GOVERNMENT POLYTECHNIC, DHENKANAL

LECTURE NOTES

ON

ENGINEERING MATHEMATICS-III

3rd SEMESTER ELECTRICAL ENGINNERING

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MATRICES

Minor – Minor is the determinate value which is obtained by deleting row & coloumn of the particular element and denoted by the symbol ...., i-rows j-coloum.

Ex : $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -2 & 8 \\ 5 & 6 & 1 \end{bmatrix}$

$$M_{21} = \begin{vmatrix} 1 & 3 \\ 6 & 1 \end{vmatrix} = 1 - 18 = -17$$

$$M_{32} = \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = 16 - 12 = 4$$

Upper triangular Matrix – A matrix is said to be upper triangular if the elements below the main diagoned are zeros.

Ex. $\begin{bmatrix} 1 & 5 & 9 \\ 0 & 3 & 7 \\ 0 & 0 & 8 \end{bmatrix}$

Elementary transformations : – The following operations three of which refer to rows are known as elementary transformations.

I. The interchange of any two rows (Rij)

II. The multiplication of any row by a non-zero scalar (kRi)

III. The addition of a constant multiple of the elements of any row to the corresponding elements of any other row (Ri + kRj)

Equivalent matrix – Two matrices A and B are said to be equivalent if one can be obtained from the other by a sequence of elementary transformations.

Rank of a matrix : A matrix is said to be of rank 'r' if

(i) It has atleast one non-zero minor of order 'r'

(ii) Every minor of order higher than 'r' varishes.

The rank of a matrix A shall be denoted by the symbol e(A).

Working Rule :

Step – I : Conver the matrix to the upper triangular form.

Step – II : The no.of non-zero rows is the rank of the matrix

Example – 1 :

Find the rank of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

Solution :

$$A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 8 \\ -3 & 1 & 2 \end{bmatrix} \to R_2 + 2R_1$$

$$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 8 \\ 0 & 0 & 4 \end{bmatrix}_{\to R_3 + R_1}$$

$$\sim \begin{bmatrix} 3 & -1 & 2 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}_{\to 2R_3 - R_2}$$

$\rho(A) = 2$

Consistency : A system of equatiars are said to be consistent if either they will have unique solution on many solution and sid to be inconsistent if they will have no solution.

$2x + 3y = 8$	$x + 2y = 5$	$x - y = 10$
$x - 2y = 4$	$2x + 4y = 10$	$3x - 3y = 15$
(unique solution)	(many soluion)	(No solution)

Consistency of a system of linear equations : -

Consider a system of m linear equations

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1x}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2x}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ am_1x_1 + am_2x_2 + \ldots\ldots + a_{mn}x_n = b_m \end{array}\right\} \quad \ldots..(1)$$

Containing the n unknows $x_1, x_2.....x_n$.

Writing the above equations in matrix form we get.

$$A = \begin{bmatrix} a_{11} \; a_{12} \; .... \; a_{1n} \\ a_{21} \quad a_{22} ....a_{2n} \\ .... \\ .... \\ a_{m1} \quad a_{m2} ....a_{mn} \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$C = [A \vdots B]$$

$$C = \begin{bmatrix} a_{11} & a_{12} \; ...... & a_{1n} & ..... & b_1 \\ a_{21} & a_{22} \; ...... & a_{2n} & ..... & b_2 \\ ...... & ...... & ...... & ...... & ...... \\ a_{m1} & a_{m2} \; ...... & a_{mn} & ... & b_m \end{bmatrix}$$

A is the co-efficient matrix and

C is called agumented matrix

Rouche's Theorem : (Without proof)

The system of equations (1) is consistant if and only if the co-efficient matrix A and the augmented matrix C are of some rank otherwise the system is inconsistent.

Procedure to test the consistency of a system of equations in x unknows.

Find the ranks of the co-efficient matrix A and the augmented matrix 'C' by reducing to the upper triangular form by elementary row operations.

(a) Consistant equations : If Rank A = Rank C

(i) Unique solution Rank A = Rank C = n

Where n = number of unknowns.

(ii) Infinite solution : Rank A = Rank C = r. r < n.

(b) Inconstant equations if Rank A ≠ Rank C

Example – 2 :

Show that the equations

$2x + 6y = -11, 6x + 20y - 6z = -3, 6y - 18z = -1$ are not consistant.

