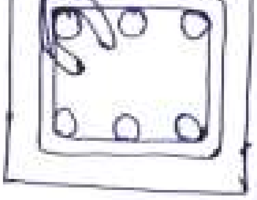
$= 0.87 f_y \times A_{sv}$

$\boxed{T = 0.87 f_y A_{sv}}$

Then,

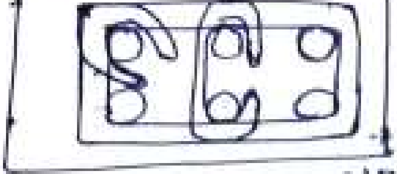
Shear force resisted by shear reinforcement or vertical stirrup $\boxed{(V_{us}) = 0.87 f_y A_{sv} \frac{d}{S_v}}$

Types of stirrup



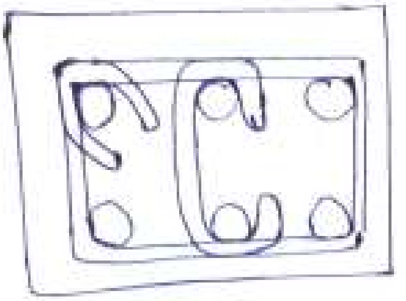
$A_{sv} = 2 \times \frac{\pi}{4} \times \phi^2$

(Two-legged stirrups)



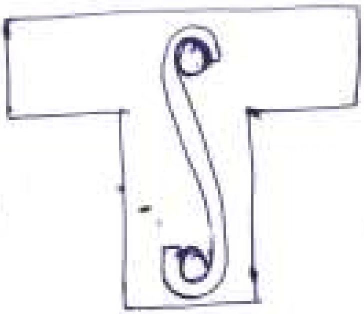
(3-legged stirrups)

$A_{sv} = 3 \times \frac{\pi}{4} \times \phi^2$



(3-legged ~~stirr~~ stirrup)

$A_{sv} = 3 \times \frac{\pi}{4} \times \phi^2$ . . .



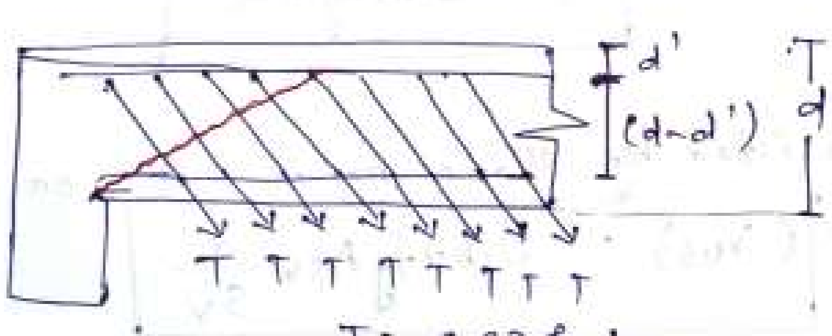
(one-legged stirrup)

Maximum spacing (S_v)

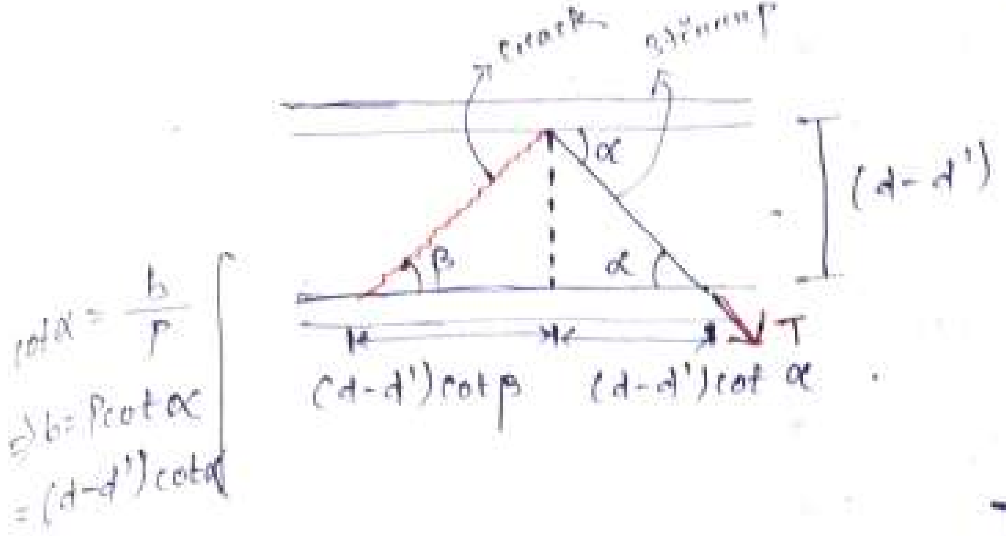
$$S_v = \min \begin{cases} 0.75\,d \\ 300\text{ mm} \end{cases}$$

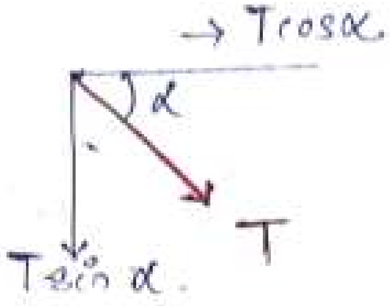
Maximum spacing should be ~~< 300mm~~ not more than 300mm.

2) Inclined stirrup



$T = 0.87 f_y A_{sv}$.





No. of stirrup crossing one crack =

$$= \frac{(d-d')\cot\beta + (d-d')\cot\alpha}{S_v}$$

$$= \frac{(d-d')(\cot\alpha + \cot\beta)}{S_v}$$

(neglect d')

$$= \frac{d(\cot\alpha + \cot\beta)}{S_v}$$

$$= \frac{d(\cot\alpha + \cot 45^\circ)}{S_v}$$

$$= \frac{d(\cot\alpha + 1)}{S_v}$$

(β = angle of crack w.r.t horizontal axis = 45°)

Vertical component of shear stress carried by one stirrup = $T\sin\alpha$

$= 0.87 f_y A_{sv} \sin\alpha$

So shear resisted by total inclined stirrups =

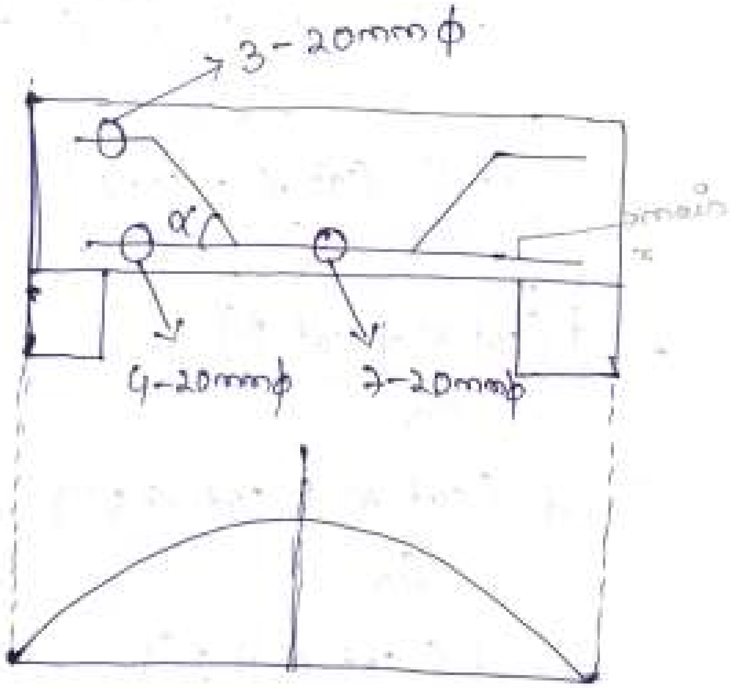
$$V_{us} = 0.87 f_y A_{sv} \sin\alpha \left(\frac{d(\cot\alpha + 1)}{S_v}\right)$$

$$\Rightarrow V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} (\sin\alpha + \cos\alpha)$$

Maximum spacing

$$S_v = \min \begin{cases} d \\ 300\,mm \end{cases}$$

3) Bent-up bar :-



* Not more than 50% of bar bent up.

