



**GOVERNMENT POLYTECHNIC, DHENKANAL**

**Programme: Diploma in Mechanical Engineering**

**Course: Design of Machine Elements (Theory)**

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**Semester: 5<sup>th</sup>**

## TH.2 DESIGN OF MACHINE ELEMENTS

Name of the Course: Diploma in MECHANICAL ENGINEERING			
Course code:		Semester	5 <sup>th</sup>
Total Period:	60	Examination	3 hrs.
Theory periods:	4 P/W	I.A:	20
Maximum marks:	100	End Semester Examination:	80

### A. RATIONALE:

Machine design is the art of planning or devising new or improved machines to accomplish specific purposes. Idea of design is helpful in visualizing, specifying and selection of parts and components which constitute a machine. Hence all mechanical engineers should be conversant with the subject.

### B. COURSE OBJECTIVES

At the end of the course the students will be able to

1. Understanding the behaviours of material and their uses.
2. Understanding the design of various fastening elements and their industrial uses.
3. Understanding the different failures of design elements.
4. Understanding the change of design to accomplish the different field of applications.
5. Design shafts, keys, couplings required for power transmission.
6. Design closed coil helical spring

### C. CHAPTER WISE DISTRIBUTION OF PERIODS

Sl.No.	Topic	Periods
01	INTRODUCTION	12
02	DESIGN OF FASTENING ELEMENTS	12
03	DESIGN OF SHAFT AND KEYS	12
04	DESIGN OF COUPLING	12
05	DESIGN OF CLOSED COIL HELICAL SPRING	12
	TOTAL	60

## **D. COURSE CONTENTS**

### **Introduction:**

Introduction to Machine Design and Classify it.  
Different mechanical engineering materials used in design with their uses and their mechanical and physical properties.  
Define working stress, yield stress, ultimate stress & factor of safety and stress –strain curve for M.S & C.I.  
Modes of Failure (By elastic deflection, general yielding & fracture)  
State the factors governing the design of machine elements.  
Describe design procedure.

### **Design of fastening elements:**

Joints and their classification.  
State types of welded joints .  
State advantages of welded joints over other joints.  
Design of welded joints for eccentric loads.  
State types of riveted joints and types of rivets.  
Describe failure of riveted joints.  
Determine strength & efficiency of riveted joints.  
Design riveted joints for pressure vessel.  
Solve numerical on Welded Joint and Riveted Joints.

### **Design of shafts and Keys:**

State function of shafts.  
State materials for shafts.  
Design solid & hollow shafts to transmit a given power at given rpm based on  
a) Strength: (i) Shear stress, (ii) Combined bending tension;  
b) Rigidity: (i) Angle of twist, (ii) Deflection, (iii) Modulus of rigidity  
State standard size of shaft as per I.S.  
State function of keys, types of keys & material of keys.  
Describe failure of key, effect of key way.  
Design rectangular sunk key considering its failure against shear & crushing.  
Design rectangular sunk key by using empirical relation for given diameter of shaft.  
State specification of parallel key, gib-head key, taper key as per I.S.  
Solve numerical on Design of Shaft and keys.

## **Design of Coupling:**

Design of Shaft Coupling  
Requirements of a good shaft coupling  
Types of Coupling.  
Design of Sleeve or Muff-Coupling.  
Design of Clamp or Compression Coupling.  
Solve simple numerical on above.

## **Design a closed coil helical spring:**

Materials used for helical spring.  
Standard size spring wire. (SWG).  
Terms used in compression spring.  
Stress in helical spring of a circular wire.  
Deflection of helical spring of circular wire.  
Surge in spring.  
Solve numerical on design of closed coil helical compression spring.

**Syllabus covered up to I.A-Chapters 1,2 &3**

## **LEARNING RESOURCES**

<b>SL.NO</b>	<b>AUTHOR</b>	<b>TITLE OF THE BOOK</b>	<b>PUBLISHER</b>
<b>01</b>	<b>PANDYA AND SHAH</b>	<b>MACHINE DESIGN</b>	<b>CHAROTAR PP</b>
<b>02</b>	<b>R.S.KHURMI &amp;J.K.GOPTA</b>	<b>A TEXT BOOK OF MACHINE DESIGN</b>	<b>S.CHAND</b>
<b>03</b>	<b>P.C.SHARMA &amp;D.K AGRAWAL</b>	<b>A TEXT BOOK OF MACHINE DESIGN</b>	<b>S.K.KATARIYA</b>
<b>04</b>	<b>V.B.BHANDARI</b>	<b>DESIGN OF MACHINE ELEMENTS</b>	<b>TMH</b>
<b>05</b>	<b>S.MD.JALAUDEEN</b>	<b>DESIGN DATA BOOK</b>	<b>ANURADHA PUBLICATION</b>

# MACHINE DESIGN

## WHAT IS A MACHINE?

A machine is the assembly of resistant bodies or links which is used to transmit available energy to do useful work.

## MACHINE DESIGN:

- Machine design is defined as the use of scientific principles, engineering techniques and imagination to create a machine or machine element economically.
- Machine Design focuses on the basic principles of following three areas for creation of new machines and improving the existing.
  - a) Mechanical behavior of material.
  - b) Mechanical parts or machine element.
  - c) Life cycle analysis.

## CLASIFFICATION OF MACHINE DESIGN

### **1. Adaptive design:-**

Here designers work is concerned with adaptation of existing designs. This type design needs no special knowledge or skill .The designer only makes minor modification in the existing designs of the product.

### **2. Development design:-**

This type of design needs scientific training and design ability in order to modify the exiting design into a new idea by adopting a new material or different method of manufacturing. In this case, though the designer starts from the existing design, but the final product will be quite different from the original product.

### **3. New design:-**

This type of design needs lo of research, technical ability and creative thinking. From this design completely new product will be found.

The designs, depending upon the methods used, may be classified as follows:

- Rational design:-This type of design depends upon mathematical formulae of principal of mechanics.
- Industrial design:-This type of design depends upon the production aspects to manufacture any machine component in the industry.
- System design:-It is the design of any complex mechanical system like a motor car.

- Element design:-It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.
- Computer aided design:-In this of design, creation, modification & analysis of a design is done by using computer system.

### **DIFFERENT MECHANICAL ENGG. MATERIALS USED IN DESIGN**

- ❖ Commonly used ferrous alloys are carbon steel, Low-alloy steel, Tool steel, Stainless steel and cast iron.
- ❖ Non ferrous alloys are Aluminum alloys, Nickel alloys, Titanium alloys.

### **Properties of Material:**

Properties of material are the characteristics of matter which differentiate one material from the other.

The major properties of material to be studied for selection of material in engineering field are:

1. Physical properties
2. Chemical properties
3. Mechanical properties

### **Physical Properties:**

A materials physical properties denote the physical state of material. Physical properties include

1. Density
2. Specific Heat
3. Thermal Expansion
4. Conductivity
5. Melting Point
6. Porosity
7. Crystal Structure
8. Appearance

## Mechanical properties of Materials:

- |                |                   |
|----------------|-------------------|
| 1. Strength    | 7. Malleability   |
| 2. Stiffness   | 8. Toughness      |
| 3. Elasticity  | 9. Hardness       |
| 4. Plasticity  | 10. Machinability |
| 5. Ductility   | 11. Creep         |
| 6. Brittleness | 12. Fatigue       |

### Strength:

- It is the ability of a material to resist the externally applied forces without breaking.
- The internal resistance offered by a part to an externally applied force is called stress.

### Stiffness:

It is the ability of materials to resist deformation under the action of load.

Mathematically:

$$\text{Stiffness (K)} = \frac{\text{Load (W)}}{\text{Deflection ( } \delta \text{)}}$$

**Unit:** KN / mm or N / mm

This property is considered during selection of material for spring manufacturing.

### Elasticity:

It is a property by virtue of which a material regains its original dimension after removal of load. Elasticity is measured by Young's Modulus or Modulus of Elasticity.

Unit – N / mm<sup>2</sup>

### Plasticity:

- It is the property by virtue of which the material does not regain its original shape after removal of load . I retains its deformed shape permanently.
- This property of the material is necessary for forging & rolling process.

### **Ductility:**

- It is a property by which materials can be drawn into wires with the application of a tensile force.
- Ductile materials have the ability to flow or elongate under load. Example : Copper , Aluminum , mild steel , nickel, zinc, tin & lead.
- The ductility of a material is commonly measured by means of percentage elongation and percentage reduction in area in a tensile test.

### **Brittleness:**

It is the ability of a material by which it can develop crack under load or it can break suddenly.

Example : Wood , Concrete , Cast iron.

### **Malleability:**

It is the property by virtue of which the material is able to be converted in to thin sheets.

Materials which are more elastic are also more malleable.

Example : Steel , Copper, Al, Brass, Bronze etc.

### **Toughness:**

- It is the property by virtue of which a material is able to resist shock or impact loading.
- Impact loading means applied load fall from a height.
- The amount of energy absorbed per unit volume within elastic limit is know as Resilience.
- In the deign of springs toughness or resilience of material is considered.

### **Hardness:**

- It is the property by which the material is able to resist scratches, marks or wear & tear.
  - It also measures the ability of a material to cut another metal.
  - Harness is independent of the weight of a material.
- Brittle materials are more hard example: Glass, cast iron , concrete.



### Machinability:

- It is the property of a material which refers to the ease with which a material can be cut.
- Machinability of a material can be measured by measuring the energy required to remove a unit volume of the material, keeping all machining parameters constant.
- It may be noted that brass can be easily machined than steel.

### Creep:

- When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep.
- This property is considered in designing internal combustion engines, boilers and turbines.
- Super alloys resist creep failure as they can withstand high temperature for a prolonged period without developing stress.

### Fatigue:

Fatigue is the property of material by virtue of which the material fails under stress less than yield stress due to cyclic nature of stress.

Fatigue failure is responsible for 90% of mechanical failure.

### **Different type of stress:-**

When external load is applied on an object resist distortion due to the applied force for which internal forces developed in the object whose magnitude is equal to externally applied force.

Stress can be defined as the internal resisting force acting on unit cross sectional area of the object.

Mathematically,

$$\text{Stress}(\sigma) = \frac{\text{Load}(F)}{\text{c/s area}}$$

Unit is  $\text{N/mm}^2$

Before discussing types of stress we will discuss about types of load.

## **LOAD:**

It is defined as any external force acting upon a machine part. The following four types of the load are:-

### **1. Dead or steady load:-**

A load is said to be a dead or steady load, when it does not change in magnitude or direction.

### **2. Live or variable load:-**

A load is said to be a live or variable load, when it changes continually.

### **3. Suddenly applied or shock loads:-**

A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.

