GOVERNMENT POLYTECHNIC, DHENKANAL

LECTURE NOTES

ON

CONTROL SYSTEM ENGINEERING

6th SEMESTER

PREPARED BY

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CH-01

[Fundamental Of Control System]

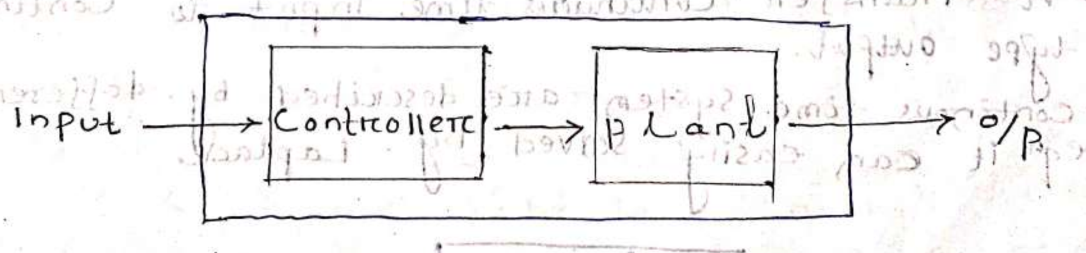
System :→
Arrangement or Combination of different physical Component that are connect together to form a entire Unit to achieve a certain objective is called System.

Control :→
The meaning of control to regulate, direct or command a system, so that desired objective
e.x - speed control of a dc motor can control by controling the Input dc voltage.

Plant :→
It is defined as the part of the system which is to be controlled or regulate It is also called process.

Controller :→
The element of the system Itself may be external to the system. It controls the plant or process.

Control system :→
The system which physical element linked in such a way that so as regulate, direct or command itself to obtain a certain objective. It must have Input, output, controller and plant.



Control system

Classification of control system :→

It is three types.

1. Natural Control System
2. Man-made Control System
3. Combinational Control System

Natural Control System :→

The system inside a human being or biological system are known as Natural Control System.

Ex- Solar system, Planetary atmosphere Circulation system.

Man-made Control System :→

Some Control system which are designed or developed by men are called man-made Control system

Ex- Automobile system.

Combinational Control System :→

Combinational Of natural Control system and manmade Control system is called Combinational Control system.

Ex- Driver driving a car

Continuous and discriptive type Control system :→

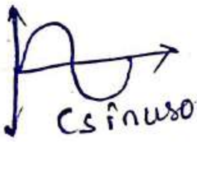
Continuous type Control system →

If all the system variable of Control system are function of time It is called Continuous control system.

→ It Transfer Continuous time input to Continuous type output.

→ continous time system are described by differential eqn it can easily solved by Laplace

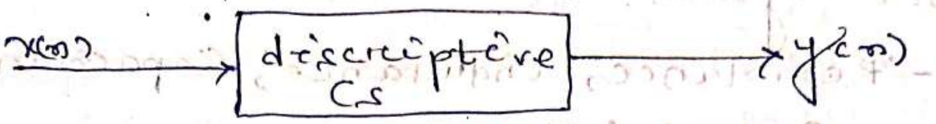
f(t) → [Continuous C/S] → g(t)



(sinusodial signal)

Discriptive time Control system :-

If one or more system variable of a Control System are Known as certain it is called Discriptive time control system.



ex - Microprocessor or computer are example of discriptive time Control system. It converts discriptive time Input to discriptive time output

→ Discriptive time System are described by differential eqn it can easily solved by Z-transform.

SISO

Single Input Single output

- If a Control System has one Input and one output it called SISO Control system.

MIMO

Multi Input multi output

- If a Control system has multi input and multi output it called as MIMO Control System.

Time varying Control system :→

If a parameter of control System vary with time the Control system is termed as time varying Control system.

e.x - Space Vehicle leaving (Satelite)

Time invarying Control System :-

If parameter of Control system at not vary with time is called time-invarying Control System.

e.x - Resistance, inductance, Capacitance.

Linear Control System :-

A control System is known as linear if, it satisfied the additive property as well as homogenous property.

→ It hold the principle of Superposition.

Additive

If $x, y \to$ domain of function "f"

$$f(x+y) = f(x) + f(y)$$

Homogenous

for a variable "x" → domain of function and any Scalar Constant (β)

$$f(\beta x) = \beta f(x)$$

Open Loop Control System :-

→ Open Loop Control system is known as without feedback control system

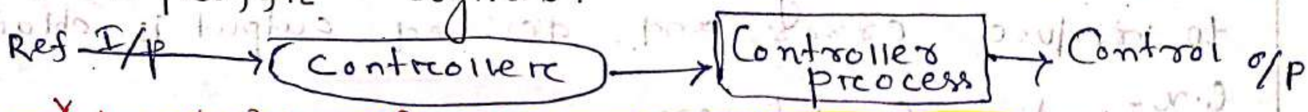
★ The open loop Control system the control action is independent of desired output.

★ In this System output is not Compare with reference input.

→ The component of open loop system are Controller and process.

→ The Controller may be amplifier, filter. depends upon the system.

Ex - Automatic Washing Machine.
- Electric Water Heater
- Traffic Signal.



Advantages of open loop Control system :→

* This system is simple in construction and design.
* This System are economic.
* It required Less maintance and not difficult.
* This system are not much trouble in Problem in Stability.
* This System is Coinvienent to use output is difficult to measure.

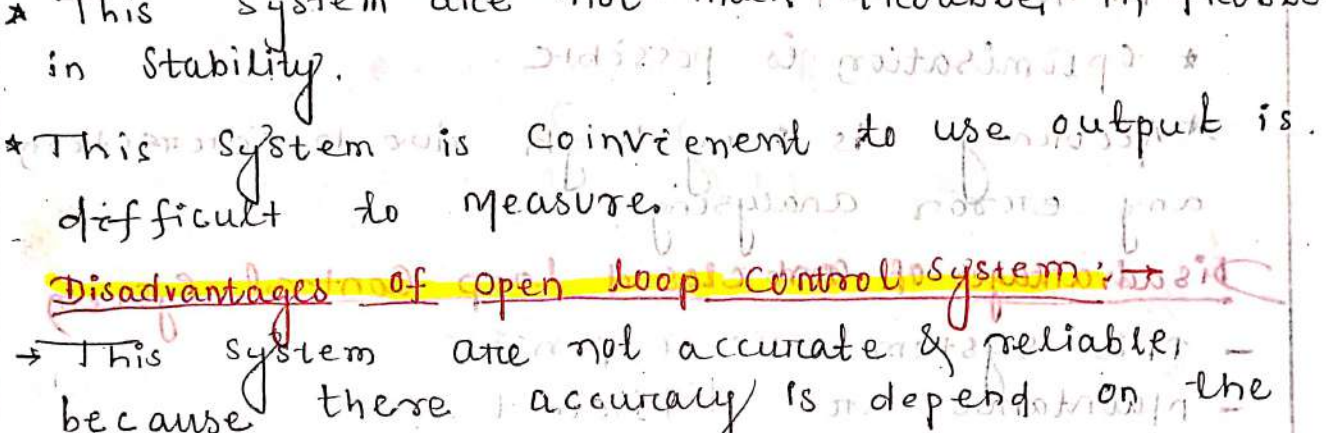
Disadvantages of open loop Control system :→

→ This system are not accurate & reliable, because there accuracy is depend on the acurately of calibration.

→ These are Slow

→ Optimatisation is not possible.

Close loop Control system :→



- Closed Loop Control System are also known as feedback Control System.
- In close loop system control action is dependent upon the desired output.

- In a closed loop system compare with reference input.
- The error signal is fed to the controller to reduce error and desired output is obtained
e.x.- 1. Air Conditioner
2. Electric Iron

Advantages of closed loop control system :→

- This system are more reliable.
* Closed loop system are faster.
* In number of variable can be handle simultaneously
* Optimisation is possible
* Accuracy is very high due to correction of any error analysing.

Disadvantages of ~~cont~~ closed loop control system

- This system are expensive
- Maintance are difficult.
- Complicated installisation.