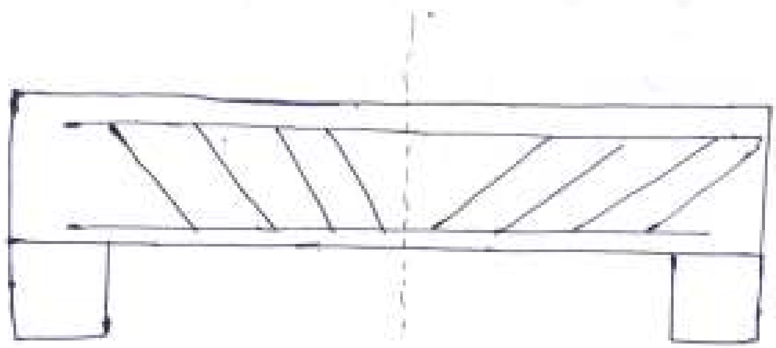
→ The S.F (V_{us}) resisted by bent up bar inclined at an angle α = vertical component of force in bar.

$$V'_{us} = T \sin\alpha$$
$$= 0.87 f_y A_{sv} \sin\alpha$$

→ when bars are bent up at same cross section.

*. If bars are bent up at different cross - section

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} (\sin\alpha + \cos\alpha)$$

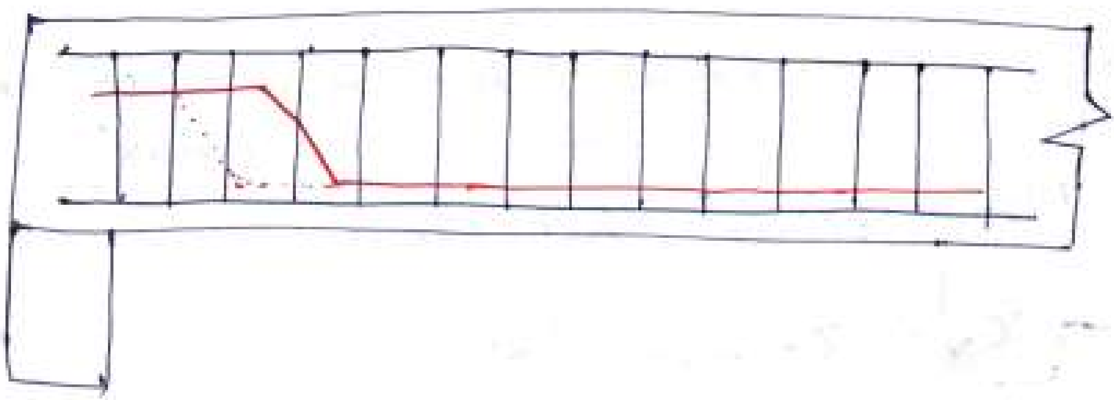


N.B :- Shear resistance contribution of bent up bars shall not be more than 50% & remaining shear force should be resisted by vertical or inclined stirrup.

→ Contribution of shear resistance of bent up bare $\not> \frac{V_{us}}{2}$

i.e. $V'_{us} \leq \frac{V_{us}}{2}$

→ The remaining S.F = $(V_{us} - V'_{us})$ is resisted by vertical stirrup.



Q1. A simply supported R.C.C. beam 250mm wide and 450mm deep (effective) is reinforced with a 4-18 mm diameter bars. Design the shear reinforcement if M20 grade of concrete and Fe 415 steel is used and beam is subjected to a shear force of 150KN at critical section at service state.

Ans:- 1) Data Given,

$b = 250\,mm$

$d = 450\,mm$

4-18mm ϕ bar

$A_{st} = 4 \times \frac{\pi}{4} \times 18^2 = 1018\,mm^2$

$f_{ck} = 20\,N/mm^2$ $\quad f_y = 415\,N/mm^2$

$V = 150\,kN$.

2) Calculation of factor shear force (V_u) :-

$V_u = V \times 1.5 = 150 \times 1.5 = 225\,KN.$

3) Calculation of nominal shear stress :-

$$\tau_v = \frac{V_u}{bd} = \frac{225 \times 10^3}{250 \times 450} \simeq 2\,N/mm^2$$

4) Calculation of maximum shear stress of concrete $(\tau_{c,max})$:-

For M20, $\tau_{c,max} = 2.8\,N/mm^2$

$\tau_v < \tau_{c,max}$ (OK)

5) <u>Calculation of design shear strength of concrete (τ_c)</u> :-

$$P_t = \frac{100\,A_{st}}{bd} = \frac{100 \times 1018}{250 \times 450} = 0.9\,\%$$

P_t		τ_c	
(x_1) 0.75	$\longrightarrow$	0.56 (y_1)	for M_{20}
(x) 0.9	$\longrightarrow$	? (y)	
(x_2) 1	$\longrightarrow$	0.62 (y_2)	

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\tau_c - 0.56 = \frac{0.62 - 0.56}{1 - 0.75}(0.9 - 0.75)$$

$$\Rightarrow \tau_c = 0.59 \ N/mm^2.$$

<u>Check</u> :-

$$\tau_v > \tau_c$$

So shear reinforcement provided.

6) <u>Design of shear reinforcement</u> :-

Shear taken by stirrups (V_{us}) $= (\tau_v - \tau_c) \cdot bd$

$$V_{us} = (2 - 0.59) \times 250 \times 450$$

$$= 158625 \ N.$$

<u>Let's use 8mm ϕ, 2-legged vertical stirrups.</u>

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.5 \ mm^2.$$

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}.$$

$$\Rightarrow S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$\Rightarrow S_v = \frac{0.87 \times 415 \times 100.5 \times 450}{158625}$$

$\Rightarrow S_v = 102.9$ mm or 103 mm.