Solution :

Writing the above equations in matrix form

$$\underbrace{\begin{bmatrix} 2 & 6 & 0 \\ 6 & 20 & -6 \\ 0 & 6 & -18 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} -11 \\ -3 \\ -1 \end{bmatrix}}_{B}, \qquad AX = B$$

$$A = \begin{bmatrix} 2 & 6 & 0 \\ 6 & 20 & -6 \\ 0 & 6 & -18 \end{bmatrix} \quad B = \begin{bmatrix} -11 \\ -3 \\ -1 \end{bmatrix} \quad C = [A : B]$$

$$C = \begin{bmatrix} 2 & 6 & 0 & : & -11 \\ 6 & 20 & -6 & : & -3 \\ 0 & 6 & -18 & : & -1 \end{bmatrix} \sim \begin{bmatrix} 2 & 6 & 0 & : & -11 \\ 0 & 2 & -6 & : & 30 \\ 0 & 6 & -18 & : & -1 \end{bmatrix} \rightarrow R_2 - 3R_1$$

$$\sim \begin{bmatrix} 2 & 6 & 0 & : & -11 \\ 0 & 2 & -6 & : & 30 \\ 0 & 0 & 0 & : & -91 \end{bmatrix} \rightarrow R_3 - 3R_2$$

The rank of C is 3

and rank of A is 2

Rank A $\neq$ Rank C.

$\therefore$ The system of equations are not consistant

Example – 3 :

Test consistency and solve :

$5x + 3y + 7z = 4$

$3x + 2by + 2z = 9$

$7x + 2y + 10z = 5$

Solution :

Writing the above equations in matrix form

$$\underbrace{\begin{bmatrix} 5 & 3 & 7 \\ 3 & 2b & 2 \\ 7 & 2 & 10 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}}_{B}, \qquad AX = B, C = [A : B]$$

$$C = \begin{bmatrix} 5 & 3 & 7 & : & 4 \\ 3 & 2b & 2 & : & 9 \\ 7 & 2 & 10 & : & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{3}{5} & \frac{7}{5} & : & \frac{4}{5} \\ 3 & 2b & 2 & : & 9 \\ 7 & 2 & 10 & : & 5 \end{bmatrix} \frac{1}{5}R_1$$

$$\sim \begin{bmatrix} 1 & \frac{3}{5} & \frac{7}{5} & : & \frac{4}{5} \\ 0 & \frac{121}{5} & \frac{-11}{5} & : & \frac{33}{5} \\ 0 & \frac{-11}{5} & \frac{1}{5} & : & \frac{-3}{5} \end{bmatrix} \begin{matrix} \\ \rightarrow R_2 - 3R_1 \\ \rightarrow R_3 - 7R_1 \end{matrix}$$

$$\sim\begin{bmatrix} 1 & \frac{3}{5} & \frac{7}{5} & : & \frac{4}{5} \\ 0 & \frac{121}{5} & \frac{-11}{5} & : & \frac{33}{5} \\ 0 & 0 & 0 & : & 0 \end{bmatrix} \rightarrow R_3 + \frac{1}{11}R_Z$$

Here Rank of A = Rank of C.

Hence the equations are consistent.

But the rank is less than 3 i.e. the number of unknows.

So its solutions are infinite

$$\begin{bmatrix} 1 & \frac{3}{5} & \frac{7}{5} \\ 0 & \frac{121}{5} & \frac{-11}{5} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4}{5} \\ \frac{33}{5} \\ 0 \end{bmatrix}$$

$$x + \frac{3}{5}y + \frac{7}{5}z = \frac{4}{5}$$

$$\frac{121}{5}y - \frac{11}{5}z = \frac{33}{5} \text{ or } 11y - z = 3$$

Let $z = k$, $11y - k = 3$ or $y = \frac{3}{11} + \frac{k}{11}$

$$x + \frac{3}{5}\left[\frac{3}{11} + \frac{k}{11}\right] + \frac{7}{5}k = \frac{4}{5} \text{ or } x = \frac{-16}{11}k + \frac{7}{11}$$

Example – 4 :

Determine the values of λ & μ so that the following equations have (i) no solution (ii) a unique solution (iii) infinite number of solutions.

$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$

Solution :

Writing the above equations in matrix form we have

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & z & \lambda \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 6 \\ 10 \\ \mu \end{pmatrix}}_{B}$$

$\therefore \quad AX = B$

$C = [A : B]$

$$C = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 3 & \lambda & : & \mu \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix} \begin{matrix} \\ \rightarrow R_Z - R_1 \\ \rightarrow R_Z - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix} \rightarrow R_3 - R_2$$

(i) There is no, solution = b $\rho(A) \neq \rho(C)$

i.e. $\lambda - 3 = 0$ or $\lambda = 3$ & $\mu - 10 \neq 0$ or $\mu \neq 10$

(ii) There is a unique solution if $\rho(A) = \rho(C) = 3$

i.e., $\lambda - 3 \neq 0$ or $\lambda \neq 3$ and μ have any value

(iii) There are infinite solution of $\rho(A) = \rho(C) = 2$

$\lambda - 3 = 10$ or $\lambda = 3$ and $\mu - 10 = 0$ or $\mu = 10$

Assignments

1. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

2. Test the consistency & solve

$4x - 5y + z = 2$

$3x + y - 2z = 9$

$x + 4y + z = 5$

3. Determine the values of a & b for which the system of equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

(i) has a unique solution (ii) has no solution (iii) has infinite solution.