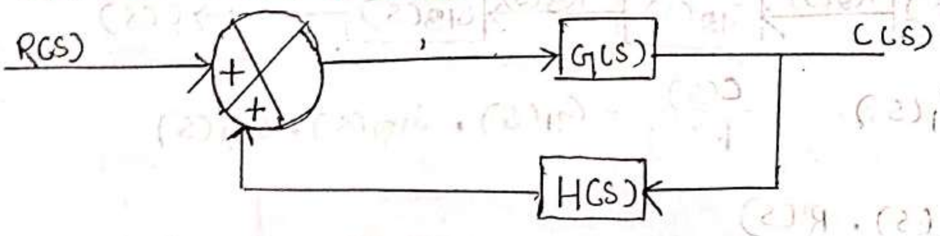
Comparision between open loop & closed loop C S

Open loop system	Closed loop system
* These are not reliable	* These are reliable
* easier to built	* difficult to built
* optimisation not possible.	* Optimisation is possible
* Cost is less	* Cost is more.
* Less stable	* Stable
* If the calibration is good it can perform accurate.	* They are accurate
* e.x - Traffic signal - Water heater	* E.x - Air conditioner - Electric iron

Effect of Feedback

closed loop

Positive feedback



$$= R(s) + H(s)\cdot$$

$$= \frac{C(s)}{G(s)} = R(s) + H(s)\cdot C(s)$$

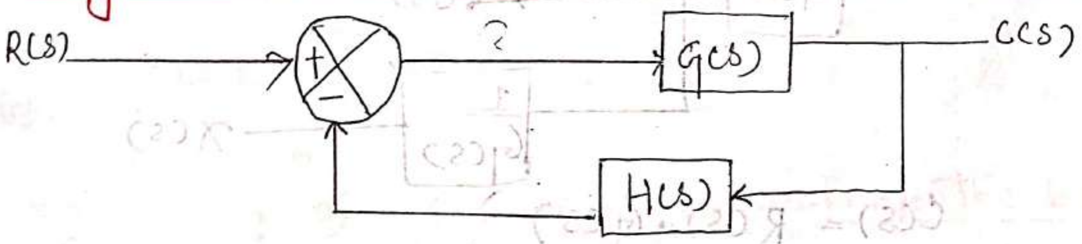
$$\Rightarrow \frac{C(s)}{G(s)} - H(s)\cdot C(s) = R(s)$$

$$\Rightarrow C(s)\left[\frac{1}{G(s)} - H(s)\right] = R(s)$$

$$\Rightarrow C(s)\left[\frac{1 - G(s)\cdot H(s)}{G(s)}\right] = R(s)$$

gain G(s) $\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)\cdot H(s)}}$

Negative feedback



$$= R(s) - H(s)$$

$$\Rightarrow \frac{C(s)}{G(s)} = R(s) - H(s)\cdot C(s)$$

$$\Rightarrow \frac{C(s)}{G(s)} + H(s)\cdot C(s) = R(s)$$

$$\Rightarrow C(s)\left[\frac{1}{G(s)} + H(s)\right] = R(s)$$

$$\Rightarrow C(s)\left[\frac{1 + G(s)\cdot H(s)}{G(s)}\right] = R(s) \Rightarrow \text{gain } G(s) = \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\cdot H(s)}}$$

CH-02

CH-02 [Mathematical Model of a system]

Transfer function :→

→ It is defined as the ratio of the Laplace transformation of output response to the laplace transformation of Input response. assuming all the Initial Condition to be Zero

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$T.F = G(s) = \frac{C(s)}{R(s)} \quad \{\text{all initial Cond}^{n} \text{ are zero}\}$$

Impulse Response :→

→ It has been proved that the Laplace Transformation of an impulse function

→ The transfer function between an input Variable and output Variable of a system is defined as the laplace transform of the impulse response.

Properties of Transfer Function:→

1. The transfer function is defined only for a linear time-invarient system. It is not defined for non-linear System.
2. The Transfer function between an input variable and an output variable of a System is defined as the Laplace transform of the impulse response.
3. All initial conditions of the system are set to zero.
4. The transfer function is independent of the input of the system.
5. Stability can be found form characteristic open ratio.

Advantages of Transfer Function:-

↳ A Transfer function is a mathematical model and it gives the gain of the system.

↳ Transfer function helps in the study of stability another of the system.

↳ The Response of the system to any input can be determined very easily.

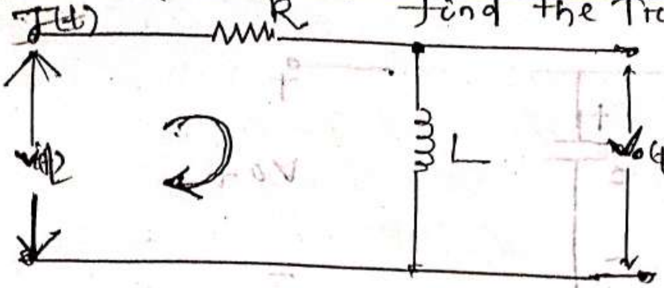
Disadvantages:-

↳ Transfer function does not take into account the initial conditions.

↳ The transfer function can be defined for linear systems only.

↳ No inferences can be drawn about the physical structure of the system.

Q) 21-01-22 Find the Transfer function of given network



$\rightarrow V_i(t) - I(t)R - L\dfrac{di(t)}{dt} = 0$

$\Rightarrow V_i(t) = I(t)R + L\dfrac{di(t)}{dt}$ ——— (1)

$\Rightarrow V_o(t) = L\dfrac{di(t)}{dt}$ ——— (2)

Taking Laplace of equation — (1)

$L\left[V_i(t)\right] = L\left[I(t)R\right] + L\left[L\dfrac{di(t)}{d(t)}\right]$

$\Rightarrow V_i(s) = R \cdot I(s) + L \cdot s \cdot I(s)$

Taking Laplace of eqn - (2)

$\Rightarrow V_o(t) = L\dfrac{di(t)}{dt}$

$\Rightarrow L\{V_o(t)\} = L\left[L\dfrac{di(t)}{dt}\right]$

$V_o(s) = Ls \cdot I_s$

Transfer Function

$= \dfrac{V_o(s)}{V_i(s)} = \dfrac{Ls \cdot I_s}{R \cdot I(s) + Ls \cdot I(s)}$

$= \dfrac{Ls \cdot \cancel{I(s)}}{\cancel{I(s)}\left[R+Ls\right]}$

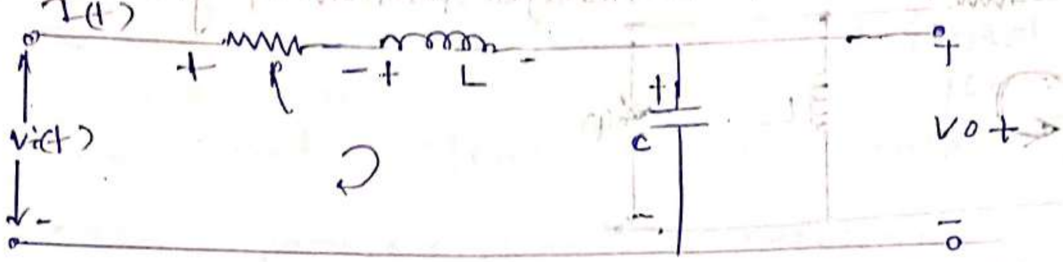
$= \dfrac{Ls}{R+Ls}$

$L\left[\dfrac{dx(t)}{d(t)}\right] = s \cdot x(s)$

$L\left[\int x(t)\,dt\right] = \dfrac{1}{s} \cdot x(s)$

$L\left[\dfrac{d^2x(t)}{d(t)^2}\right] = s^2 x(s)$

Find the Transfer function of given network:



$\Rightarrow V_i(t) - I(t)R - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt = 0$

$\Rightarrow V_i(t) = I(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int I(t)\cdot dt = 0$

Taking Laplace

$= L\left[V_i(t)\right] = L\left[I(t)\cdot R\right] + L\left[L\cdot\frac{di(t)}{dt}\right] + L\left[\frac{1}{C}\int I(t)\cdot dt\right]$

$V = L\frac{di}{dt}$

$C = \frac{1}{V}\int V(t)\,dt$

$I = C\frac{dv}{dt}$

$V = \frac{1}{C}\int i(t)\,dt$

$V_i(s) = I(s)R + L(s)\cdot I(s) + \frac{1}{Cs}\times I(s)$

$V_o(t) = \frac{1}{C}\int I(t)\cdot dt$

Taking Laplace: $\Rightarrow V_o(s) = \frac{1}{Cs}\times I(s)$

$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs\times I(s)}{I(s)R + L(s)\cdot I(s) + \frac{1}{Cs}\times I(s)}$

$= \frac{\frac{1}{Cs}\times \cancel{I(s)}}{\cancel{I(s)}\left[R + L(s) + \frac{1}{Cs}\right]}$

$= \frac{1/\cancel{Cs}}{\frac{RCs + Ls^2C + 1}{\cancel{Cs}}}$

$= \frac{1}{Ls^2C + RCs + 1}$

Poles & Zeroes of Transfer Function :-

↳ Poles & Zeros of transfer function are the frequencies for which value of the denominator and numerator of transfer function becomes zero respectively.