### **4. Impact load:-**

A load is said to be an impact load, when it is applied from certain height with some initial velocity.

## **Different types of Stresses**

- **Yield stress:-**

It is defined as the maximum stress at which increase in elongation occurs without increase in load. After yield point on removal of the load the material will not be able to recover its original shape and size. Stress corresponding to yield point is known as yield point stress.

- **Ultimate stress:-**

The stress, which attains its maximum value is known as ultimate stress. It is obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

- **Working Stress :-**

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

(**Note:** By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactorily)

### Factor of Safety:

It is defined, as the ratio of the maximum stress to the working stress.  
Mathematically,

$$\text{Factor of safety} = \frac{\text{(Maximum Stress)}}{\text{(Working or design stress)}}$$

In case of ductile material example mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress.

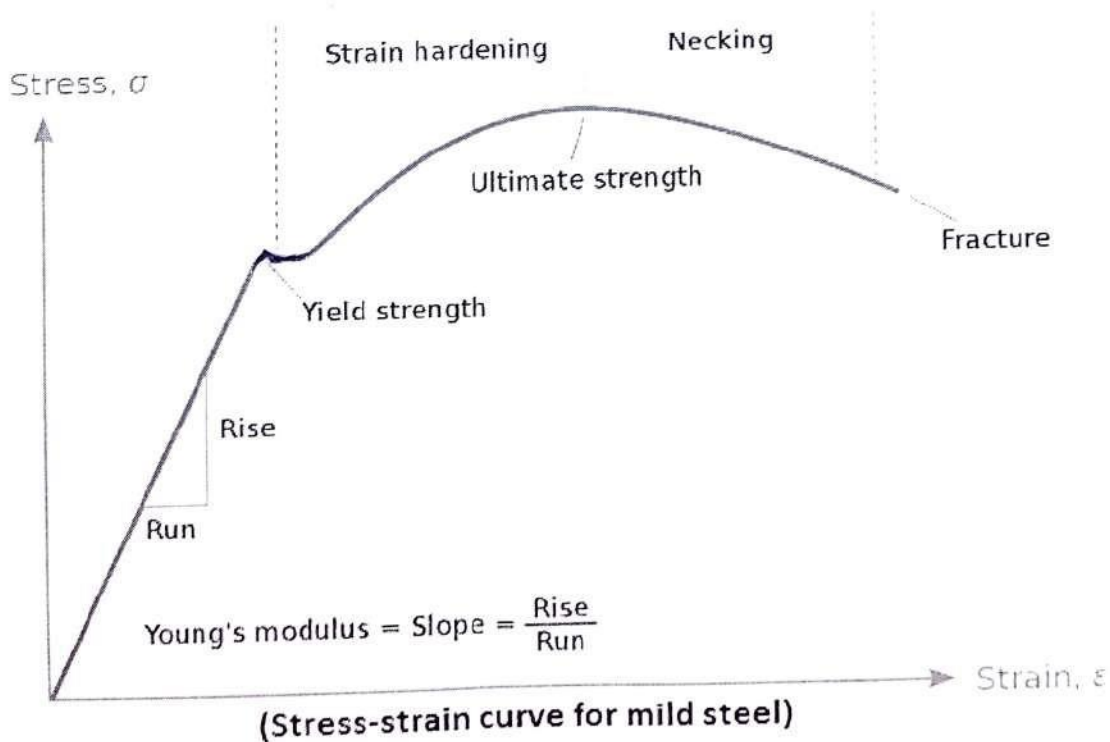
In such cases,

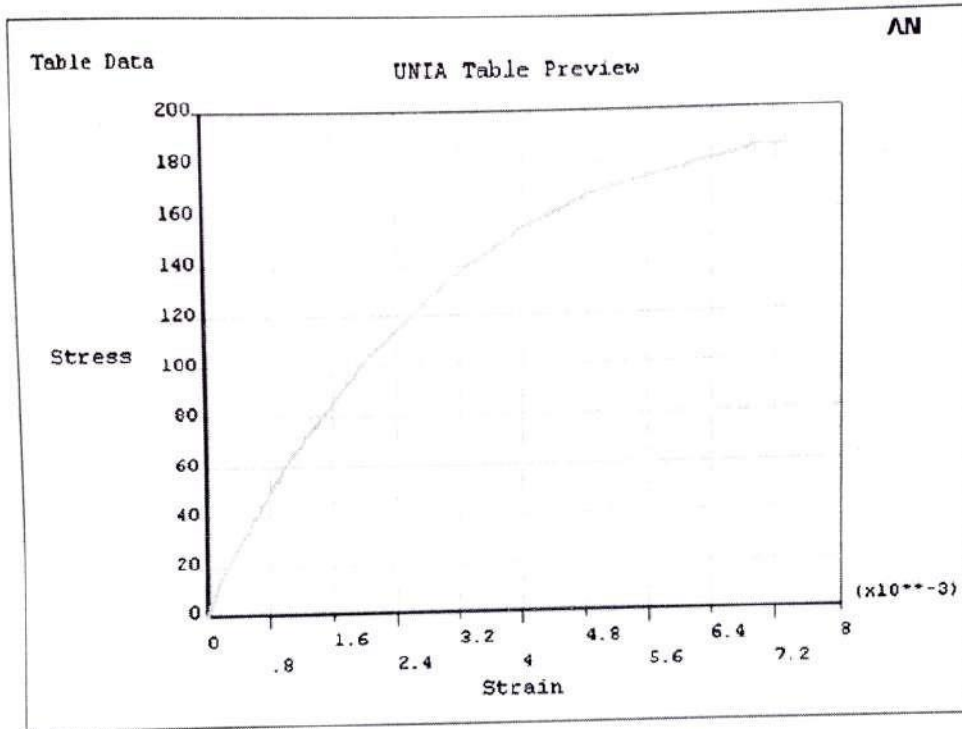
$$\text{Factor of safety} = \frac{\text{(Yield point stress)}}{\text{(Working or design stress)}}$$

In case of brittle materials example cast iron, the yield point is not well defined as for ductile materials.

Therefore, the factor of safety for brittle material is based on ultimate stress.

$$\therefore \text{Factor of safety} = \frac{\text{(Ultimate stress)}}{\text{(Working or design stress)}}$$





(Stres-strain curve for cast iron)

### Mode of Failure to be considered in Machine design

#### 1. By Elastic deflection:

- In the transmission system, the shaft which support gears are subjected to load which causes deflection of shaft.
- The maximum force which can be applied on the shaft is limited by the permissible elastic deflection.
- Lateral or torsional rigidity is the criteria for designing such components.
- Most effective method of decreasing the defection of a member is by changing its shape or c/s dimension.

## **2. Yielding:**

- Mechanical component made of ductile material loses its engineering usefulness due to a large amount of plastic deformation or yielding of a considerable portion of the member.
- To avoid failure due to yielding working stress for ductile material is always less than they yield stress.

## **3. Failure by fracture:**

- Sudden fracture of brittle materials.
- Fracture of cracked or flawed members.
- Progressive fracture due to cyclic load (fatigue) and due to low temperature.

## **4. By Fracture:**

- Fracture occurs at much below stress than yield stress if the component has to work under cyclic load.
- Brittle materials cease to function suddenly due to fracture without any plastic deformation.

## **State the factors governing the design of machine elements:**

### **1. Strength:**

A machine part shouldn't fail under the effect of forces acting on it. It should have sufficient strength to avoid failure due to fracture or yielding.

### **2. Rigidity:**

Machine component should be rigid enough, so that it will not deflect or bend due to forces or moment acting on it. A transmission shaft is designed on the basis of lateral rigidity & torsional rigidity.

### **3. Wear resistance :**

A machine component should be wear resistant because wear reduces accuracy of machine tool along with its life cycle. Surface hardening will increase the wear resistance of the machine component.

#### **4. Safety:**

The shape and dimension of the machine part should ensure safety to the operator of the machine.

#### **5. Minimum dimension and weight :**

A machine should have minimum possible dimension & weight which will reduce the material cost.

#### **6. Conformance to the standard:**

Machine part should conform to the national & international standards covering the dimension , profile & material.

#### **7. Minimum life cycle cost:**

Total cost i.e to be paid for purchasing the parts, operating & maintaining it for its life span should be minimum.

#### **Describe design procedure:**

The general procedure to solve a design problem is as follows:

##### **1. Recognition of need :**

First of all the need, aim or purpose for which the machine is to be designed should be recognized.

##### **2. Mechanism:**

Select the possible mechanism or group of mechanisms which will give the desired motion.

##### **3. Analysis of forces:**

Find the forces acting on each member of the machine and the energy transmitted by each member.

##### **4. Material selection:**

Select the material best suited for each member of the machine.

##### **5. Design of elements:**

Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used . it should be kept in mind that each member should not deflect or deform more than the permissible limit.

**6. Modification:**

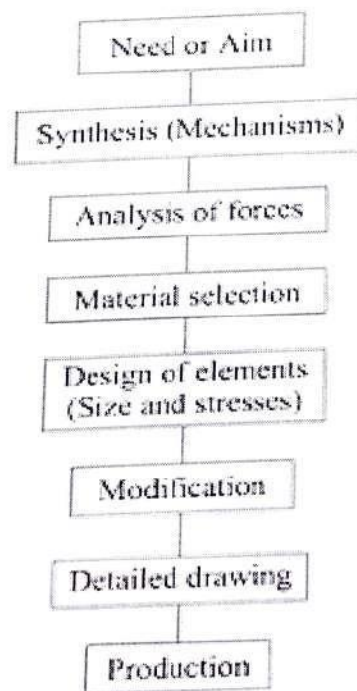
Modify the size of the member to reduce overall cost.

**7. Detailed drawing:**

Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes.

The detailed drawing along with material of each component.

The flow chart for the general procedure in machine design is shown in fig.



General procedure in Machine Design

## Design of machine elements

Joint- A mechanical joint is a type of joint that connects one or more mechanical surfaces. Machines such as lathes and milling machines contains hundreds or thousands of individual moving parts. In areas where these components meet, a mechanical joint is to be used. The defining characteristic of mechanical joints is that they are used exclusively to connect multiple mechanical parts.

Mechanical joints are of two types.

Permanent and temporary joints.