Check for Spacing :-

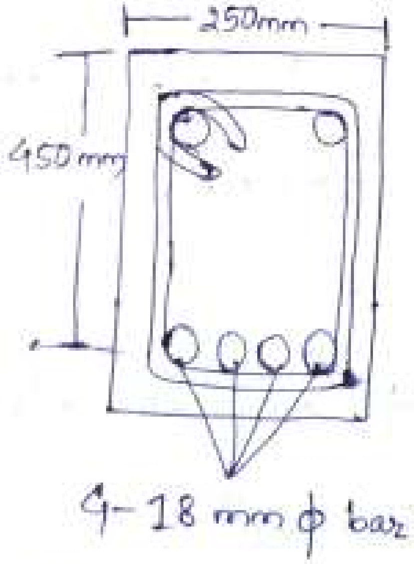
$$S_v = \min \begin{cases} 0.75\,d = 0.75 \times 450 = 337.5 \text{ mm} \\ 300 \text{ mm} \\ 103 \text{ mm} \end{cases}$$

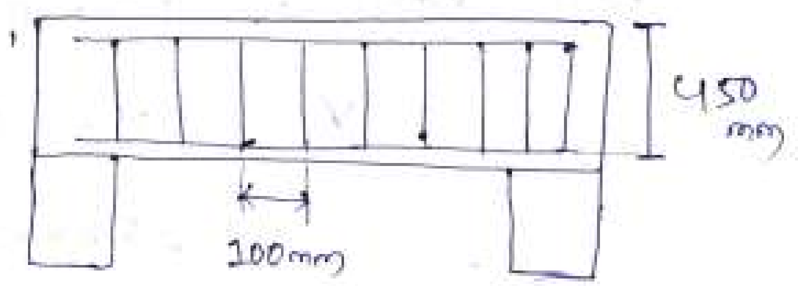
$S_v = 103$ mm

8mm ϕ
10mm ϕ
12 mm ϕ

So, provide 2-legged 8mm ϕ vertical stirrups @ 100mm centre to centre throughout the length of the beam.
(b/c 103 is odd) so, 100 even no. is taken

⇒ Detailing of reinforcement :-





2-legged 8 mm ϕ @ 100 mm c/c

Bond and development length (LSM)

→ Bond in reinforced concrete refers to the adhesion between reinforcement steel and surrounding concrete.

↦ It's responsible fore transfere of axial force from reinforced steel to concrete, and provide composite action.

↦ If the bond is inadequate, slipping of reinforced steel will occure and destroy the composite action.

Mechanism of bond :-

(1) Adhesion Resistance / chemical adhesion :-

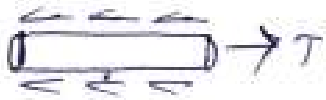
↦ It is due to gum like property of the substance, formed after setting of concrete.

↦ Due to hydration.
i.e. C-S-H gel.

(2) Frictional resistance /:-

↦ It is due to friction between steel and concrete.

↦ Due to roughness or surface roughness of reinforcement.



(3) Mechanical resistance / Mechanical interlock :-

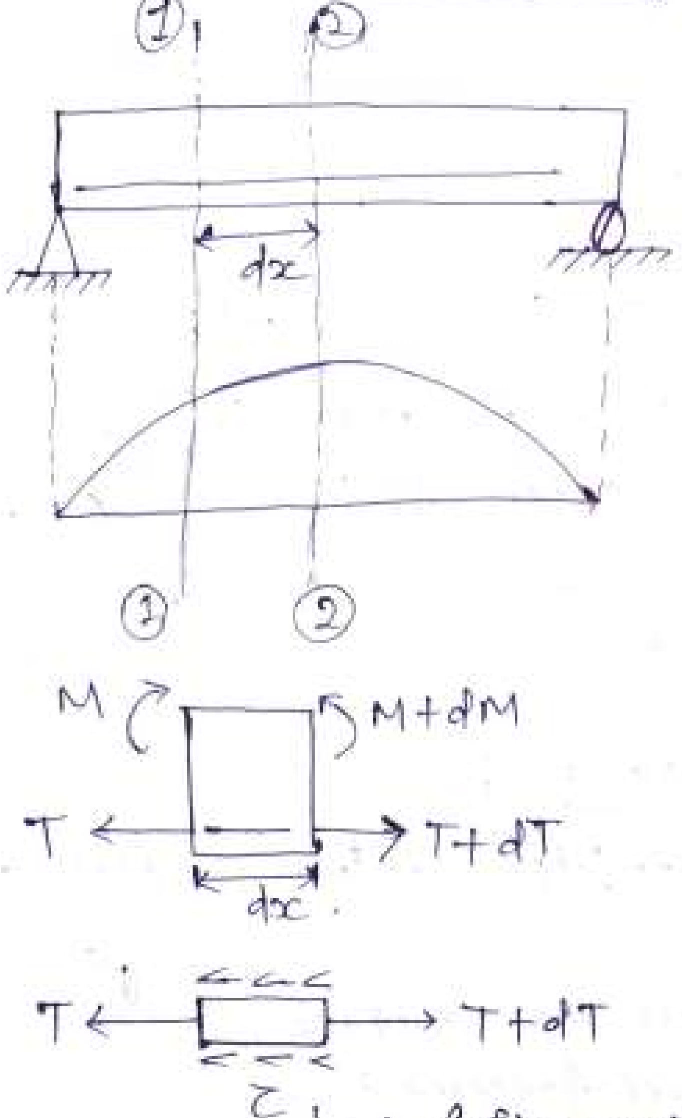
↦ It is provided by the corrugations or ribs present on the surface of the deformed bars.

Types :-

(1) Flexural bond :-

↦ It's arises in flexural member on account of sheare or a variation in Bending moment which cause a variation in axial tension along length of reinforced bar.

$$V = \frac{dM}{dx}$$



τ ↳ (flexural bond stress)

At section (1)-(1), $M = T \times (LA)$ ——— (I)

Section (2)-(2), $M + dM = (T + dT) \times (LA)$ ——— (II)

$\Sigma M = 0$.

$$(M + dM) - M = (T + dT)(LA) - (T)(LA)$$

$$\Rightarrow dM = dT \cdot LA$$

$$\Rightarrow \boxed{dT = \frac{dM}{LA}} \text{ ——— (III)}$$

$\Sigma H = 0$.

$$(T + dT) - T - (\tau \times \text{surface area}) = 0$$

$$dT = \tau \times \text{surface area}$$

$$= \tau \times (\text{circumference} \times \text{Length})$$

$$= \tau \times (\pi \phi \cdot dx)$$

$$\Rightarrow \boxed{dT = \tau(n \pi \phi \, dx)} \text{ ——— (IV)}$$

$\sigma = \frac{F}{A}$

$\Rightarrow F = \sigma \times A$.

Compare equ^n (III) & (IV)

$$\frac{dM}{LA} = \tau(n \pi \phi) \cdot dx.$$

$$\Rightarrow \boxed{\tau = \frac{(dM/dx)}{(n \pi \phi) \cdot (LA)}} \text{ ——— (V)}$$

$$\boxed{\tau = \frac{V}{(\Sigma P) \cdot Z}} \text{ ——— (VI)}$$

τ → Flexural bond stress

N.B

1) $\tau \propto V$

⇒ S.F ↑ ⇒ τ ↑

⇒ Flexural bond stress is maximum at support. (∵ S.F is maxm. at support).

2) $\tau \propto \frac{1}{(\Sigma P)}$

⇒ τ ↑ ⇒ (ΣP) ↓

⇒ So use small 'ϕ' bar in larger ~~member~~ number rather that using large 'ϕ' bar in less number.

Design bond stress (τ_{bd})

→ $\tau < \tau_{bd}$

→ Value of 'τ' calculated above should be less than design bond stress (τ_{bd}).

Cl. 26.2.1.1

Grade	τ_{bd} (N/mm²)
M20 →	1.2
M25 →	1.4
M30 →	1.5
M35 →	1.7
≥ M40 →	1.9

N.B :-

↦ τ_{bd} is increased by 60% in case of HYSD bar.

↦ τ_{bd} is increased by 25% in case of bars in compression.