↳ Transfer function of a control system can also represented as.

$$G(s) = \frac{C(s)}{R(s)} = \frac{C_0 s^n + C_1 s^{n-1} + \dots + C_{n-1} s + C_n}{R_0 s^m + R_1 s^{m-1} + \dots + R_{m-1} s + R_m}$$

$$= K \frac{(s - z_1)(s - z_2)(s - z_3) \dots (s - z_n)}{(s - p_1)(s - p_2)(s - p_3) \dots (s - p_m)}$$

Where, K = gain factor of the transfer function.

↳ $z_1, z_2, z_3 \dots z_n$ are roots of the numerator polynomial.

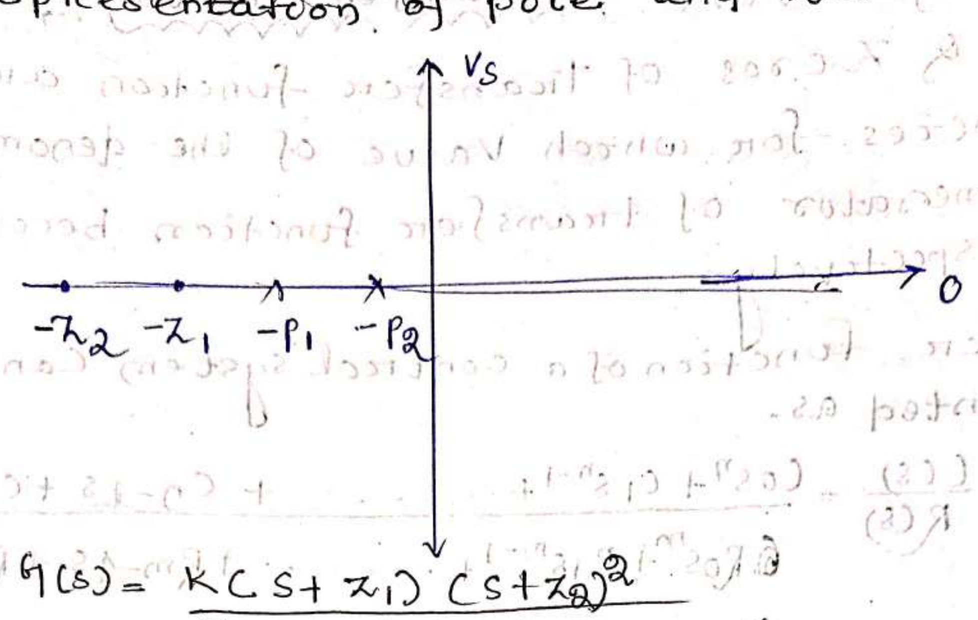
↳ As for these polynomial, the transfer function becomes zero, these roots are called zeros of the transfer function.

↳ $p_1, p_2, p_3 \dots p_m$ → The value of T.f becomes Infinite Thus the roots of denominator are called the poles of the function.

↳ Zero of a transfer function are needed are termed Circuit Zero,

↳ pole of a transfer function are defined as the value of magnitude of the transfer function becomes infinity.

Representation of pole and zero :→



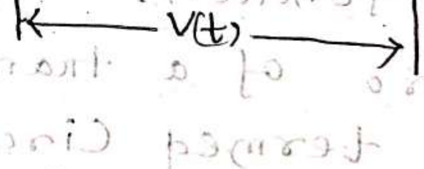
$$G(s) = \frac{K(s+z_1)(s+z_2)^2}{(s+p_1)(s+p_2^2)}$$

Mathmatical Modeling of Electrical system:-

↳ Most of the electrical systems can be modelled by three basic elements.

1. Resistor.
2. capacitor
3. Inductor.

Resistor :→

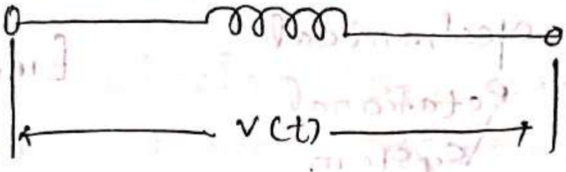


↳ The mathmatical model is given by the ohm's law relationship.

$$V(t) = i(t)R$$

$$i(t) = \frac{V(t)}{R}$$

Inductor (L):→

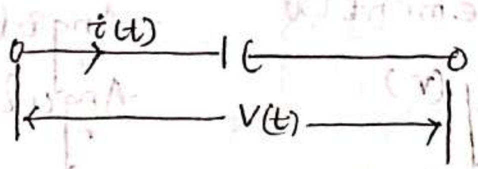


↳The input, output relations ar given by Faraday's Law.

$$v(t) = L\,\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int v\,dt$$

Capacitor:→



$$v(t) = \frac{1}{C}\int i\,dt$$

$$i(t) = C\,\frac{dv}{dt}$$

Analogous System:→

↳Comparing equations for the mechanical translational system or for the mechanical rotational system and for the Series electrical System.

↳Such systems whose differential equations are of identical form are called analogous system.

Analogous quantities in force - voltage analogy-

Mechanical translational system	Mechanical rotational system	Electrical System.
Force (F)	Torque (T)	Voltage.
Mass (M)	Moment of inertia J	Inductance
Viscous friction coefficient(f) -	viscous friction of coefficient (f)	Resistance
Spring Stiffness (K) =	Torsional Spring Stiffness K	Capacitance
Displacement (x) =	Angular displacement (θ)	Charge
velocity (v) =	Angular Velocity (ω)	Current

Analogous quantities in force current analogy:

Mechanical translational system	Mechanical Rotational system	Electrical system
Force(F)	Torque T	Current i
Mass (M)	Moment of Inertia J	Capacitance C
Viscous friction Coefficient(f)	Viscous friction Coefficient (f)	Resistance 1/R
Spring stiffness (K)	Torsional Spring (K)	Inductance 1/L
Displacement (x)	Angular displacement θ	→ magnetic flux Linkage
Velocity (v)	Angular Velocity(ω)	→ Voltage(e)