Temporary Joining	Permanent Joining
A temporary joint can be dismantled without breaking the assembled parts.	A permanent joint cannot be dismantled without breaking parts.
Temporary joining is beneficial where frequent assembly and disassembly are required.	Permanent joining is beneficial where joint is intended to stay fixed for longer period.
Strength of temporary joint is comparatively lower.	Permanent joint offers stronger joining.
Temporary joints are not leak-proof.	Most permanent joining processes provide leak-proof joints.
Temporary joining processes offer easy and cost efficient inspection, repair and maintenance as parts can be dismantled without breaking.	Inspection, repair and maintenance are difficult when structures are joined permanently as disassembly is not possible without breaking.
Examples of various temporary joining techniques: <ul style="list-style-type: none"><li>• Fasteners</li><li>• Press fit</li><li>• Cotter joint</li><li>• Knuckle joint.</li></ul>	Examples of various permanent joining techniques: <ul style="list-style-type: none"><li>• Welding</li><li>• Brazing and soldering</li><li>• Riveting</li><li>• Coupling</li></ul>

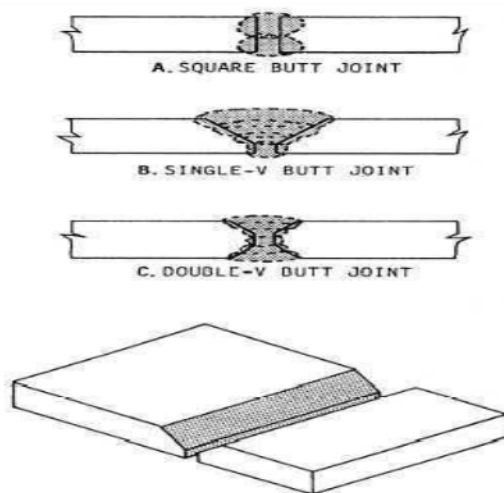


Welded joint-

The joints that are made using welding process is known as welded joint.

Welded joints are classified into two categories.

- 1) Butt joint –A butt joint is a joint where two pieces of metal are placed together in the same plane, and the side of each metal is joined by welding. A butt weld is the most common type of joint that is used in the fabrication of structures and piping systems. It's fairly simple to prepare, and there are many different variations that can be applied to achieve the desired result.
  - ✓ When thickness of plate is less than 5 mm, there is no need to bevel the edges of the plate and it is called square butt joint.
  - ✓ When the thickness of the plates is between 5 to 25 mm, The the edges of the plates are beveled before welding. The edge of the plate takes the shape of letter V. It is called V joint or single welded V joint.
  - ✓ When the thickness of the plates is more than 20 mm, the edges of the plates are



machines to take the shape of U. It is done only from one side. It is called U joint.

- ✓ When the thickness of the plates is more than 30 mm, double welded V joint is used.

Lap joint - A lap welded joint is a joint in which two or more materials that are overlapped on top of one another. The edge of one material is melted and fused with the surface of another material. A lap weld is categorized in the fillet weld category.

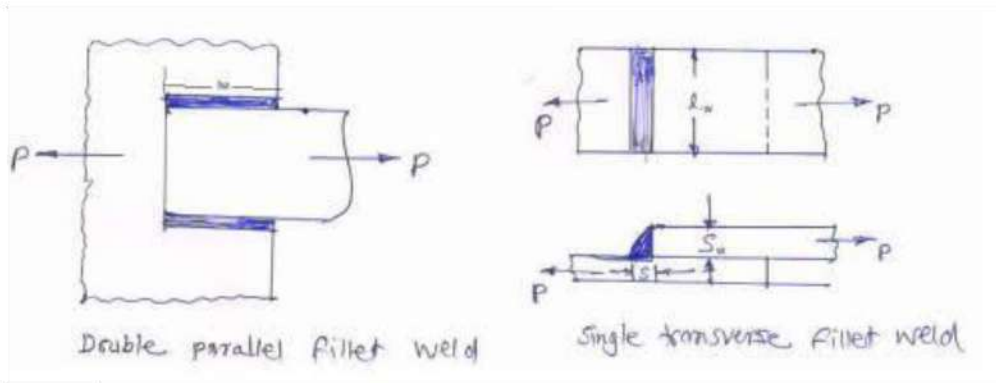
Lap welds are commonly used in welding processes that involve automation. The lap joint, which is used when making a lap weld, is very forgiving to varying part dimensions. That is because satisfactory lap weld fit-up is easily achieved and the member whose flat surface is being welded to the edge of the other material acts as a backing material that reduces the risk of blowing through the weld joint.

## Types of lap weld/fillet weld

- ✓ Longitudinal (parallel) and transverse fillet weld.

When the applied force is parallel to the length of the weld is called longitudinal fillet weld.

When the applied force is perpendicular to the length of the weld is called transverse fillet weld.



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**Advantages and Disadvantages of Welded Joints over Riveted Joints**

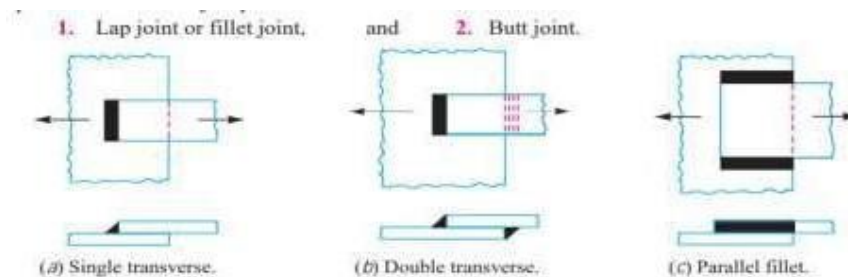
**Advantages**

1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (i.e. circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

**Disadvantages**

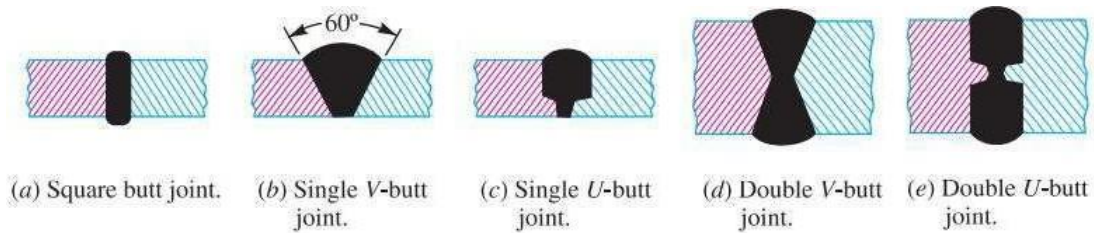
1. As there is an uneven heating and cooling during fabrication, therefore the members gets distorted or additional stresses can be developed.
2. It requires a highly skilled labour and supervision.
3. As there is no provision kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

**Types of Weld joint.**

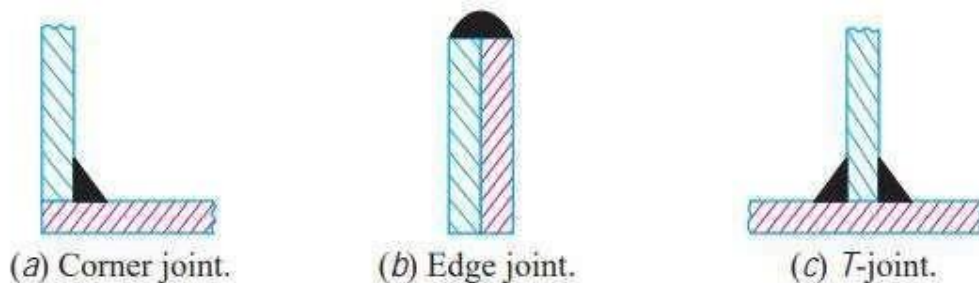


**Type of Lap joint(plate over plate joints)**

### Butt joint(End on end plate joints)



### Special type of weld joint



1. A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

Solution. Given: \*Width = 100 mm ; Thickness = 10 mm ; P = 80 kN = 80 × 10<sup>3</sup> N;

$\tau = 55 \text{ MPa} = 55 \text{ N/mm}^2$

Let l = Length of weld, and s = Size of weld = Plate thickness = 10 mm ... (Given)

We know that maximum load which the plates can carry for double parallel fillet weld (P),

$$80 \times 10^3 = 1.414 \times s \times l \times \tau$$

$$= 1.414 \times 10 \times l \times 55 = 778 l$$

$$\therefore l = 80 \times 10^3 / 778 = 103 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run,

we have  $l = 103 + 12.5 = 115.5 \text{ mm}$  (Ans.)

2. A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 MPa.

Solution. Given: \*Width = 100 mm ; Thickness = 12.5 mm ; P = 50 kN = 50 × 10<sup>3</sup> N ;  $\tau = 56 \text{ MPa} = 56 \text{ N/mm}^2$ .

Let l = Length of weld,

s = Size of weld = Plate thickness = 12.5 mm ... (Given)

We know that the maximum load which the plates can carry for double parallel fillet welds

$$P = 50 \times 10^3 = 1.414 s \times l \times \tau$$

$$= 1.414 \times 12.5 \times l \times 56$$

$$= 990 l$$

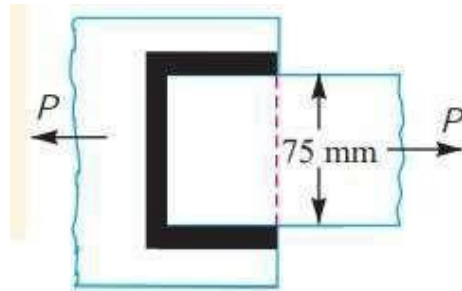
$$\therefore l = 50 \times 10^3 / 990$$

$$= 50.5 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run,

we have  $l = 50.5 + 12.5 = 63 \text{ mm}$  (Ans.)

3. A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. 10.15. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld.



Solution. Given : Width = 75 mm ; Thickness = 12.5 mm ;  $\sigma_t = 70 \text{ MPa} = 70 \text{ N/mm}^2$  ;  $\tau = 56 \text{ MPa} = 56 \text{ N/mm}^2$ .

The effective length of weld ( $L_1$ ) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$$\therefore L_1 = 75 - 12.5 = 62.5 \text{ mm}$$

Length of each parallel fillet for static loading

Let  $L_2 =$  Length of each parallel fillet.

We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} \\ = 75 \times 12.5 \times 70 = 65\,625 \text{ N}$$

Load carried by single transverse weld,

$$P_1 = 0.707 s \times L_1 \times \sigma_t \\ = 0.707 \times 12.5 \times 62.5 \times 70 \\ = 38\,664 \text{ N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 1.414 s \times L_2 \times \tau \\ = 1.414 \times 12.5 \times L_2 \times 56 = 990 L_2 \text{ N}$$

Load carried by the joint ( $P$ ),

$$65\,625 = P_1 + P_2 \\ = 38\,664 + 990 L_2 \\ L_2 = 27.2 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run,

we have  $L_2 = 27.2 + 12.5$

$$= 39.7 \text{ say } 40 \text{ mm (Ans.)}$$

Lecture note 5

Sub-Design of Machine elements

Sem-5<sup>th</sup> Sem Mechanical Diploma

### Eccentrically Loaded Welded Joints

An external load whose line of action is parallel but does not coincide with the centroidal axis of the component/structure on which load is applied is known as an eccentric load. Whereas, the perpendicular distance between the centroidal axis and line of action of the load is known as eccentricity.