*. 1) Plain bars in tension = τ_{bd}

2) HYSD bar in tension = $1.6\,\tau_{bd}$

3) Plain bar compression = $1.25\,\tau_{bd}$

4) HYSD bar in compression $= (1.6 \times 1.25)\,\tau_{bd}$
$= 2\,\tau_{bd}$

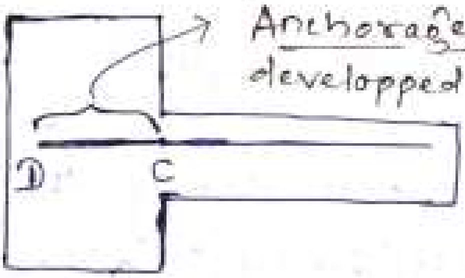
Eg:- M20, HYSD Bar in tension

$$\tau_{bd} = 1.2 \times 1.6 = 1.92\ N/mm^2$$

Anchorage bond stress / Developpment B.S (bond stress)

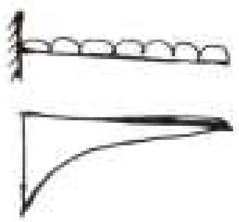
↦ It's arises over the length of anchorage provided for a bar or near the end of a reinforcing bar to resist the pulling out of bar (in tension) or pushing in of bar (in compression).

Provided length is called Anchorage length and developped Bond stress is called Anchorage bond stress.



$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$\Rightarrow$ $M \propto \sigma$



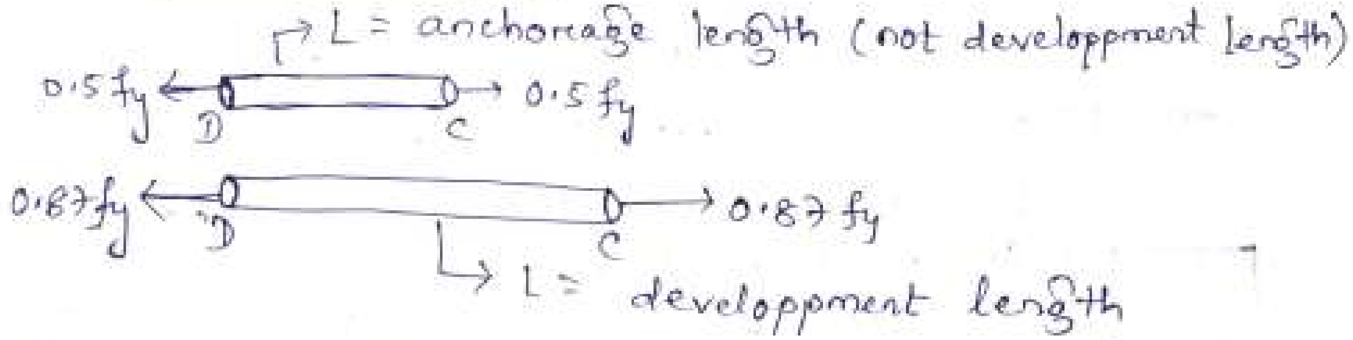
Developpment Length (L_d)

↦ It is the length of embedment necessary to developp the full tensile strength of the bar (0.87 fy).

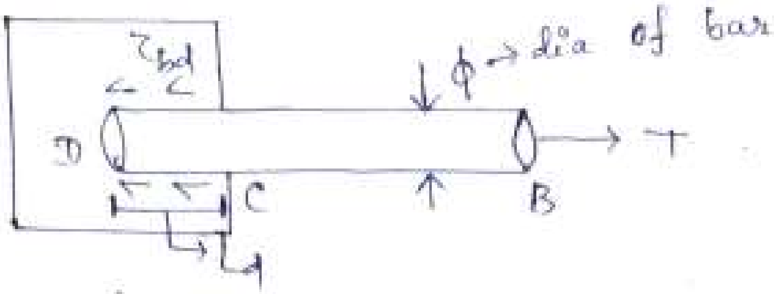
*. Maximum stress in steel = 0.87 fy

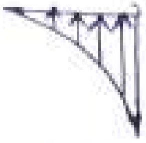
↦ So, it is that minimum length of bar required on either side of point of maximum steel stress in order to transfer the bar force to the surrounding concrete through bond and without slip so that to prevent the bar 'pulling out' under tension or 'pushing in' under compression.

↦ If the required bar embedment cannot be provided due to practical difficulties - bends, hooks or mechanical anchorage can be used to suppliment with 'equivalent embedded length'.



Developpment length → if the bare carry the maximum stress (0.87 fy) after point C.





(Actual parabolic bond stress distribution)

(Assumed rectangular or constant bond Stress distribution.)

Maximum force that can be applied on bar = F × A

$$= (0.87 f_y)\left(\frac{\pi}{4} \times \phi^2\right) \text{ ——— (i)}$$

Maximum force transferred from steel to concrete

= σ × Surface area

= (τ_{bd}) × (circumference × length)

$$= (\tau_{bd} \times \pi\phi \times L_d) \text{ ——— (ii)}$$

$$(0.87 f_y)\left(\frac{\pi}{4}\phi^2\right) = \tau_{bd} \cdot \pi\phi \cdot L_d$$

$$\Rightarrow \boxed{L_d = \frac{(0.87 f_y)\,\phi}{4\,\tau_{bd}}} = \frac{\sigma\,\phi_s}{4\,\tau_{bd}}$$

→ Plain bar in tension.

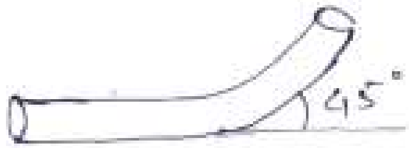
Cl. 26.2.2.1 (Pg - 43)

Provision of bend and hook :-

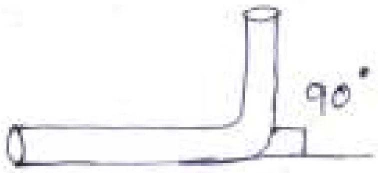
↦ For each 45° bend, equivalent anchorage of 4ϕ is taken subjected to maximum of 16ϕ ~~bar~~ for standard bar.

(standard bar)

Equivalent anchorage

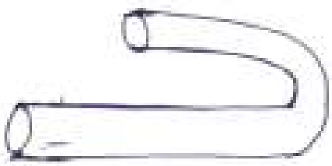


↦ 45° bend → 4ϕ.



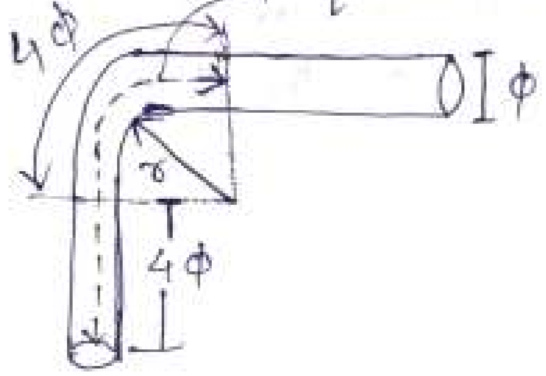
↦ 90° bend → $4\phi + 4\phi = 8\phi$.

↦ 135° bend → $4\phi + 4\phi + 4\phi = 12\phi$



↦ 180° bend → $4\phi \times 4\phi = 16\phi$.