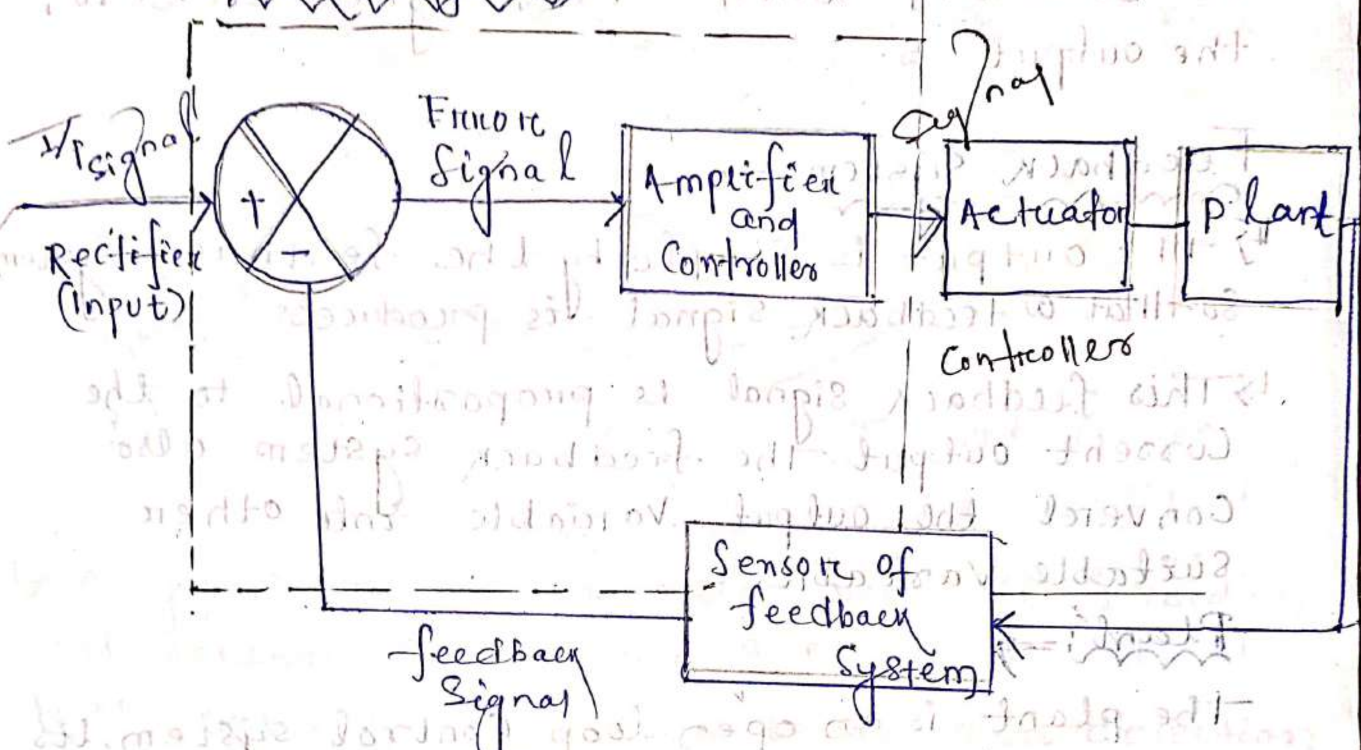
→ The Concept of analogous system is a useful technique for the study of various systems like electrical, mechanical, thermal, hydraulic etc.

Chapter-03 Control system Components :→

↳ Components of Control system :→

↳ Error detector amplifier and Controler actuator plant and sensor of feedback system are the basic Components of automatic Control system.

Block Diagram



Reference Input :→

↳ The reference Input becomes an input Signal Propertional to the desired output automatic Control system.

Error Detector :→

↳ The error detector is a become the reference I/p and feed signal. an error is produced by if it there is a difference.

↳ example- synchrous, IVDT.

Actuator:→

↳ The function of the fan is formaly the Controller output and Convert the required from of energy. which is acpect for the plant output is on the input of the plant.

↳ If there is difference between reference input and feedback signal, the process will be continued when error signal is zero, the output s.

Feedback System:→

↳ The output is simple by the feedback System So that a feedback signal is producess.

↳ This feedback signal is proportional to the Current output. The feedback system also Convert the output Variable into other Suitable Variable.

Plant:→

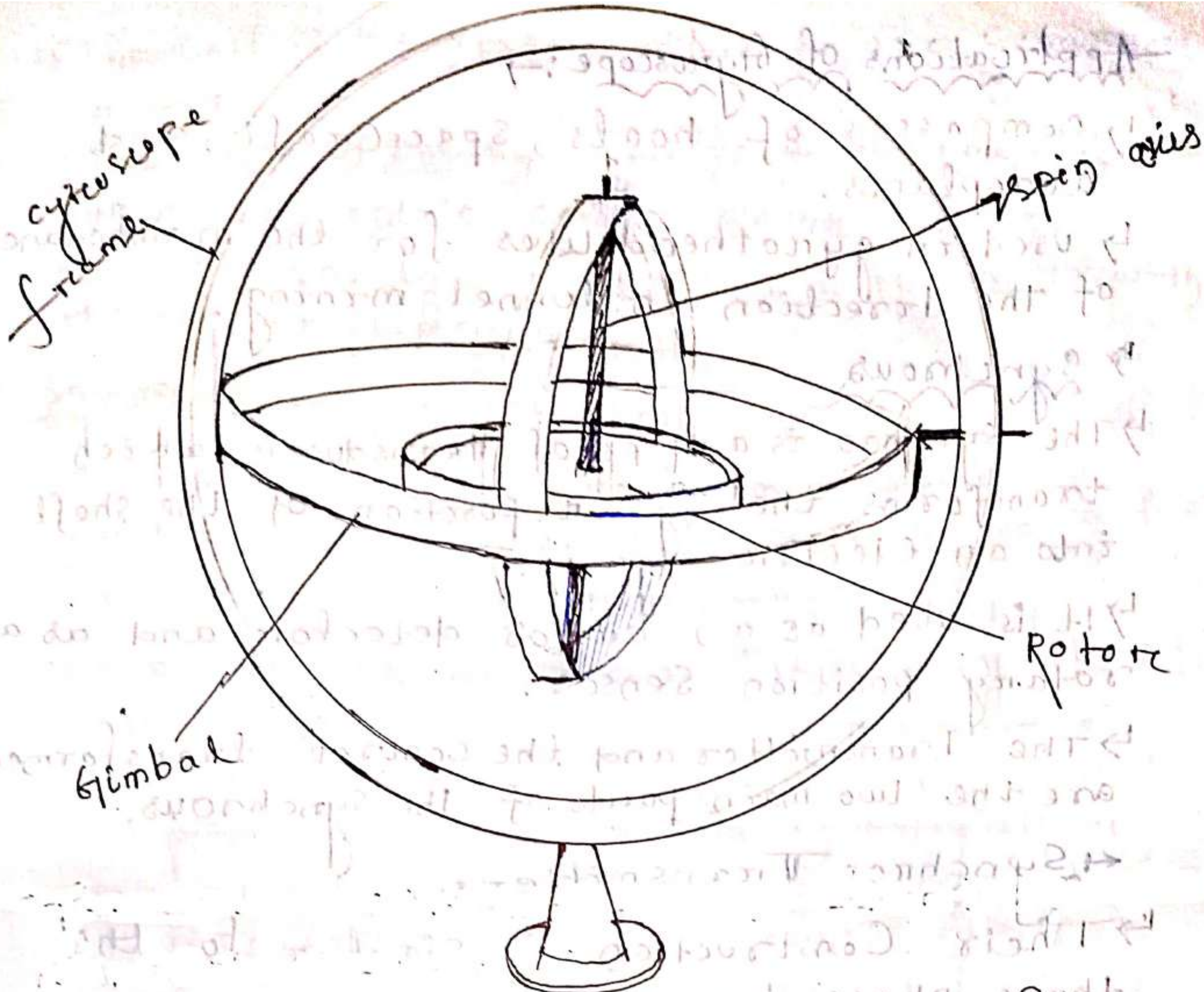
The plant is an open loop Control system, its output is controlled closed loop system.

Gyroscopes:→

↳ A Gyroscope is a device used for measuring or maintaining orientation and angular Velocity

↳ It is a spinning wheel or disc in which the axis of rotation is free to assume any orientation by itself.

↳ When rotating, the orientation of this axis is unaffected by tilting or rotation of the mounting according to the conservation of angular momentum.



↳ A gyroscope in operation, Note the freedom of rotation in all three axes.

↳ The rotor will maintain its spin axis direction regardless of the orientation of the outer frame.

* Parts of Gyroscope :-
  * Spin axis
  * Gimbal
  * Rotor.
  * Gyroscope frame.

* Working principle :→

↳ Gyroscope is based on gravity and is explained as the product of angular momentum which is experienced by the torque on a disc to produce a gyroscopic precession in the spinning wheel.