#### 1. Eccentrically loaded transverse fillet joint:

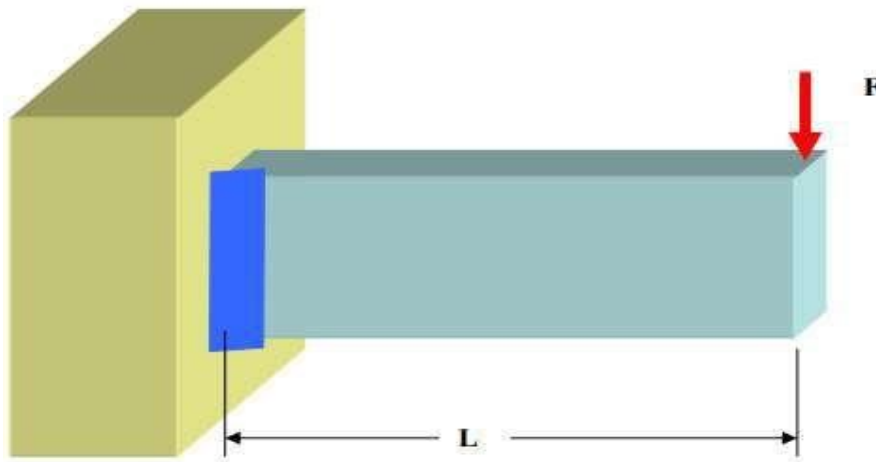


Figure 11.2.1: Eccentrically loaded welded joint

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force P or F acting at the welds, and
2. Bending stress due to the bending moment P (or F) × e.

(a) direct shear stress of magnitude  $\frac{F}{2bt}$ ,

where b = length of the weld

t = thickness at the throat

(b) Indirect shear stress due to bending of the beam

the magnitude of the shear stress is

$$\tau = \frac{FLy}{I_y}$$

$$I_y = tb^3/12$$

$$\tau_{\max} = \sqrt{\left(\frac{F}{2bt}\right)^2 + \left(\frac{3FL}{tb^2}\right)^2}$$

For Shown eccentrically welded joint.

In order to design a safe welded joint

$$\tau_{\max} \leq S_s$$

Where  $S_s$  = shear stress of the weld.

**2. Eccentrically loaded parallel fillet joint:**

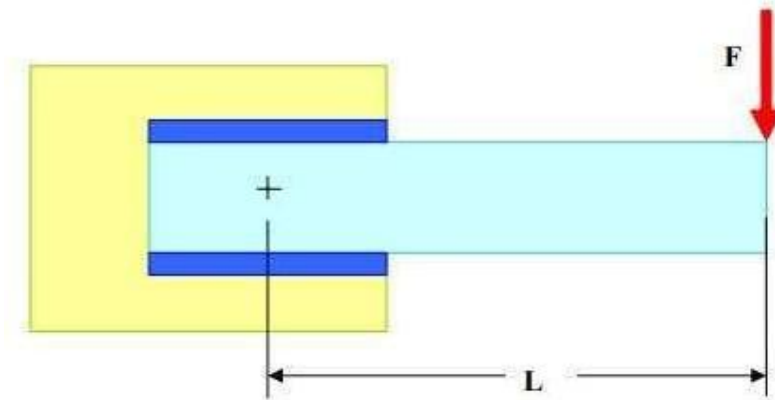


Figure 11.2.3: Eccentrically loaded parallel fillet joint

(a) Direct shear of magnitude  $\frac{F}{2lt}$

where  $l$  = length of the weld

$t$  = thickness of the throat.

**(b) Indirect shear stress owing to eccentricity of the loading.**

The shear stress at a point at a distance  $r$  from the centroid is given by

$$\tau = cr$$

where the proportionality constant =  $c$  is to be calculated using the moment equilibrium equation.

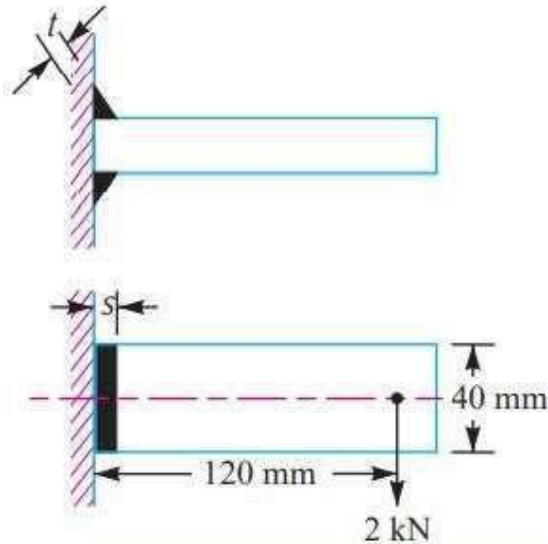
$$C = FL/J$$

$J$  = polar moment of inertia of throat section about the centroid.

The net shear stress at a point is calculated by vector addition of the two kinds of shear stresses

The net stress is calculated using trigonometry or resolution of vectors. And then it is equated with the given stress value to get the required thickness.

1. A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.



Solution. Given:  $P = 2\text{ kN} = 2000\text{ N}$  ;  $e = 120\text{ mm}$  ;  $l = 40\text{ mm}$  ;  $\tau_{\text{max}} = 25\text{ MPa} = 25\text{ N/mm}^2$

Let  $s$  = Size of weld in mm,

and  $t$  = Throat thickness.

The joint, as shown will be subjected to direct shear stress due to the shear force,

$P = 2000\text{ N}$

and bending stress due to the bending moment of  $P \times e$ .

We know that area at the throat,

$$\begin{aligned} A &= 2t \times l \\ &= 2 \times 0.707 s \times l \\ &= 1.414 s \times l \\ &= 1.414 s \times 40 \\ &= 56.56 \times s \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Shear stress, } \tau &= 2000 / 56.56 \times s \\ &= (35.4 / s) \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Bending moment, } M &= P \times e \\ &= 2000 \times 120 \\ &= 240 \times 10^3 \text{ N-mm} \end{aligned}$$

Section modulus of the weld metal through the throat,

$$\begin{aligned} Z &= \frac{t \times l^2}{6} \times 2 \quad \dots(\text{For both sides weld}) \\ &= \frac{0.707 s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242} \end{aligned}$$



$$Z = \frac{s \times l^2}{4.242} = \frac{s (40)^2}{4.242} = 377 \times s \text{ mm}^3$$

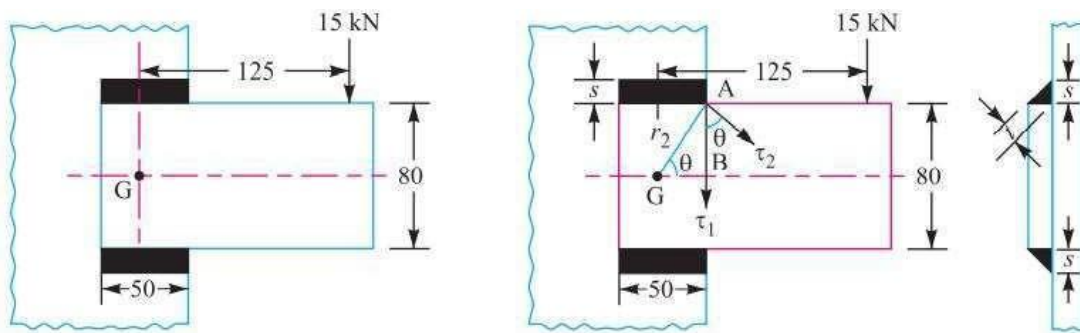
$$\therefore \text{Bending stress, } \sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s} \text{ N/mm}^2$$

We know that maximum shear stress ( $\tau_{max}$ ),

$$25 = \frac{1}{2} \sqrt{(\sigma_b)^2 + 4 \tau^2} = \frac{1}{2} \sqrt{\left(\frac{636.6}{s}\right)^2 + 4 \left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$$\therefore s = 320.3 / 25 = 12.8 \text{ mm Ans.}$$

2.A bracket carrying a load of 15 kN is to be welded as shown in Fig. 10.28. Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.



All dimensions in mm.

Solution. Given :  $P = 15 \text{ kN} = 15 \times 10^3 \text{ N}$ ;  $\tau = 80 \text{ MPa} = 80 \text{ N/mm}^2$ ;  $b = 80 \text{ mm}$ ;  $l = 50 \text{ mm}$ ;  $e = 125 \text{ mm}$

Let  $s =$  Size of weld in mm,

and  $t =$  Throat thickness.

We know that the throat area,

$$A = 2 \times t \times l = 2 \times 0.707 s \times l = 1.414 s \times l = 1.414 \times s \times 50 = 70.7 s \text{ mm}^2$$

$\therefore$  Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{15 \times 10^3}{70.7 s} = \frac{212}{s} \text{ N/mm}^2$$

From design Data book we can find the polar moment of inertia of this type of section as

$$J = \frac{tI(3b^2 + l^2)}{6} = \frac{0.707 \text{ s} \times 50 [3(80)^2 + (50)^2]}{6} \text{ mm}^4$$

$$= 127\,850 \text{ s mm}^4 \quad \dots (\because t = 0.707 \text{ s})$$

From the picture, we find that  $AB = 40 \text{ mm}$  and  $BG = r_1 = 25 \text{ mm}$

$\therefore$  Maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(40)^2 + (25)^2} = 47 \text{ mm}$$

Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{15 \times 10^3 \times 125 \times 47}{127\,850 \text{ s}} = \frac{689.3}{\text{s}} \text{ N/mm}^2$$

and  $\cos \theta = \frac{r_1}{r_2} = \frac{25}{47} = 0.532$

We know that resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2 \tau_1 \times \tau_2 \cos \theta}$$

$$80 = \sqrt{\left(\frac{212}{\text{s}}\right)^2 + \left(\frac{689.3}{\text{s}}\right)^2 + 2 \times \frac{212}{\text{s}} \times \frac{689.3}{\text{s}} \times 0.532} = \frac{822}{\text{s}}$$

$\therefore s = 822 / 80 = 10.3 \text{ mm}$  **Ans.**

Lecture note -6

Sub- Design of machine elements

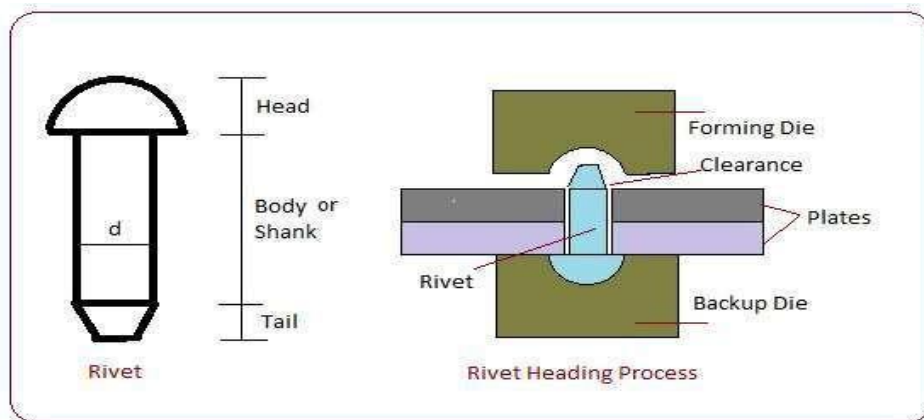
Sem- 5<sup>th</sup> sem Diploma Mechanical engg.