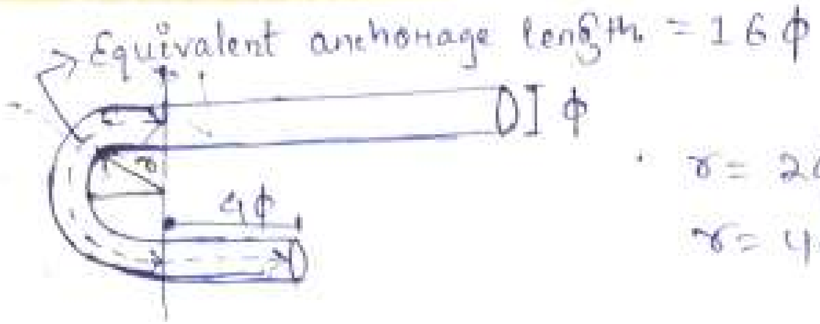
Equivalent anchorage length = 8ϕ.



$r = 2\phi$ (mild steel)

$r = 4\phi$ (HYSD bar)

(Anchoring bars in tension)



r = 2φ (mild steel)
r = 4φ (HYSD bar)

Anchorage bars in compression (Pg-43)

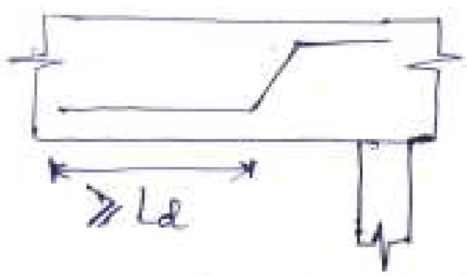
- The anchorage length of straight bar in compression shall be equal to the development length of bars in compression as specified in 26.2.1. The projected length of hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall only be considered for development length.

↦ For compression, the anchorage length is equal to Ld. The compression bar required no special anchorage arrangement.

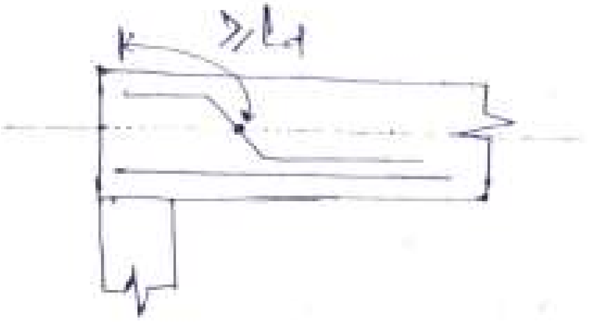
Anchorage for shear reinforcement.

(a) Inclined bars :-

(1) In tension zone, from the end of the sloping or inclined portion of the bar



(2) In the compression zone, from the mid depth of the beam.



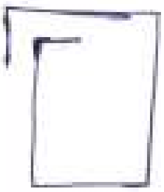
(b) Stirrups :-

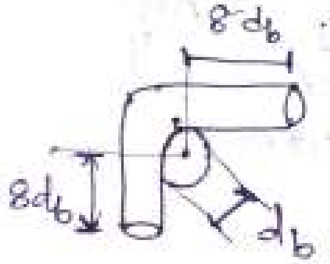
(1) Condition (1) → bent through an angle of at least 90°.

(2) Condition (2) → bent through an angle of 135°.

(3) Condition (3) → bent through an angle of 180°.

(1)

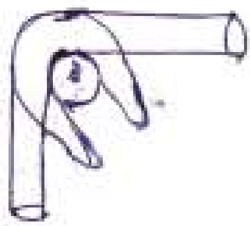




→ (90° bend)

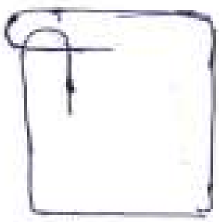
d_b - dia bar

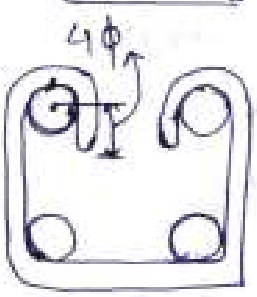
(2)

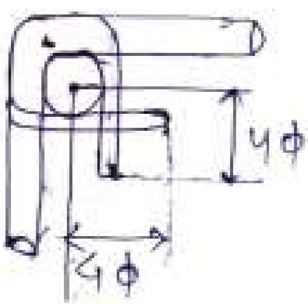


→ (135° bend)

(3)





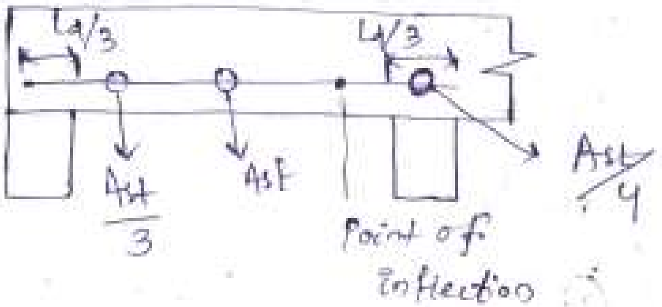


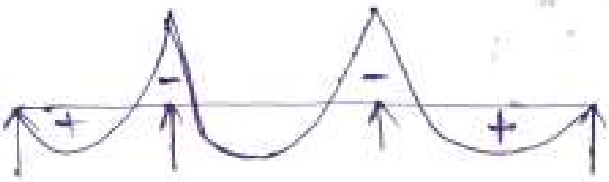
→ 180° hook

Cl- 26.2.3.3 → (Pg-44)

Positive moment reinforcement

(a) At least one-third the positive moment reinforcement in simple members and one-~~four~~ fourth the positive moment reinforcement in continuous member shall extended along the same face of the member into the support, to a length equal to $L_d/3$.





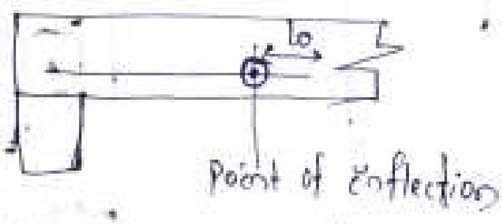
c) Centre of Simply Support

1) $L_d \leq 1.3 \frac{M_1}{V} + l_0$

for compressive reaction at support or confinement of concrete.

At point of inflection

$L_d \leq \frac{M_1}{V} + l_0$.



$l_0 = \max \begin{cases} d \\ 12\phi \end{cases}$

Analysis and Design of Slab.

Types of slab :-

(1) one-way slab

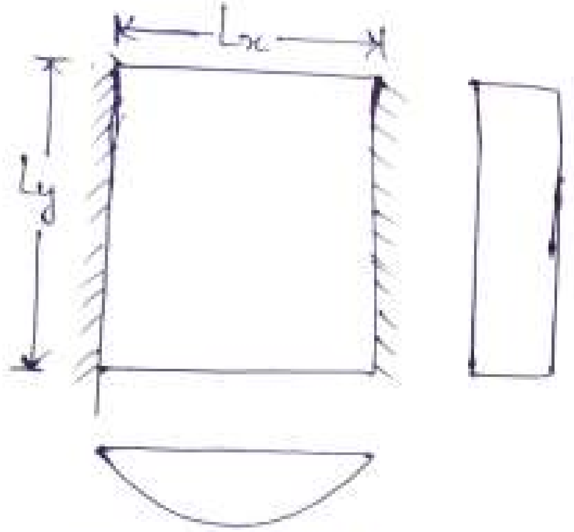
↦ Slab spanning in one direction

↦ Slab supported only on two opposite side is called one-way slab.

↦ In one-way slab, bending takes place only along shorter span.

↦ So main reinforcement is provided along shorter span.