Applications of Gyroscope :→

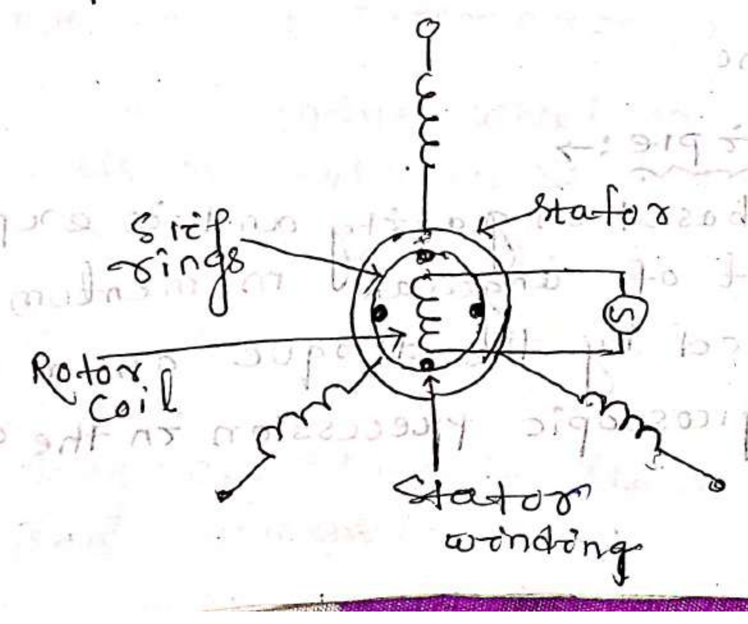
- ↳ Compasses of boats, SpaceCraft, and aeroplanes.
- ↳ used in gyrotheodolites for the maintenance of the direction in tunnel mining.

Synchrous

- ↳ The synchro is a type of transducer which transforms the angular position of the shaft into an electric signal.
- ↳ It is used as an error detector and as a rotary position sensor.
- ↳ The Transmitter and the control transformer are the two main parts of the synchrous.

Synchro Transmitter :→

- ↳ Their Construction is similar to the three phase alternator.
- ↳ The stator of the synchros is made of steel for reducing the iron losses.
- ↳ The Stator is slotted for housing the three phase windings.
- ↳ The axis of the stator winding is kept 120° apart from each other.



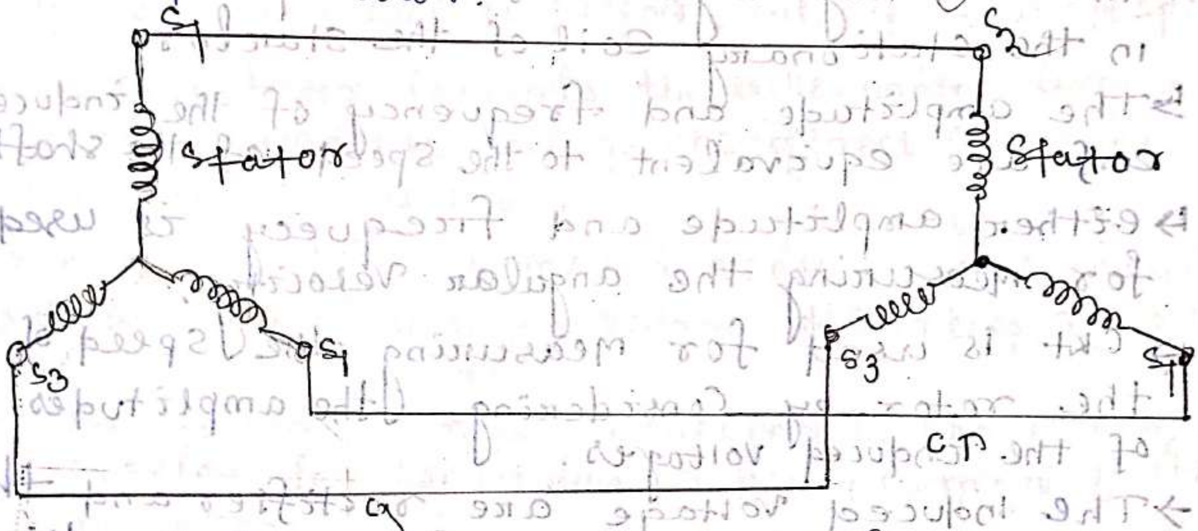
Constructional feature of (Synchro transmitter)

↳ The coil of the stator windings are connected in star.
↳ The root of the synchros is a dumbbell in shape and a concentric coil is wound on it.
↳ The AC voltage is applied to the rotor with the help of slip rings.

Synchro Control Transformer:→

A Synchro Control transformer is used in conjuction with a synchro transmitter to act as error sensor of Mechanical Components.

↳ except that the rotor is cylindrically shaped so that the air gap flux is uniformly distributed around the rotor.



↳ The essential to a control transformer since its rotor terminals are usually connected to an amplifie

↳ The cylindrical shape of the rotor of the synchro ~~rotor~~ control transformer helps to keep the change of impedance in the rotor coil ckt with the change of angular position.

Application

- For the application of detection of the error
- Used as rotory position sensors by using Synchro TX and CT.

Tachometer:→

↳ It is an electromechanism device production o/p voltage proportional to its shaft speed tachometers can be used as an analog speed indicator velocity feedback device or signal integrator.

↳ It can be Ac or DC tachometer.

Ac Tachometer:→

↳ The Ac Tachometer has stationary armature & rotating magnetic field.

↳ The Commutator & brushes are absent in Ac Tachometer.

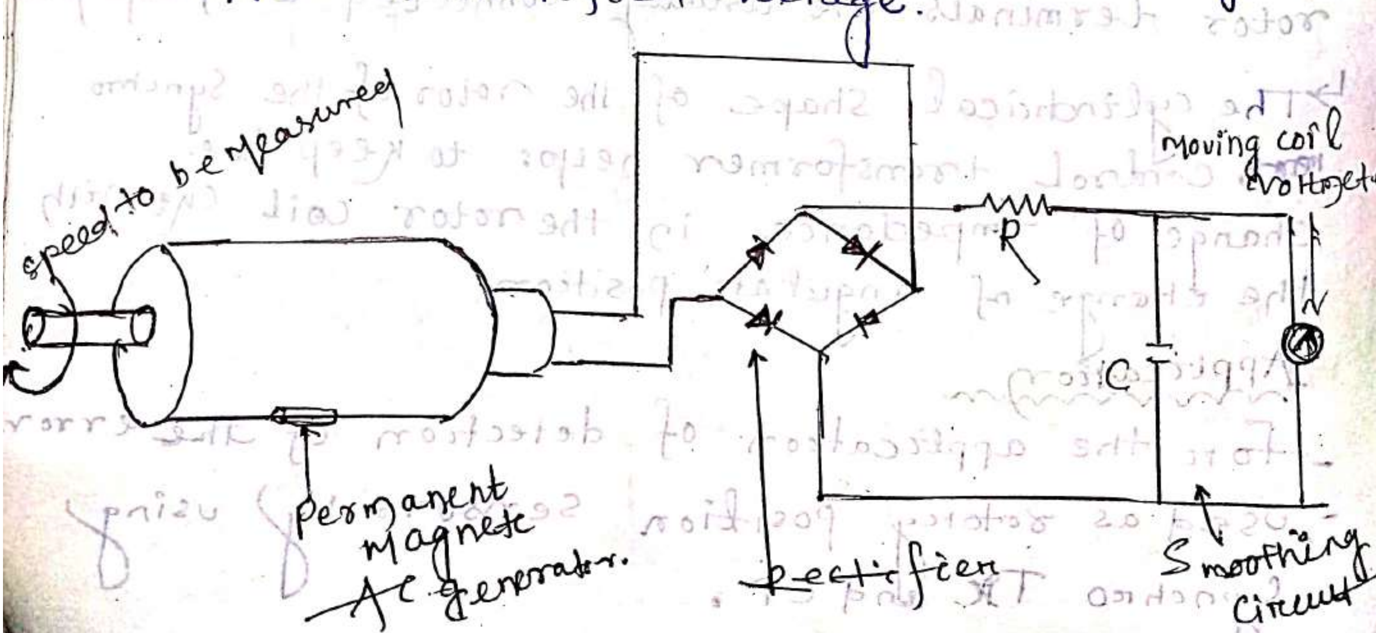
↳ The rotating magnetic field induces the emf in the stationary coil of the stator.