### Riveted joints

A riveted joint is a permanent joint with mainly two components (parts to be joined) which are held together by a rivet with the head at top and tail at the bottom.

Geometry of rivet

In simple language rivet is short cylindrical bar with integral head at top.



### **Rivet Heading Process(Riveting):**

- Rivet heading process is done with the help of forming die and backup die which keep a rivet in between them and by application of force, the rivet is set in the parts to be joined.
- Equal and opposite force makes rivet to deform and tail part of the rivet is converted to head at the bottom so, the complete rivet is seated in the plates.
- In this riveting process, the tail of rivet is converted to 'head' which is sometimes called 'shop head'.
- For riveting parts to be joined are first drilled with the help of drilling machine. Clearance is taken into consideration while riveting because by pressing application diameter of the rivet is somewhat increased.
- Usually clearance is considered as per following:  
If diameter of rivet,  $d = 12$  to  $24$  mm, Clearance,  $C = 1.5$  mm  
If diameter of rivet,  $d = 24$  to  $48$  mm, Clearance,  $C = 2$  mm

### **Rivet material:**

- Usually rivet is made up of wrought iron or soft steel due to lower hardness which is necessary to have easy deformation during riveting.

- Sometimes copper, aluminum are used in corrosive environment. Only material requirements are ductility, toughness and hardness.

**Manufacturing process-** Rivets are made from rolled bars by process of cold heading or cold upsetting.

## Types of riveted joints:

There are mainly two types of riveted joints, based on the rivet arrangement.

1.Lap joints

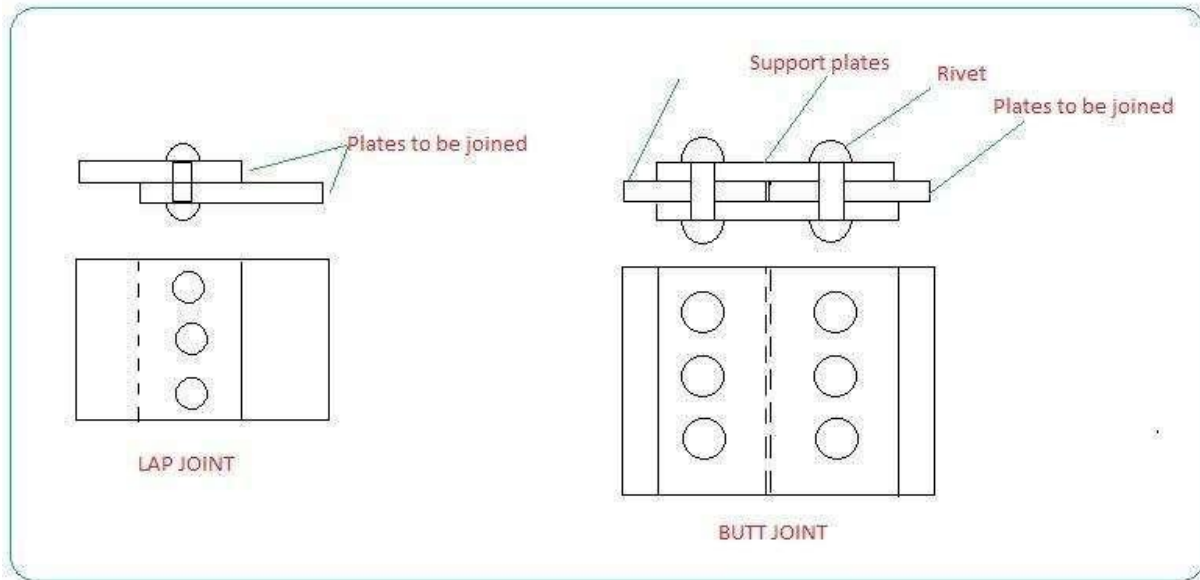
2.Butts joints

- A Lap Joint Is a joint in which one Plate is kept over the other And the Two Plates Are Riveted together.

When The Joint Is Made With Only One Row Of Rivets, It Is Called A Single-riveted Lap Joint.

. A butt joint is a joint in which the two plates to be connected are kept in alignment butting (touching) each other and a cover plate is placed on one side or on both the sides of the two main plates to be joined . The cover plate(s) are then riveted to the main plates. . At least two rows of rivets, one in each connected plate, are necessary to make the joint. The butt joints may be single strap butt joint or double strap butt joints

Both joints are also sub-classified into single riveted and double riveted. Sometimes based on joints strengths, triple-riveted are also possible. Single riveted means one row of the rivet in joint.



### Advantages of riveted joints:

- Cheaper fabrication cost
- Low maintenance cost
- Dissimilar metals can also be joined, even non-metallic joints are possible with riveted joints.
- Ease of riveting process.

### Disadvantages of riveted joints:

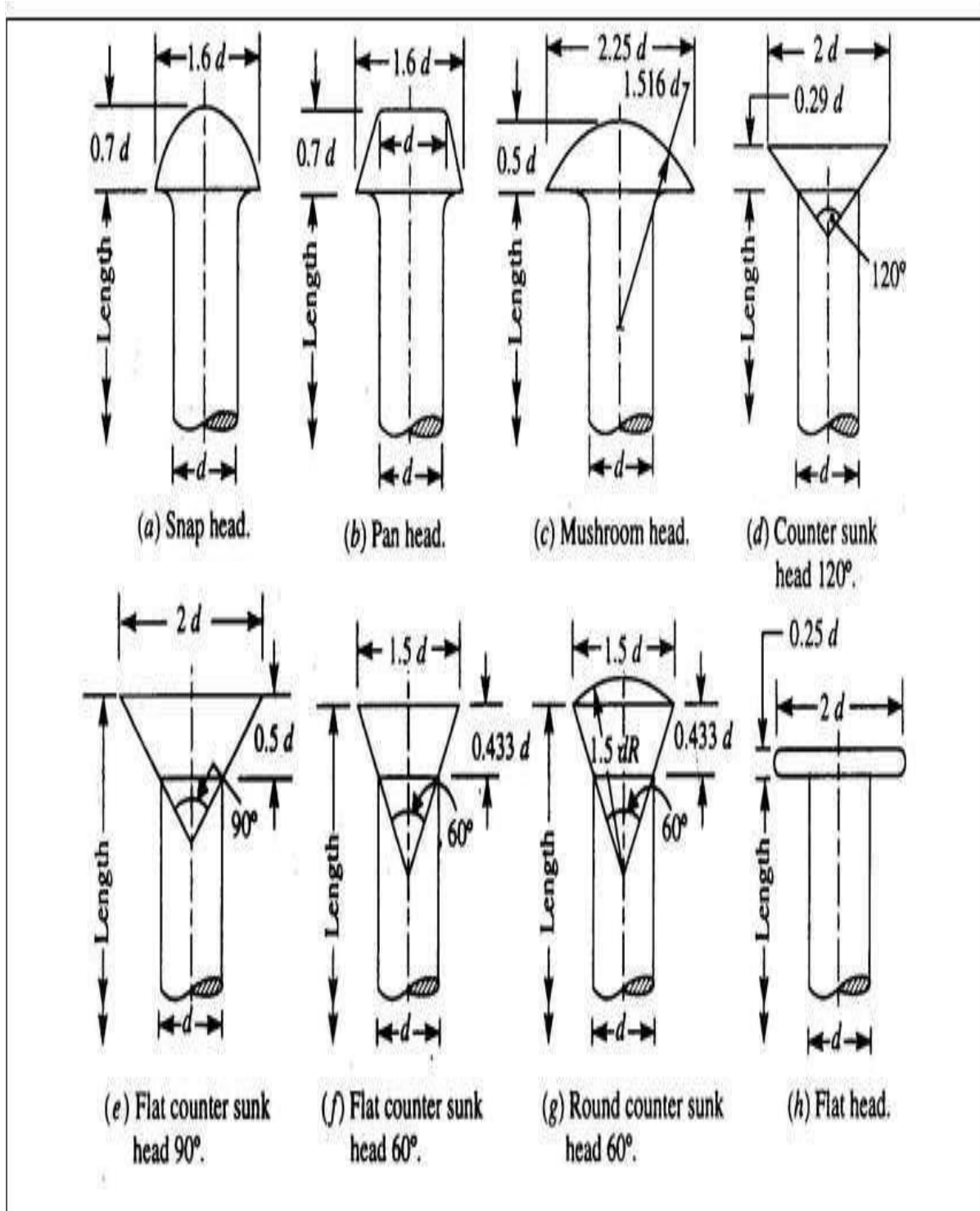
- Skilled workers required
- Leakage may be a problem for this type of joints, but this is overcome by special techniques.