* In one way slab $\boxed{\frac{l_y}{L_x} \geq 2}$



(Parabolic deflection)

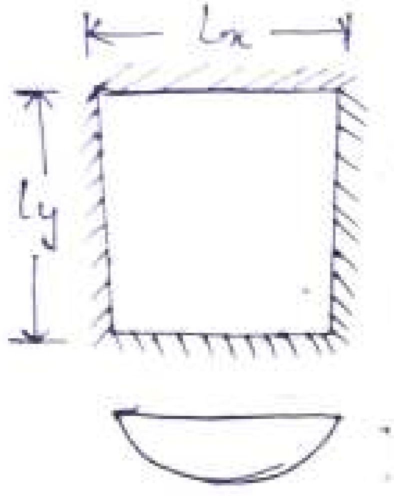
Lx → length of shorter span
ly → length of longer span

↦ Moment is greater in shorter span.
So, ~~reinf.~~ ~~Mx~~ reinforcement corresponding to Mx is provided in bottom.

(2) Two-way slab.

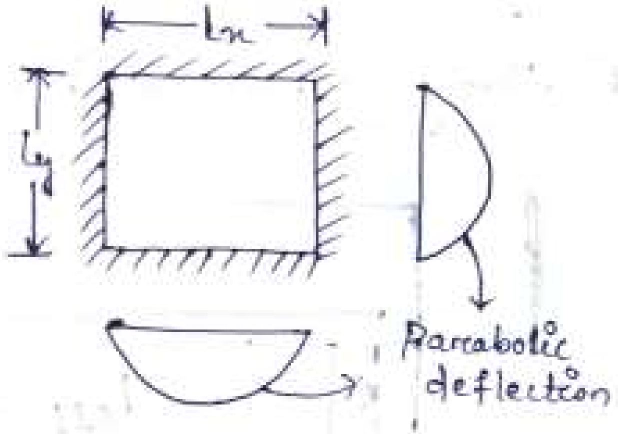
- This slab is supported on all its 4 sides.
- Span Slab spanning in two direction because bending occur in both direction.
- So main reinforcement is provided in both side.

* $\frac{l_y}{l_x} \leq 2$



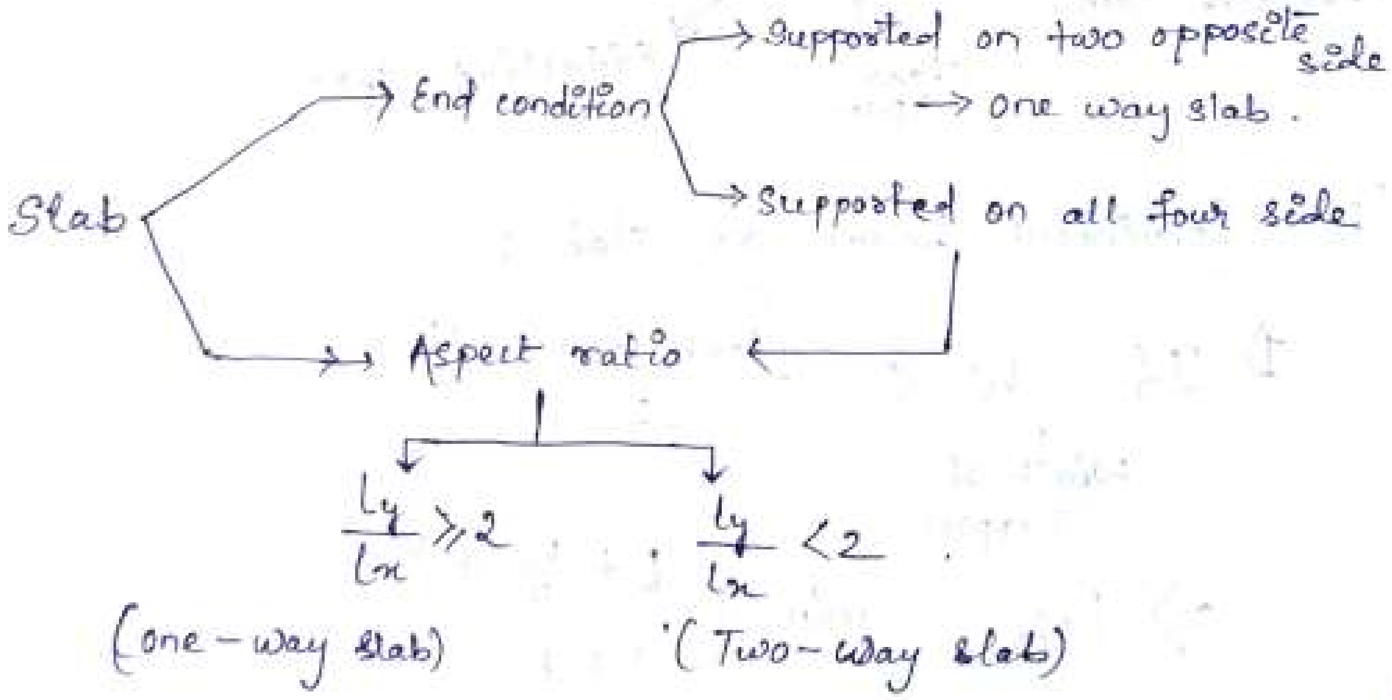
$\left(\frac{l_y}{l_x} \geq 2\right)$

(one-way slab)



$\left(\frac{l_y}{l_x} < 2\right)$

(Two-way slab)



IS codal provision :- (-Tabl

(1) Nominal cover :- (Table 16)

↦ Minimum 20mm, which can be reduce to 15mm or less for mild exposure and bare dia upto 12mm.

$$\text{Nominal cover} = \min. \begin{cases} 20\text{ mm} \\ \phi \end{cases}$$

(2) Effective Span :- (Cl.22.2)

(a) Simply supported ; beam or slab :

$$l_{eff} = \min \begin{cases} c/c \text{ support} = L + \frac{w}{2} + \frac{w}{2} \\ L + d \end{cases}$$

w → width of Support

l_{eff} → effective span

L → Clear Span

d → Effective span.

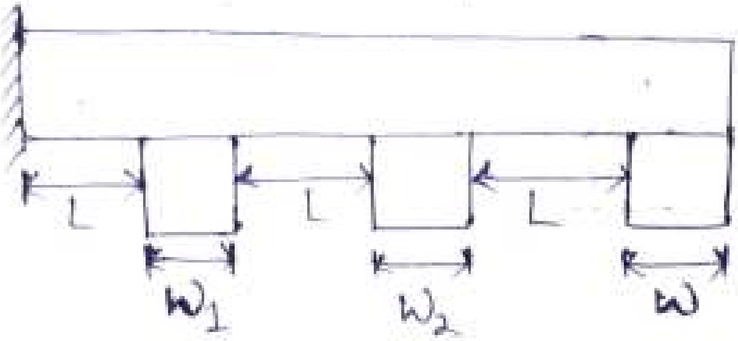
(b) Contineous beam or slab :-

1) If $w < \frac{\text{Cleare span (L)}}{12}$

w → width of support

$$\Rightarrow L_{eff} = \min \begin{cases} L + \frac{w}{2} + \frac{w}{2} \\ L + d \end{cases}$$