↳ The amplitude and frequency of the induced emf are equivalent to the speed of the shaft.

↳ either amplitude and frequecy is used for measuring the angular velocity.

↳ Ckt is used for measuring the speed of the rotor by considering the amplitudes of the induced voltages.

→ The induced voltage are rectifier and then passes to capacitor filter for smoothening the ripples of rectified voltage.



DC Tachometer Generator :→

→ Main parts of the DC Tachometer - permanent magnet, armature, Commutator, brushes, Variable resistor and moving coil voltmeter.

↳ The machine whose speed is to be measured is coupled with the shaft of the DC tachometer.

Working Principle :→

↳ When the closed conductor moves in the magnetic field, emf induced in the conductor.

↳ The magnitude of the induced emf depends on the flux link with the conductor and speed of the shaft.

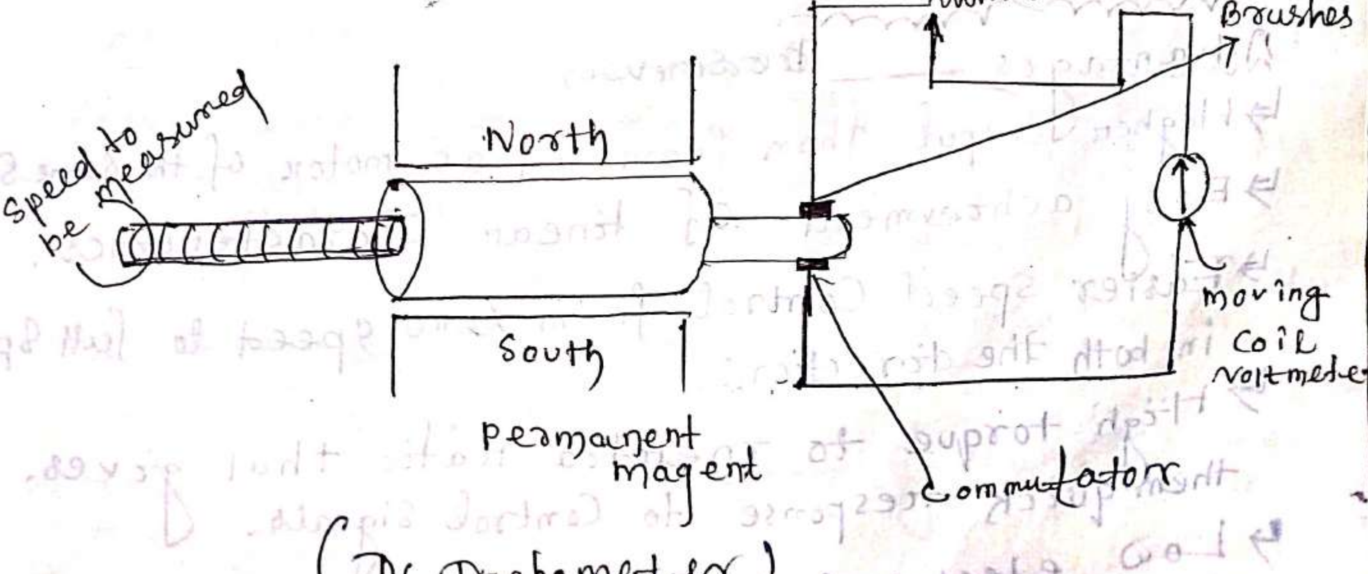
↳ The rotation induces the emf in the coil the magnitude of the induced emf is proportional to the shaft speed.

↳ The commutator converts the alternating current of the armature coil to the direct current with the help of the brushes.

↳ The moving coil voltmeter measures the induced emf.

→ The polarity of induced voltage determines the direction of motion of the shaft.

→ The resistance ~~of~~ is connected in series with the voltmeter for controlling the heavy current of the armature.



(DC Tachometer)

Servo Motors:-

↳ The Control system which are used to Control the position or time derivatives of position, i.e velocity and acceleration are called Servomechanism

↳ The motors which are used in automatic Control systems are called servomotors.

↳ The servomotors are used to Convert an electrical signal applied to them into an angular displacement of the shaft.

↳ In general, a servomotor should have the following features:-

1. Linear Relationship between speed and electric Control signal.
2. Steady State stability.
3. Wide range of speed Control.
4. Linearity of mechanical Characteristics througout the ntire speed range.
5. Low Mechanical and Electrical inertia.
6. Fast response.

DC Servomotors:-

Advantages:-

↳ Higher output than from an ac motor of the same size

↳ Easy achievment of linear Characteristics.

↳ Easier speed Control from zero speed to full speed in both the directions.

↳ High torque to inertia ratio that gives them quick response to Control signals.

↳ Low electrical & Mechanical time Constants

DC Servomotors

- Sliding Contact motors with Commutator and brushes
  - Permanent Magnet motors
    - Cylindrical armature motors.
    - Disc armature Motors.
    - Moving Coil Motors.
  - Electromagnetic field motors.
    - Armature Controlled motors.
    - Field Controlled motors.
    - Series Motors.
    - Split field motors.
- brushless or Contact Less motors with SCR / transistor Commutator.

Permanate Magnetic DC motors :-

↳ Permanate Magnetic used in these motor to replace the field winding to produce the required magnetic field.

↳ Permanent magnet motors are economical for power ratings upto a few Kilowatts

Advantages of Permanent Magnet Motors :-

↳ A Simpler and more reliable motor because the field Power Supply is not required.

↳ Higher Operating efficiency as the motor has no field losses.

↳ Field flux is less affected by temperature rise.

↳ Higher torque/inertia ratio

→ A more Linear Torque / speed Curve.

Disadvantages permanent magnet motor :→

↳ The magnets deteriorate with time and demagnetized with large current transients.

↳ These drawbacks are eliminated by high-grade magnetic material such as Ceramic Magnets & near earth magnets.

↳ The Cost of these materials is very high.

Electromagnetic field Motors :→

↳ Electromagnetic motors are economical for higher power ratings generally above 1KW.

↳ This type of servomotor is similar to a Conventional Dc motor Constructially :-

- The number of Slots and Commutator ~~Strength~~ Segments is large to improve commutation.
- Compoles and Compensating windings are provided to eliminate sparking.
- The diameter to Length ratio is kept low to reduce inertia.
- Oversize shafts are employed to withstand the high torque stress.

↳ In this type of motor, the torque and Speed may be controlled by varying the armature Current and field current.

↳ In armature controlled mode of operation, the field Current is held Constant and the armature current is varied to control the torque

↳ It except for minor differences in Constructional features, a dc Servomotor is essentially an ordinary dc motor.

Armature-Controlled DC Servomotor:

↳ An armature-Controlled DC servomotor is a dc Shunt motor designed to satisfy the requirement of a servomotor. If the field Current constant.

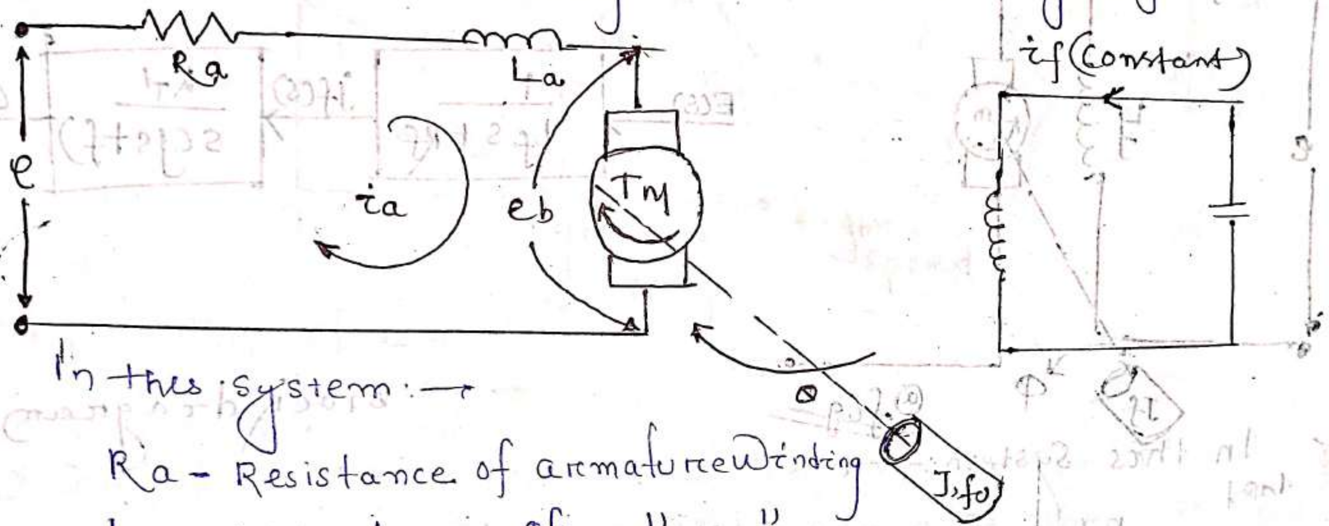
↳ Speed $\propto$ armature voltage

↳ Torque $\propto$ armature current.