### Applications of riveted joints:

- Boiler shells
- Structures members and bridges parts

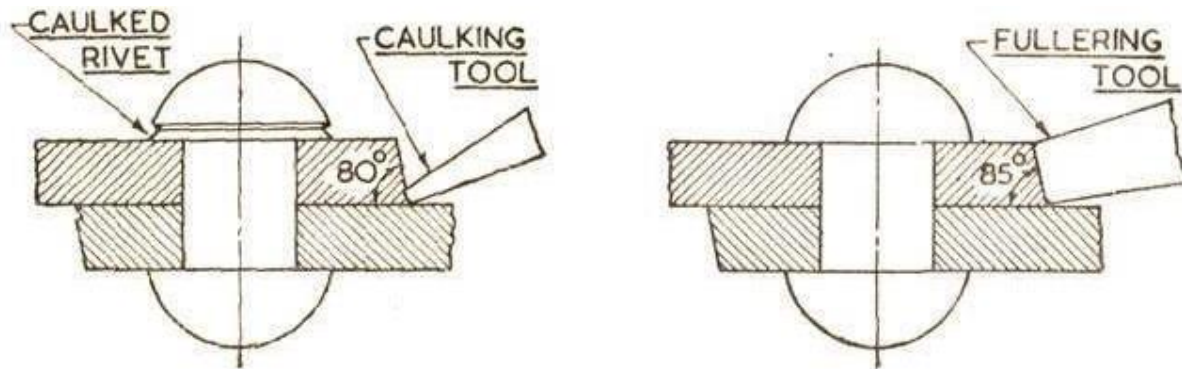
- Railway wagons and coaches
- Buses and trucks

Types of rivets(according to rivet heads)



### Process of producing leak proof joints

The riveted joints are used to produce leak proof joints in case of boilers and ship building works. These joints are made by the processes namely caulking and fullering. The edges of the plates are hammered and driven-in by a caulking tool or a fullering tool. The caulking tool is in the shape of a blunt chisel. Leakage through the hole is prevented by the caulking operation on the edge of the rivet-head. The thickness of the fullering tool is about the same as that of the plates. To facilitate these operations the edges of the plates are usually machined to an angle of about  $80^\circ$  before joining them together. This angle is increased to about  $85^\circ$  after the fullering process.



**Fig.6.6.Caulking and Fullering**

Lecture note-7

Sub- Design of machine elements

Sem- 5<sup>th</sup> Mechanical diploma engg.

### **Important Formulae of Riveted joints**

#### **Tearing strength or Ultimate tearing resistance of the plate per pitch**

$$P_t = (p - d)t \times \sigma_t$$

#### **Shearing Strength or Ultimate shearing resistance of the rivets per pitch**

$$P_s = n \times \pi/4 \times d^2 \times \tau$$

.....(single shear)

$$= n \times 2 \times \pi/4 \times d^2 \times \tau \text{ ..... (Theoretically, in double shear)}$$

$$= n \times 1.875 \times \pi/4 \times d^2$$

$\times \tau$  .....(In double shear, according to IBR)

#### **Crushing strength or Ultimate crushing resistance of the rivets per pitch**

$$P_c = n \times d \times t \times \sigma_c$$

Strength of the riveted joint = Least of  $P_t$ ,  $P_s$  and  $P_c$

#### **Strength of the un-riveted or solid plate per pitch length**

$$P = p \times t \times \sigma_t$$

#### **Efficiency of the riveted joint**

$$\eta = \text{Least of } (p_t, p_s \text{ \& } p_c) / (p \times t \times \sigma_t)$$

#### **Design of rivet joints:**

The design parameters in riveted joints are  $d$ ,  $p$ , and  $m$ .

#### **Diameter of the hole (d):**

When thickness of the plate ( $t$ ) is more than 8 mm, Unwin's formula is used,

$$d = 6\sqrt{t} \text{ mm.}$$

Otherwise is obtained by equating crushing strength to the shear strength of the joint. In a double riveted zigzag joint, this implies



$$P_c = n \times d \times t \times \sigma_c = P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$

(valid for  $t < 8$  mm)

### Pitch ( p ):

Pitch is designed by equating the tearing strength of the plate to the shear strength of the rivets. In a double riveted lap joint, this takes the following form.

$$\sigma_t \times (p-d) t = 2 \times \frac{\pi}{4} d^2 \times \tau$$

But  $p \geq 2d$  in order to accommodate heads of the rivets.

### Margin ( m ):

$$m = 1.5d$$

In order to design boiler joints, a designer must also comply with Indian Boiler Regulations (I.B.R.).

$$(p_b \text{ usually } 0.33 + 0.67d \text{ mm})$$

1. A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume  $\sigma_t = 80$  MPa ;  $\tau = 60$  MPa ; and  $\sigma_c = 120$  MPa. State how the joint will fail and find the efficiency of the joint.

Solution. Nature of Joint: double riveted lap joint with zig-zag riveting

Given data :  $t = 13$  mm ;  $\sigma_t = 80$  MPa = 80 N/mm<sup>2</sup>;  $\tau = 60$  MPa = 60 N/mm<sup>2</sup>;  $\sigma_c = 120$  MPa = 120 N/mm<sup>2</sup>

#### 1. Diameter of rivet

Since the thickness of plate is greater than 8 mm,

$$\text{therefore diameter of rivet hole, } d = 6\sqrt{t} = 6\sqrt{13} = 21.6 \text{ mm}$$

From design data book, the standard size of the rivet hole ( d ) is 23 mm and the corresponding diameter of the rivet is 22 mm.

#### Pitch of rivets

Let  $p$  = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting , therefore there are two rivets per pitch length, i.e.  $n = 2$ .

Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

$$P_t = (p - d) t \times \sigma_t = (p - 23) 13 \times 80 = (p - 23) 1040 \text{ N} \dots \dots \dots (i)$$

and shearing resistance of the rivets,  $P_s = n \times \pi/4 d^2 \times \tau$

$$= 2 \times \pi/4 (23)^2$$

$$60 = 49 864 \text{ N} \dots \dots \dots (ii) \dots$$

(There are two rivets in single shear)

From equations (i) and (ii),

$$p - 23 = 49864/1040 = 48 \text{ or}$$

$$p = 48 + 23 = 71 \text{ mm}$$

The maximum pitch is given by,  $p_{\max} = C \times t + 41.28 \text{ mm}$

From design data book, we find that for 2 rivets per pitch length, the value of C is 2.62.

$$\therefore p_{\max} = 2.62 \times 13 + 41.28 = 75.28 \text{ mm}$$

Since  $p_{\max}$  is more than  $p$ , therefore we shall adopt  $p = 71 \text{ mm}$  Ans.

**Distance between the rows of rivets**

We know that the distance between the rows of rivets (for zig-zag riveting),

$$p_b = 0.33 p + 0.67 d = 0.33 \times 71 + 0.67 \times 23 \text{ mm} = 38.8 \text{ say } 40 \text{ mm Ans.}$$

**Margin**

We know that the margin,

$$m = 1.5 d = 1.5 \times 23 = 34.5 \text{ say } 35 \text{ mm Ans.}$$

**Tearing resistance of the plate**

$$P_t = (p - d) t \times \sigma_t = (71 - 23) 13 \times 80 = 49 920 \text{ N}$$

**Shearing resistance of the rivets**

$$P_s = n \times \pi/4 \times d^2 \times \tau = 2 \times \pi/4 (23)^2 \times 60 = 49 864 \text{ N}$$

**Crushing resistance of the rivets**

$$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71 760 \text{ N}$$

The least of  $P_t$ ,  $P_s$  and  $P_c$  is  $P_s = 49 864 \text{ N}$ .

Hence the joint will fail due to shearing of the rivets.

**Strength of the un riveted plate per pitch length**

$$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73840 \text{ N}$$

$$\text{Efficiency of joint} = \text{Least of } (P_t, P_s \text{ and } P_c) / P = 49 864 / 73840$$

Efficiency of the joint = 0.675 or 67.5%

Lecture notes

Subject-Design of machine elements

Sem-5<sup>th</sup> Mechanical diploma

### Design of shaft

Shaft is a common machine element which is used to transmit rotary motion or torque. It generally has circular cross-section and can be solid or hollow. Shafts are supported on the bearings and transmit torque with the help of gears, belts and pulleys etc. Shafts are generally subjected to bending moment, torsion and axial force or a combination of these three. So the shafts are designed depending upon the combination of loads it is subjected to. Spindle stub and axle are some important types of shaft. Small shaft is called spindle. Shaft integral part of the prime mover is called stub shaft. An axle is a non-rotating member that carries no torque and is used to support rotating wheels, pulleys etc. And therefore it is subjected to bending moment only.

### Shaft Materials

Hot-rolled plain carbon steel is the least expensive material used for shafts. These essentially require machining to remove the scales of hot rolling process. Cold rolled plain carbon steel provides better yield strength and endurance strength but the cold working induces residual stresses. Surface is smooth in this case and amount of machining therefore is minimal. It is used for general purpose transmission shafts. When a shaft is to work under severe loading and corrosive conditions and require more strength, alloy steels are used, generally having Ni, Cr, Mo and V as alloying elements. Alloy steels are expensive. Sometimes shafts are heat treated to improve hardness and shock resistance and surface hardening techniques are also used if high wear resistance is the requirement. As the shafts transmitting power are subjected to fatigue loading, therefore higher factor of safety of 3 to 4 is used on the basis of yield strength for static load analysis.

### Design of shafts

Shafts are designed on the basis of strength or rigidity or both. Design based on strength is to ensure that stress at any location of the shaft does not exceed the material yield stress. Design based on rigidity is to ensure that maximum deflection (because of bending) and maximum twist (due to torsion) of the shaft is within the allowable limits. Rigidity consideration is also very important in some cases for example position of a gear mounted on the shaft will change if

the shaft gets deflected and if this value is more than some allowable limit, it may lead to high dynamic loads and noise in the gears.

In designing shafts on the basis of strength, the following cases may be considered:

- (a) Shafts subjected to torque
- (b) Shafts subjected to bending moment
- (c) Shafts subjected to combination of torque and bending moment
- (d) Shafts subjected to axial loads in addition to combination of torque and bending moment

### Shafts Subjected to Torque

Maximum shear stress developed in a shaft subjected to torque is given by,

$$\tau = \frac{T r}{J} \leq [\tau]$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

$$= \frac{\pi d^4}{32} \quad \text{for solid shafts with diameter } d$$

$$= \frac{\pi (d_o^4 - d_i^4)}{32} \quad \text{for hollow shafts with } d_o \text{ and } d_i \text{ as outer and inner}$$

diameter.

r = Distance from neutral axis to the outer most fibre = d/2 (or d<sub>o</sub>/2)

So dimensions of the shaft subjected to torque can be determined from above relation for a known value of allowable shear stress, [τ].

1. Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Ans-Given : P = 20 kW = 20 × 10<sup>3</sup> W ; N = 200 r.p.m.

τ<sub>u</sub> = 360 MPa = 360 N/mm<sup>2</sup> ; Factor of safety. = 8

$$k = d_i / d_o = 0.5$$

We know that the allowable shear stress,

$$\tau = 360/8$$

$$\tau = 45 \text{ N/mm}^2$$

#### **Diameter of the solid shaft**

Let  $d$  = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft ( $T$ ),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84 d^3$$

$$\therefore d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm } \mathbf{Ans.}$$

#### **Diameter of hollow shaft**

Let  $d_i$  = Inside diameter, and

$d_o$  = Outside diameter.