2) If $w >$ min. $\begin{cases} \frac{L}{12} \\ 600\text{ mm} \end{cases}$



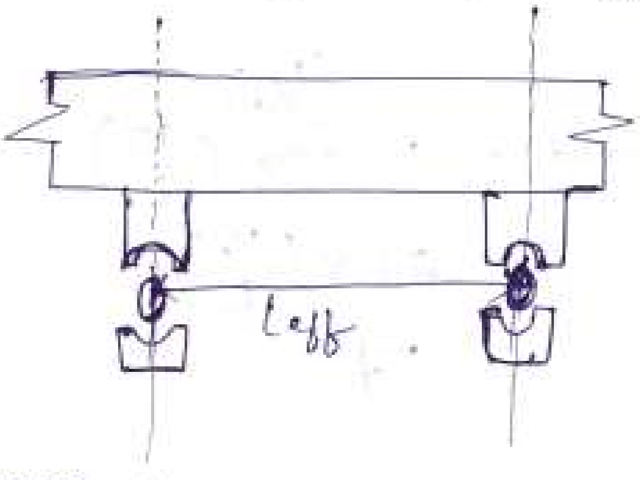
a) For one end fixed and other continious

$$\boxed{L_{eff} = L}$$

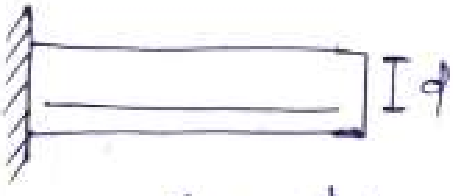
b) One end continious & other discontinious.

$$L_{eff} = \min \begin{cases} L + \frac{d}{2} \\ L + \frac{w}{2} \end{cases}$$

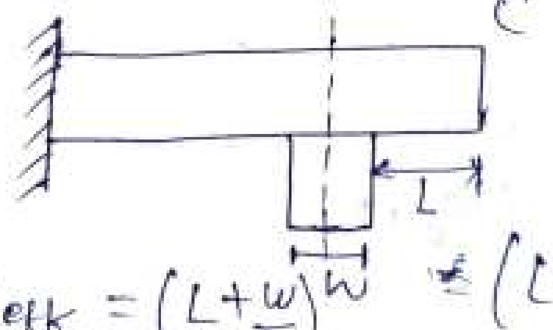
c) Roller or Rocker bearing



3) <u>Cantilever</u>

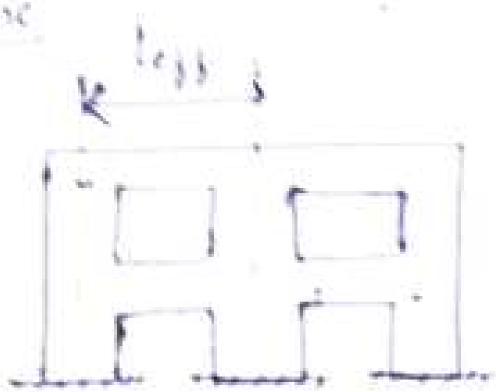


$$L_{eff} = \left(L + \frac{d}{2}\right)$$



$$L_{eff} = \left(L + \frac{w}{2}\right) \nless \left(L + \frac{w}{2}\right)$$

4) Frame



l_{eff} = c/c distance of support.

① Cl.23.2.1 (Pg-37)

Control of deflection

(a) One-way slab :- For span (L) upto 10m

	(L/d)
Cantilever →	7
Simply supported →	20
Continuous →	26

(b) For spans above 10m

$L > 10m$

	(L/d)
Simply supported →	$20 \times \left(\frac{10}{L}\right)$
Continuous →	$26 \times \left(\frac{10}{L}\right)$
Cantilever →	7

(c) Depending on the area and the stress of steel for tension reinforcement, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained. (Pg-38) (Fig 4)

$$f_s = 0.58 f_y \frac{A_{st}\text{ required}}{A_{st}\text{ provided}}$$

(bar) 5.6 ⇒ 6
required → provided

✱ For trial, use Fe 415 steel and $P_t = (0.4 - 0.5)\%$.

Then $f_s = 0.58 \times 415 \times \frac{A_{st}\text{ required}}{A_{st}\text{ provided}}$ (taking, Ast require = Ast provide.)

$\simeq 240$

So, modification factor = 1.25

(l/d)

For simply suported = $20 \times 1.25 = 25$ (22-28)

Continuous = $26 \times 1.25 = 32$.

② Cl. 24.1 (Pg-39) (Note)

2) Two way slabs of shorter spans (upto 3.5m) with mild steel reinforcement...

$L_{\text{shorter span}} \leq 3.5\text{m}$

$\text{Load} \leq 3\text{kN/m}^2$

	Fe 250	Fe415 (80% reduce)
Simply supported	$\left(\frac{L_x}{D}\right) = 35$	$\left(\frac{L_x}{D}\right) = 35 \times 0.8 = 28$
Continuous	$\left(\frac{L_x}{D}\right) = 40$	$\left(\frac{L_x}{D}\right) = 40 \times 0.8 = 32$

③ Cl. 26.5.2.2 (Pg-48)

Maximum diameter of reinforcement.

$$\phi_{max} \ngtr \frac{D}{8}$$

D → overall depth of slab or total thickness of slab.

(4) Minimum dia of reinforcement :-

Main reinforcement :-

Fe 250 => ϕ_{min} = 10mm.

Fe 415 => ϕ_{min} = 8mm

Secondary or distribution reinforcement :

ϕ_{min} = 6mm

(5) Cl. 26.5.2.1 (minimum reinforcement)

1) For Mild steel (Fe 250), minimum reinforcement should not be less than ≮ 0.15 % gross area (total area).

2) For HYSD, minimum reinforcement ≮ 0.12 % of gross area.

Minimum reinforcement is provided to resist the crack.

(6) Cl. 26.3.3 (b) :- (Pg - 46)

Spacing of reinforcement.

1) For main bar, $s = \min \begin{cases} 3d \\ 300mm \end{cases}$

2) For distribution bar, $s = \min \begin{cases} 5d \\ 300\ mm \end{cases}$

(450mm is revised to 300mm)

7) Load on Slab

1) Self or unit weight of RCC = $25\ kN/m^3$

2) Floor finishes and partition = $1.5\ kN/m^2$

On floor:

3) Imposed load (LL) or live load for residential building = $2\ kN/m^2$

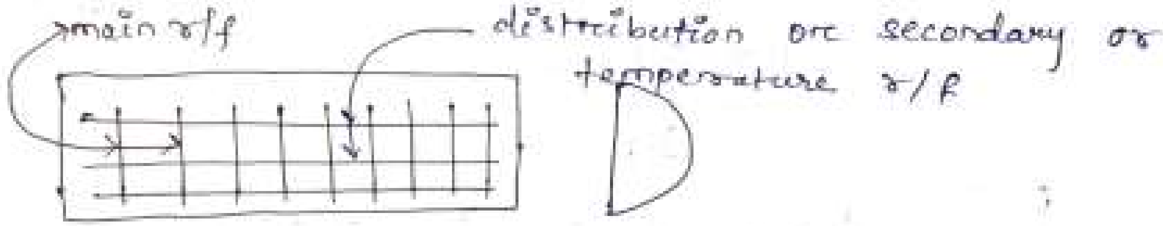
4) Live load (LL) for office building = $3\ kN/m^2$

Roof:

5) LL with access = $1.5\ kN/m^2$

6) LL without access = $0.75\ kN/m^2$

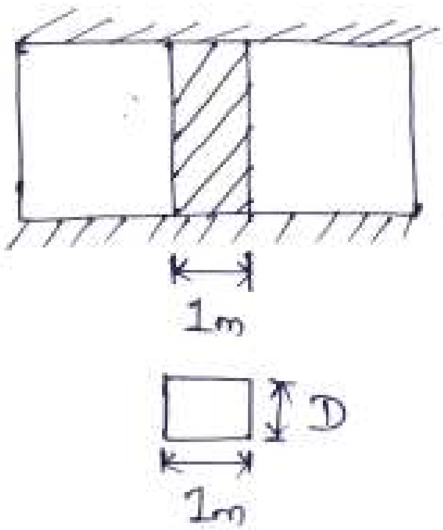
8)



Distribution r/f :-

- To protect the slab against cracking due to creep & shrinkage.
- To keep the main bar in position
- Main reinforcement are always provided at bottom.