↳ Torque & speed Can be Controlled by armature Voltage.

↳ The armature voltage is Controlled by a Variable resistance.

↳ But in Large motors in order to reduce power loss, armature Voltage is Controlled by thyristor.



In this system:→

Ra - Resistance of armature winding

La - Inductance of " "

Ia = Armature Current

if = field Current

e = applied voltage

eb = back emf

TM = Torque developed by motor

θ = Angular displacement

J = equivalent moment of inertia of motor and load referred to motor Shaft.

fo = equivalent Viscous friction Coefficient of motor and load referred to motor Shaft

Field - Controlled DC Servomotor

↳ A field Controlled DC Servomotor is a dc Shunt motor designed to satisfy the requirement of a Servomotor.

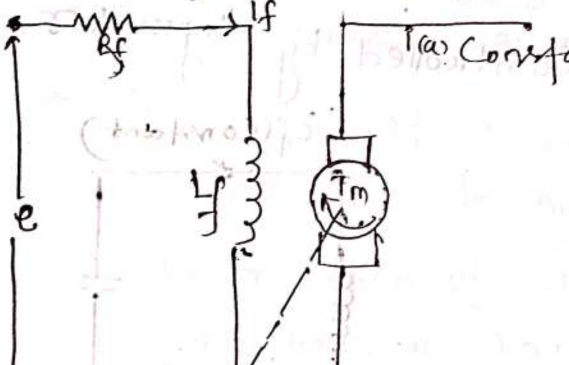
↳ The armature is ~~required~~ supplied with a Constant Current or voltage.

↳ Armature voltage Constant

↳ Torque α field flux

↳ field current is proportional to flux

↳ The torque of the motor is controlled by Controlling the field current.



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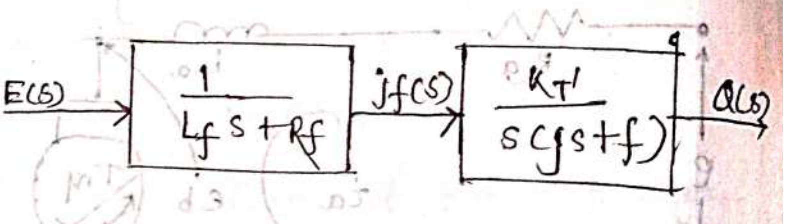


Block diagram

In this system:

R_f = field winding resistance

L_f = field winding Inductance

e = field Control voltage

I_f = field Current

T_M = torque developed by motor

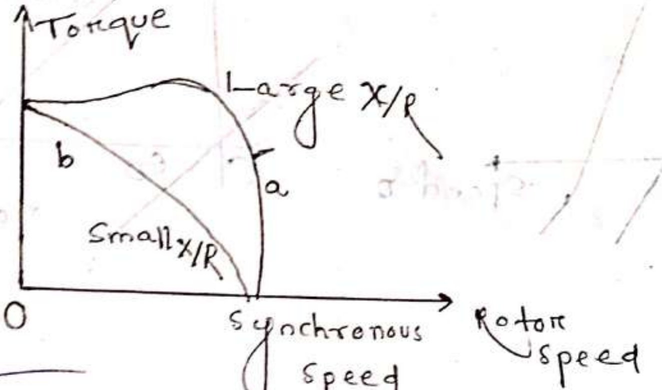
J = equivalent moment of inertia of motor and load referred to motor Shaft.

Q = Angular displacement

f = equivalent viscous friction Coefficient of motor & load referred to motor Shaft.

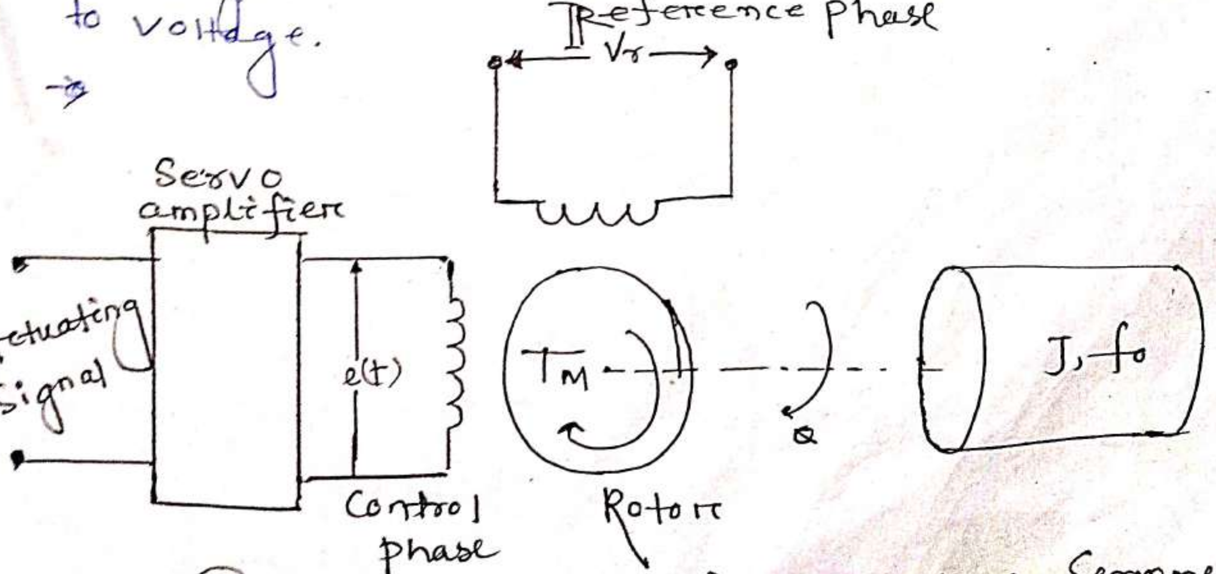
AC Servomotors :→

- An ac Servomotor is basically a two phase Induction motor except for Certain special design features.
- A two phase induction motor Consists of two ways from a normal Induction motor.
- The rotor of the servomotor is built with high resistance so that its X/R ratio is small which results in linear speed-torque characteristics,
- The excitation voltage applied to two stator windings should have a phase difference of 90°



Working of an ac Servomotor :→

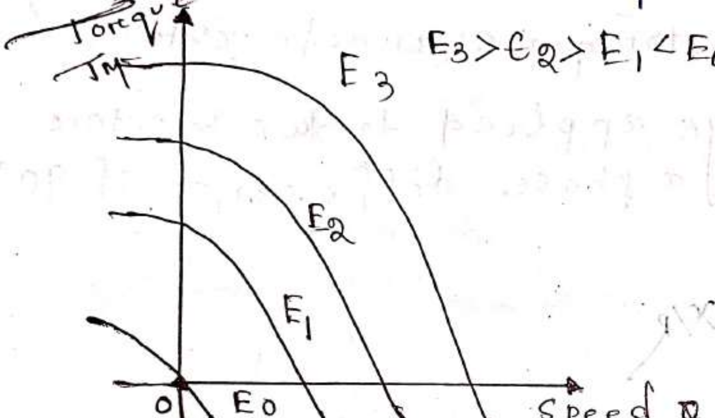
- One of the phase known as the reference phase is excited by a Constant voltage, and the other phases known as the Control phase, is energized by a voltage which is 90° out of phase with respect to voltage.