We know that the torque transmitted by the hollow shaft ( $T$ ),

$$\begin{aligned} 955 \times 10^3 &= \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \\ &= \frac{\pi}{16} \times 45 (d_o)^3 [1 - (0.5)^4] = 8.3 (d_o)^3 \end{aligned}$$

$$\therefore (d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm } \mathbf{Ans.}$$

and  $d_i = 0.5 d_o = 0.5 \times 50 = 25 \text{ mm } \mathbf{Ans.}$

Lecture notes

Subject-Design of machine elements

Sem-5<sup>th</sup> Mechanical diploma

## Design of shaft

### Shafts Subjected to Bending Moment

Maximum bending stress developed in a shaft is given by,

$$\sigma_b = \frac{M y}{I} \leq [\sigma_t]$$

where M = Bending Moment acting upon the shaft,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation

$$= \frac{\pi d^4}{64} \quad \text{for solid shafts with diameter } d$$

$$= \frac{\pi (d_o^4 - d_i^4)}{64} \quad \text{for hollow shafts with } d_o \text{ and } d_i \text{ as outer and inner diameter.}$$

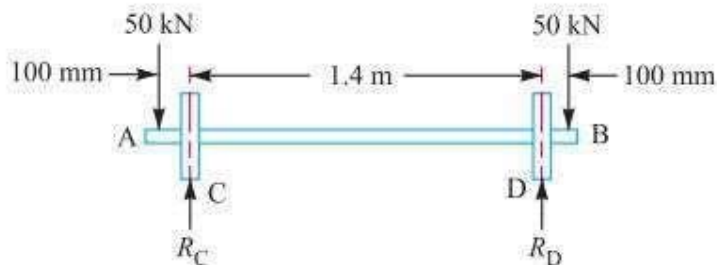
y = Distance from neutral axis to the outer most fibre = d / 2 (or d<sub>o</sub>/2)

So dimensions of the shaft subjected to bending moment can be determined from above relation for a known value of allowable tensile stress.

1. A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Ans-

Given :  $W = 50 \text{ kN} = 50 \times 10^3 \text{ N}$  ;  $L = 100 \text{ mm}$  ;  $x = 1.4 \text{ m}$  ;  $\sigma_b = 100 \text{ MPa} = 100 \text{ N/mm}^2$



Maximum bending moment- load  $\times$  distance

$$\begin{aligned}M &= W \times l \\ &= 50 \times 10^3 \times 100 \\ &= 5 \times 10^6 \text{ N-mm}\end{aligned}$$

We know that the maximum bending moment ( $M$ ),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82 d^3$$

$$\therefore d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm Ans.}$$

### Shafts Subjected to Combination of Torque and Bending Moment

When the shaft is subjected to combination of torque and bending moment, principal stresses are calculated and then different theories of failure are used. Bending stress and torsional shear stress can be calculated using the above relations.

$$\begin{aligned}\tau &= \frac{T r}{J} = \frac{T \frac{d}{2}}{\frac{\pi}{32} d^4} = \frac{16 T}{\pi d^3} \\ \sigma_b &= \frac{M y}{I} = \frac{M \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32 M}{\pi d^3}\end{aligned}$$

### Maximum Shear Stress Theory

Maximum shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \sqrt{\left(\frac{16 M}{\pi d^3}\right)^2 + \left(\frac{16 T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2} \leq [\tau]$$

$\sqrt{M^2 + T^2}$  is called equivalent torque,  $T_e$ , such that

$$\tau_{max.} = \frac{T_e r}{J} \leq [\tau]$$

### Maximum Principal Stress Theory

Maximum principal stress is given by,



$$\sigma = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \frac{16 M}{\pi d^3} + \sqrt{\left(\frac{16 M}{\pi d^3}\right)^2 + \left(\frac{16 T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}] \leq [\sigma_t]$$

$[M + \sqrt{M^2 + T^2}]$  is called equivalent bending moment,  $M_e$ , such that

$$\sigma = \frac{M_e y}{I} \leq [\sigma_t]$$

1. A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Ans-

**Solution.** Given :  $M = 3000 \text{ N-m} = 3 \times 10^6 \text{ N-mm}$  ;  $T = 10\ 000 \text{ N-m} = 10 \times 10^6 \text{ N-mm}$  ;  
 $\sigma_{tu} = 700 \text{ MPa} = 700 \text{ N/mm}^2$  ;  $\tau_u = 500 \text{ MPa} = 500 \text{ N/mm}^2$

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let  $d =$  Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment ( $T_e$ ),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36 d^3$$

$$\therefore d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$\begin{aligned}M_e &= \frac{1}{2} \left( M + \sqrt{M^2 + T^2} \right) = \frac{1}{2} (M + T_e) \\ &= \frac{1}{2} (3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}\end{aligned}$$

We also know that the equivalent bending moment ( $M_e$ ),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46 d^3$$

$$\therefore d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 86 \text{ say } 90 \text{ mm Ans.}$$

Sub-Design of machine elements

Sem-5<sup>th</sup> sem mechanical Diploma

## Design of coupling-

### Coupling-

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

### Use of Coupling

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.

### Requirements of a Good Shaft Coupling

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
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### Types of Shafts Couplings

1. Rigid coupling. It is used to connect two shafts which are perfectly aligned. e.g- sleeve coupling, flange coupling, compression coupling
2. Flexible coupling-It is used to connect two shafts having both lateral and angular misalignment. E.g-Bushed, Oldham couplings

#### Sleeve or Muff-coupling-

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. So all the elements must be strong enough to transmit the torque.

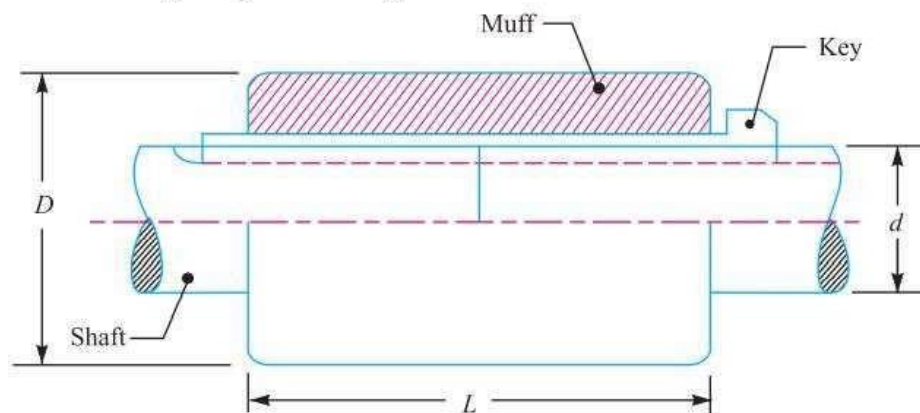
The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve,  $D = 2d + 13 \text{ mm}$  and

length of the sleeve,  $L = 3.5 d$

where  $d$  is the diameter of the shaft.

### Design for sleeve



Let  $T$  = Torque to be transmitted by the coupling, and  
 $\tau_c$  = Permissible shear stress for the material of the sleeve which is cast iron.  
 The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \quad \dots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

### Design for key

The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = L/2$$

$$= 3.5d/2$$

The torque transmitted is

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots \text{(Considering crushing of the key)}$$

Q) Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Ans

**Solution.** Given :  $P = 40 \text{ kW} = 40 \times 10^3 \text{ W}$ ;  
 $N = 350 \text{ r.p.m.}$ ;  $\tau_s = 40 \text{ MPa} = 40 \text{ N/mm}^2$ ;  $\sigma_{cs} = 80 \text{ MPa} = 80 \text{ N/mm}^2$ ;  $\tau_c = 15 \text{ MPa} = 15 \text{ N/mm}^2$

### 1. Design for shaft

Let  $d$  = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2 \pi N} = \frac{40 \times 10^3 \times 60}{2 \pi \times 350} = 1100 \text{ N-m} \quad \text{or}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted ( $T$ ),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86 d^3$$

$$\therefore d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm} \quad \text{Ans.}$$

Using Design data book the diameter of sleeve is taken .

## 2. Design for sleeve

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm Ans.}$$

and length of the muff,

$$L = 3.5 d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm Ans.}$$

So the torque transmitted is

$$\begin{aligned} 1100 \times 10^3 &= \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[ \frac{(125)^4 - (55)^4}{125} \right] \\ &= 370 \times 10^3 \tau_c \\ \therefore \tau_c &= 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2 \end{aligned}$$

It is less than the given value i.e 15 Mpa. So design is safe.

Design of key-

From design data book we can get directly value of keys

$$w = t = 18 \text{ mm}$$

$$l = L/2$$

$$= 97.5 \text{ mm}$$

Now ,calculating the induced crushing stress and shear stress

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots \text{ (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots \text{ (Considering crushing of the key)}$$

Considering Crushing strength we have

$$\begin{aligned} 1100 \times 10^3 &= l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs} \\ \sigma_{cs} &= 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2 \end{aligned}$$

It is less than the given value i.e 80 Mpa.

Considering shear strength we have,

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$
$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

It is less than the given value i.e 40 Mpa .

So design of coupling is safe.

## Lecture note-11

Sub- Design of machine elements

Sem- 5<sup>th</sup> Mechanical (Diploma)

### Keys and key ways

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

### Types of keys

1. Sunk keys
2. Saddle keys
3. Tangent keys
4. Round keys
5. Splines

### Sunk key

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. Rectangular sunk key. A rectangular sunk key is shown in the picture. The usual proportions of this key are  
Width =  $w = d/4$   
Thickness =  $t = 2w/3 = d/6$   
Diameter of shaft or Hole in the hub.



2. Square sunk key- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal.

$$W = t = d/4$$

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section

uniform in width and thickness throughout. It can be taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. . Gib-head key. It is a rectangular sunk key with a head at one end known as gib head.

It is usually provided to facilitate the removal of key.

$$W = d/4$$

$$t = 2w/3 = d/6$$

5. Feather key. A key attached to one member of a pair and which permits relative axial

movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



### Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and 2. Hollow saddle key.

A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft.

A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key

is shaped to fit the curved surface of the shaft.