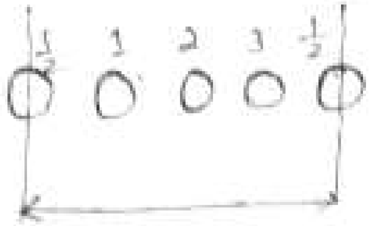
9)



Slab is a combination of beam

So, take unit width of slab i.e. **b = 1m**

(10) calculation of spacing



1000 mm = 1m

$$S = \frac{b}{\text{no. of bars}}$$

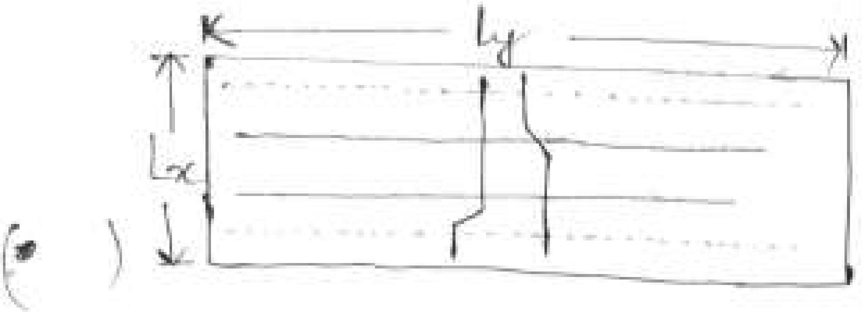
E.g:- $S = \frac{1000}{4} = 250$

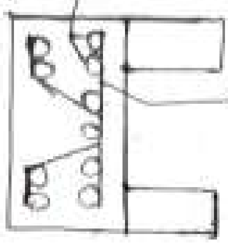
$$n = \frac{Ast}{Ast/\text{one bar}}$$

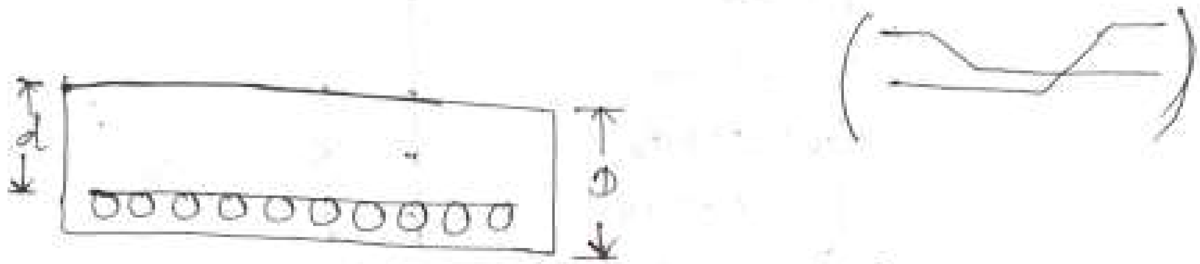
$$S = \frac{b \times Ast/\text{one bar}}{Ast}$$

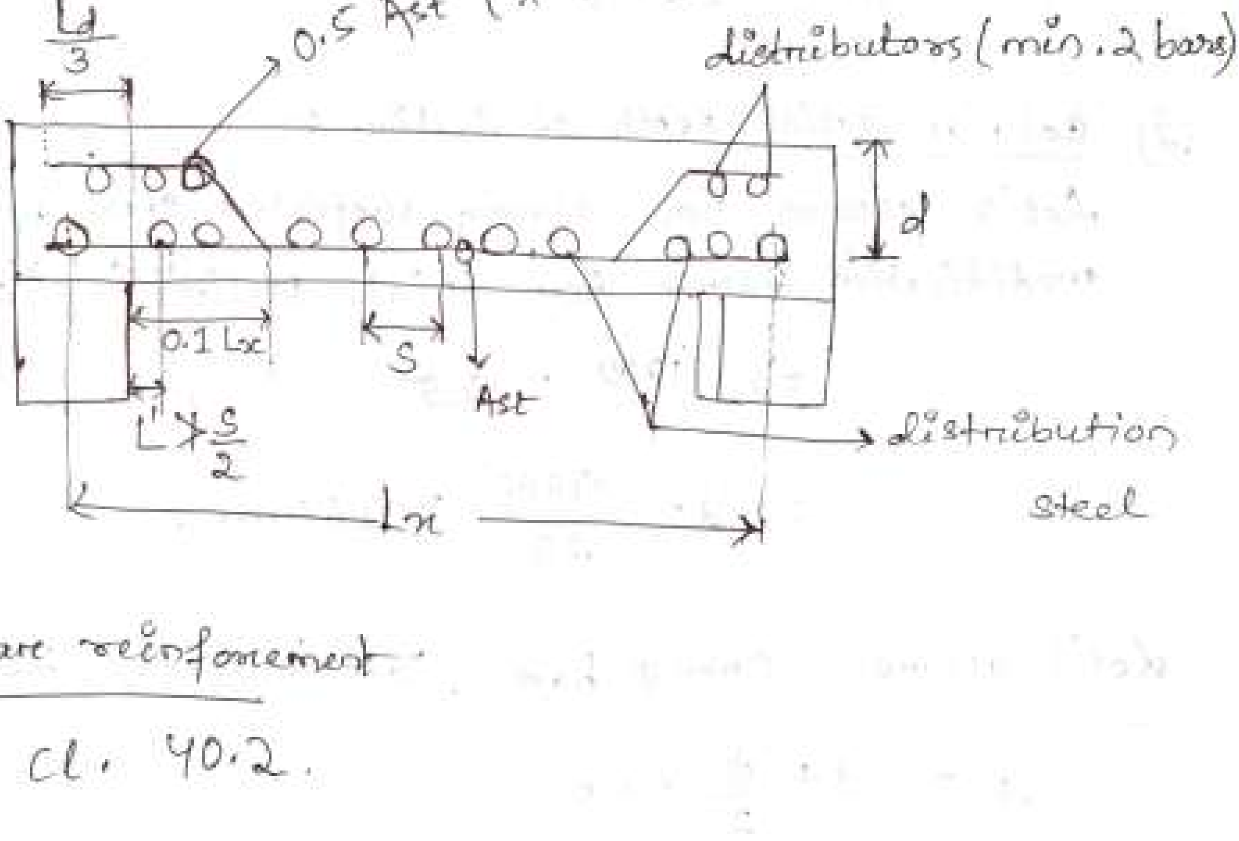
$$S = \frac{\text{Area of one bar} \times 1000}{\text{Total } Ast/\text{required}}$$

Detailing of reinforcement









Sheare reinforcement

Cl. 40.2.