### Tangent Keys

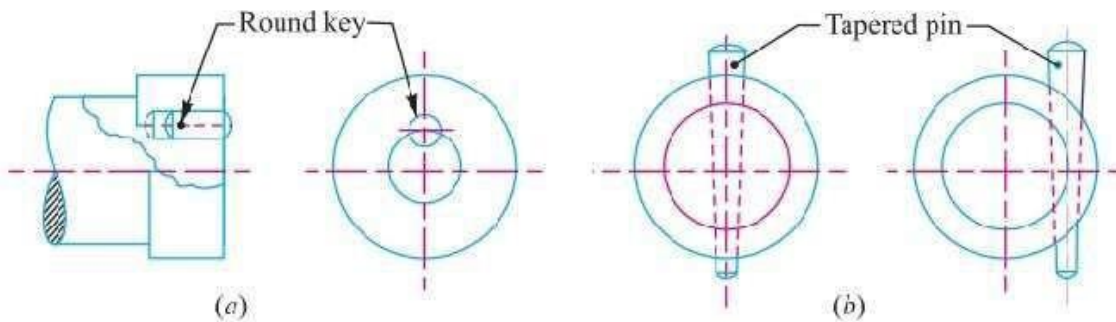
The tangent keys are fitted in pair at right angle. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

### Round keys

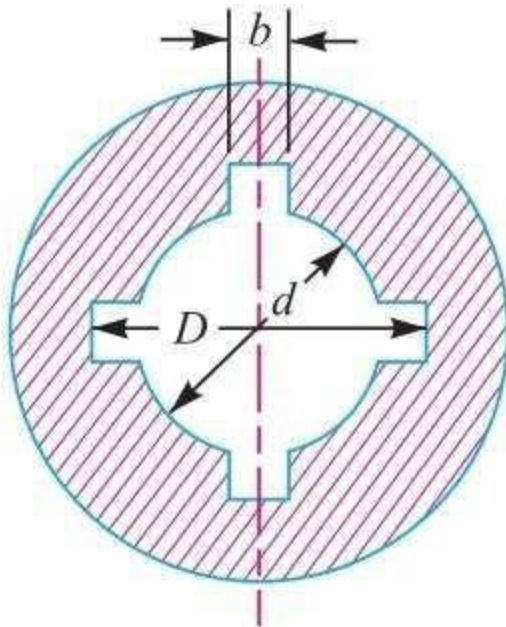
These are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

### Splines

When keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as splined shafts. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.



**Fig. 13.7.** Round keys.



$$D = 1.25 d \text{ and } b = 0.25 D$$

**Fig. 13.8. Splines.**

### Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key.

1. Forces ( $F_1$ ) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered

key driven in place. These forces produce compressive stresses in the key which are difficult

to determine in magnitude.

2. Forces ( $F$ ) due to the torque transmitted by the shaft. These forces produce shearing and

compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting

of the shaft within the hub.

### Strength of a Sunk Key

Let  $T$  = Torque transmitted by the shaft,

$F$  = Tangential force acting at the circumference of the shaft,

$d$  = Diameter of shaft,

$l$  = Length of key,

$w$  = Width of key.

$t$  = Thickness of key, and

$\tau$  and  $\sigma_c$  = Shear and crushing stresses for the material of key.

If we consider the shearing of the key, the tangential shearing force acting at the circumference of the

shaft,

$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$

Torque transmitted by the shaft,

$T = F \times d/2$

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \dots(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots[\text{Equating equations (i) and (ii)}]$$

or 
$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \quad \dots(iii)$$

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \dots(v)$$

...(Taking  $\tau_1$  = Shear stress for the shaft material)

From equations (iv) and (v), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571 d \times \frac{\tau_1}{\tau} \quad \dots \text{(Taking } w = d/4) \quad \dots(vi)$$

1.Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

Ans-

**Solution.** Given :  $d = 50 \text{ mm}$  ;  $\tau = 42 \text{ MPa} = 42 \text{ N/mm}^2$  ;  $\sigma_c = 70 \text{ MPa} = 70 \text{ N/mm}^2$

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key,  $w = 16 \text{ mm}$  **Ans.**

and thickness of key,  $t = 10 \text{ mm}$  **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let  $l =$  Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800 \text{ l N-mm} \quad \dots(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 (50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \dots(ii)$$

From equations (i) and (ii), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750 \text{ l N-mm} \quad \dots(iii)$$

From equations (ii) and (iii), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$$l = 117.7 \text{ say } 120 \text{ mm} \text{ **Ans.**}$$

A design data book should be used to have the proper dimension of key.

2. A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Ans-

**Solution.** Given :  $d = 45 \text{ mm}$  ;  $\sigma_{yt}$  for shaft =  $400 \text{ MPa} = 400 \text{ N/mm}^2$  ;  $w = 14 \text{ mm}$  ;  
 $t = 9 \text{ mm}$  ;  $\sigma_{yt}$  for key =  $340 \text{ MPa} = 340 \text{ N/mm}^2$  ;  $F.S. = 2$

Let  $l =$  Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100 (45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted ( $T$ ),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775 \, l$$

$$\therefore l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key ( $T$ ),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213 \, l$$

$$\dots \left( \text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.} \right)$$

$$\therefore l = 1.8 \times 10^6 / 17\,213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

$$l = 104.6 \text{ say } 105 \text{ mm Ans.}$$

Effect of Keyways- The keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. In other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.



$$e = 1 - 0.2 \left( \frac{w}{d} \right) - 1.1 \left( \frac{h}{d} \right)$$

$e$  = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway.

$w$  = Width of keyway,

$d$  = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

3. A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Ans-

**Solution.** Given :  $P = 15 \text{ kW} = 15 \times 10^3 \text{ W}$  ;  $N = 960 \text{ r.p.m.}$  ;  $d = 40 \text{ mm}$  ;  $l = 75 \text{ mm}$  ;  
 $\tau = 56 \text{ MPa} = 56 \text{ N/mm}^2$  ;  $\sigma_c = 112 \text{ MPa} = 112 \text{ N/mm}^2$

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2 \pi N} = \frac{15 \times 10^3 \times 60}{2 \pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let  $w$  = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted ( $T$ ),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3 w$$

$$\therefore w = 149 \times 10^3 / 84 \times 10^3 = 1.8 \text{ mm}$$

This width of keyway is too small. The width of keyway should be at least  $d/4$ .

$$\therefore w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm Ans.}$$

Since  $\sigma_c = 2\tau$ , therefore a square key of  $w = 10 \text{ mm}$  and  $t = 10 \text{ mm}$  is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2 \left( \frac{w}{d} \right) - 1.1 \left( \frac{h}{d} \right) = 1 - 0.2 \left( \frac{w}{d} \right) - 1.1 \left( \frac{t}{2d} \right) \quad \dots (\because h = t/2)$$
$$= 1 - 0.2 \left( \frac{10}{40} \right) - \left( \frac{10}{2 \times 40} \right) = 0.8125$$

$\therefore$  Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56 (40)^3 \times 0.8125 = 571\,844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47 \text{ Ans.}$$

## Design of coupling-

### Coupling-

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

### Use of Coupling

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.

### Requirements of a Good Shaft Coupling

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
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#### Sleeve or Muff-coupling-

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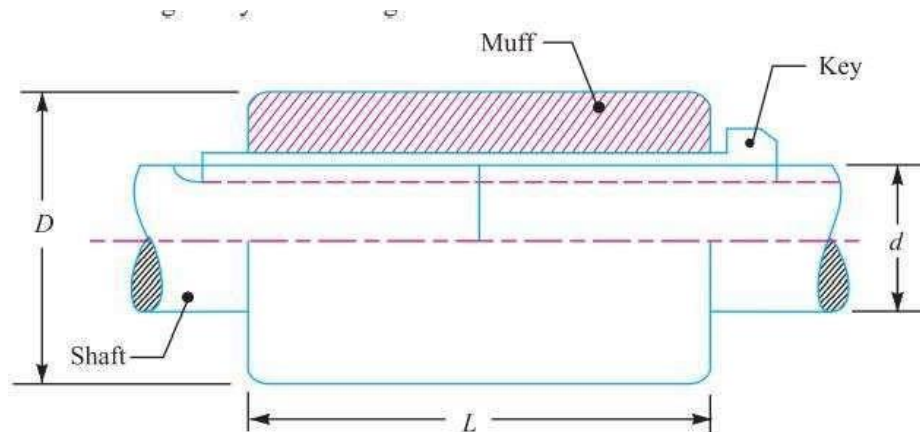
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Outer diameter of the sleeve,  $D = 2d + 13 \text{ mm}$  and

length of the sleeve,  $L = 3.5 d$

where  $d$  is the diameter of the shaft.

### Design for sleeve



Let  $T$  = Torque to be transmitted by the coupling, and  
 $\tau_c$  = Permissible shear stress for the material of the sleeve which is cast iron.  
 The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \quad \dots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

### Design for key

The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = L/2$$

$$= 3.5d/2$$

The torque transmitted is

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots \text{(Considering crushing of the key)}$$

Q) Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Ans

**Solution.** Given :  $P = 40 \text{ kW} = 40 \times 10^3 \text{ W}$ ;  
 $N = 350 \text{ r.p.m.}$ ;  $\tau_s = 40 \text{ MPa} = 40 \text{ N/mm}^2$ ;  $\sigma_{cs} = 80 \text{ MPa} = 80 \text{ N/mm}^2$ ;  $\tau_c = 15 \text{ MPa} = 15 \text{ N/mm}^2$

### 1. Design for shaft

Let  $d$  = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2 \pi N} = \frac{40 \times 10^3 \times 60}{2 \pi \times 350} = 1100 \text{ N-m} \quad \text{or}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted ( $T$ ),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86 d^3$$

$$\therefore d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm} \quad \text{Ans.}$$

Using Design data book the diameter of sleeve is taken .

## 2. Design for sleeve

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm Ans.}$$

and length of the muff,

$$L = 3.5 d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm Ans.}$$

So the torque transmitted is

$$\begin{aligned} 1100 \times 10^3 &= \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[ \frac{(125)^4 - (55)^4}{125} \right] \\ &= 370 \times 10^3 \tau_c \\ \therefore \tau_c &= 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2 \end{aligned}$$

It is less than the given value i.e 15 Mpa. So design is safe.

Design of key-

From design data book we can get directly value of keys

$$w = t = 18 \text{ mm}$$

$$l = L/2$$

$$= 97.5 \text{ mm}$$

Now ,calculating the induced crushing stress and shear stress

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots \text{ (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots \text{ (Considering crushing of the key)}$$

Considering Crushing strength we have

$$\begin{aligned} 1100 \times 10^3 &= l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs} \\ \sigma_{cs} &= 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2 \end{aligned}$$

It is less than the given value i.e 80 Mpa.

Considering shear strength we have,

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$
$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

It is less than the given value i.e 40 Mpa .

So design of coupling is safe.

Lecture note

Sem-5<sup>th</sup> Mechanical

Sub-Design of machine elements

### Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. In the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called surge.

The natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 n} \sqrt{\frac{6Gg}{\rho}} \text{ cycles/s}$$

where

$d$  = Diameter of the wire,

$D$  = Mean diameter of the spring,

$n$  = Number of active turns,

$G$  = Modulus of rigidity,

$g$  = Acceleration due to gravity, and

$\rho$  = Density of the material of the spring.

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The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.