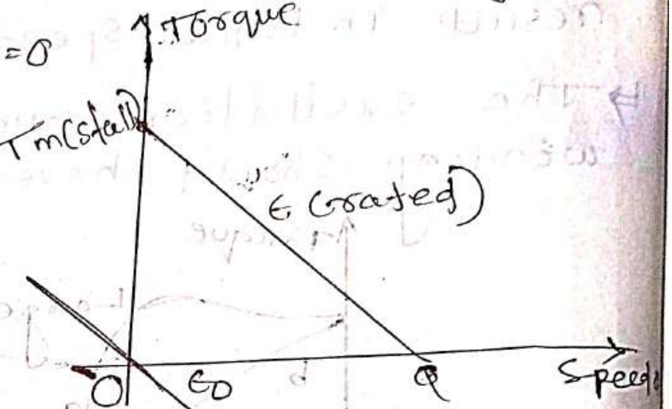


(Shemetic diagram of a two phase Servometer)

↳ The control signals in control system are usually of low frequency, in the range of 0 to 20 Hz.
↳ For production of rotating magnetic field, the control phase voltage must be of the same frequency.

↳ The torque-speed curves of ac servomotors are nonlinear except in the low speed region.



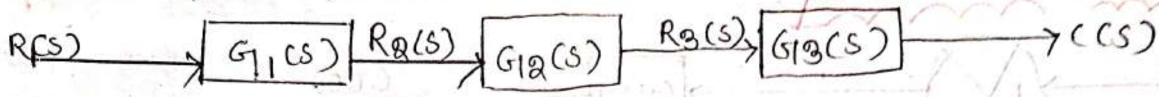


Ch. 04

Block Diagram Algebra & Signal Flow graphs

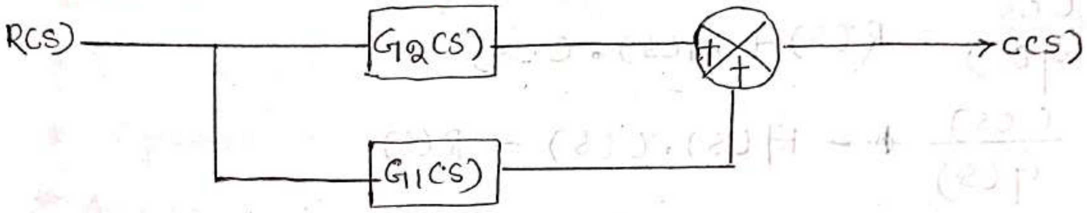
Rules for block diagram Reduction :→

1. For blocks in cascade/series :→



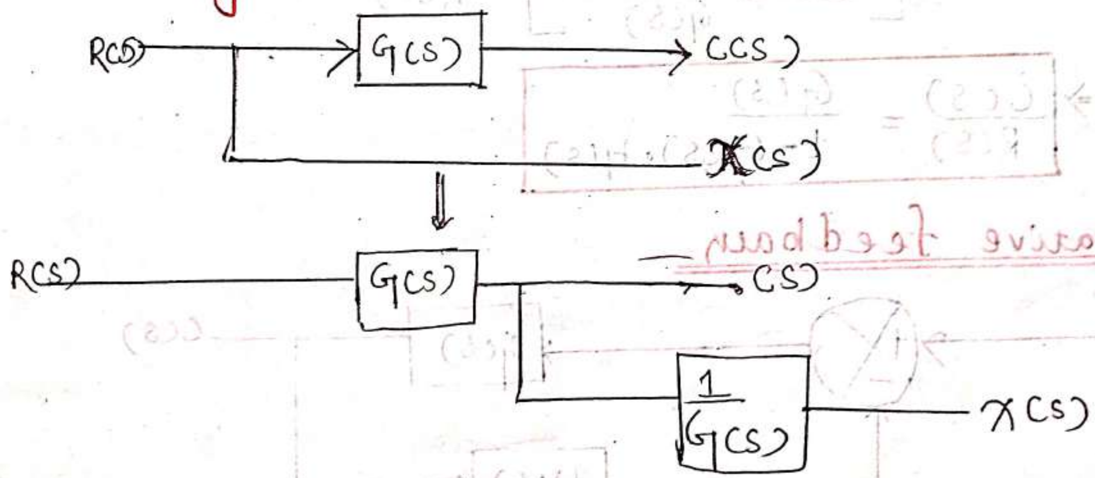
$$\frac{C(S)}{R(S)} = G_1(S) \cdot G_2(S) \cdot G_3(S)$$

2. For Blocks in parallel :→



$$\frac{C(S)}{R(S)} = G_1(S) + G_2(S)$$

3. Shifting a Take-off point before the block :-

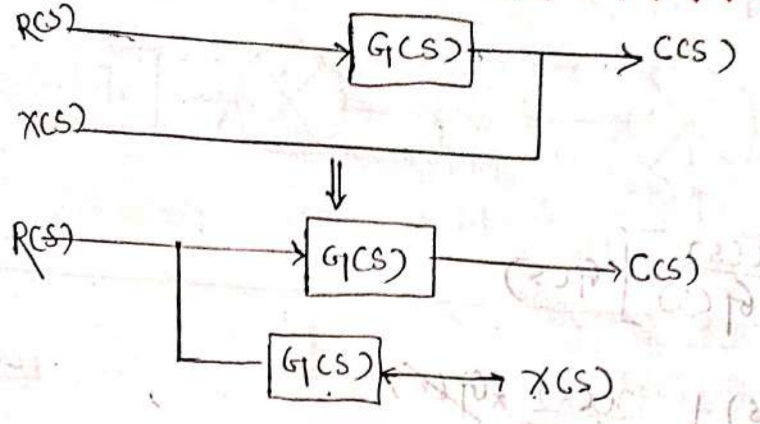


$$C(S) = R(S) \cdot G(S)$$

$$X(S) = \frac{1}{\cancel{G(S)}} \times R(S) \cdot \cancel{G(S)}$$

$$= R(S)$$

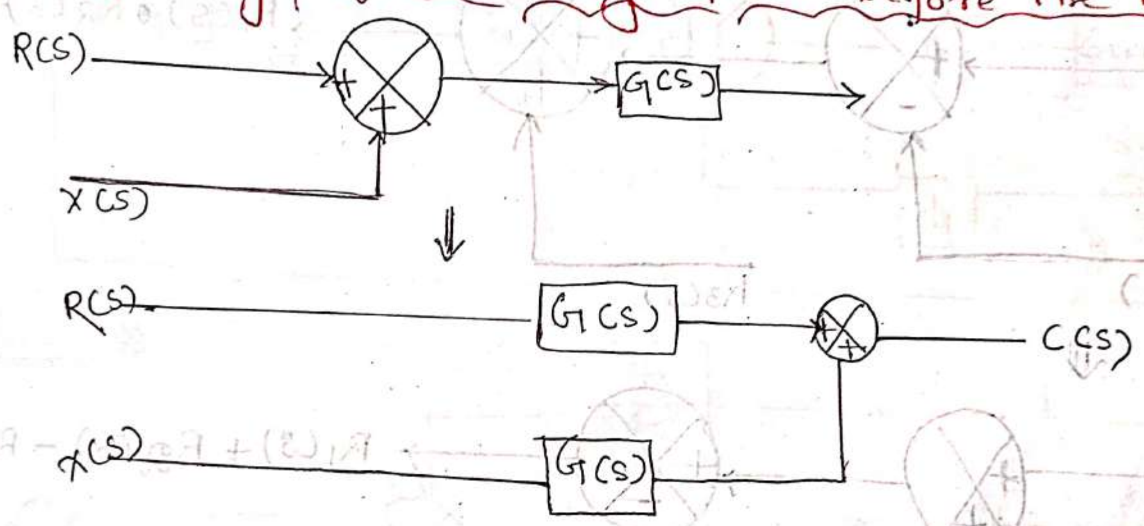
4. Shifting of take off point after the block :→



$C(S) = R(S) \cdot G(S)$

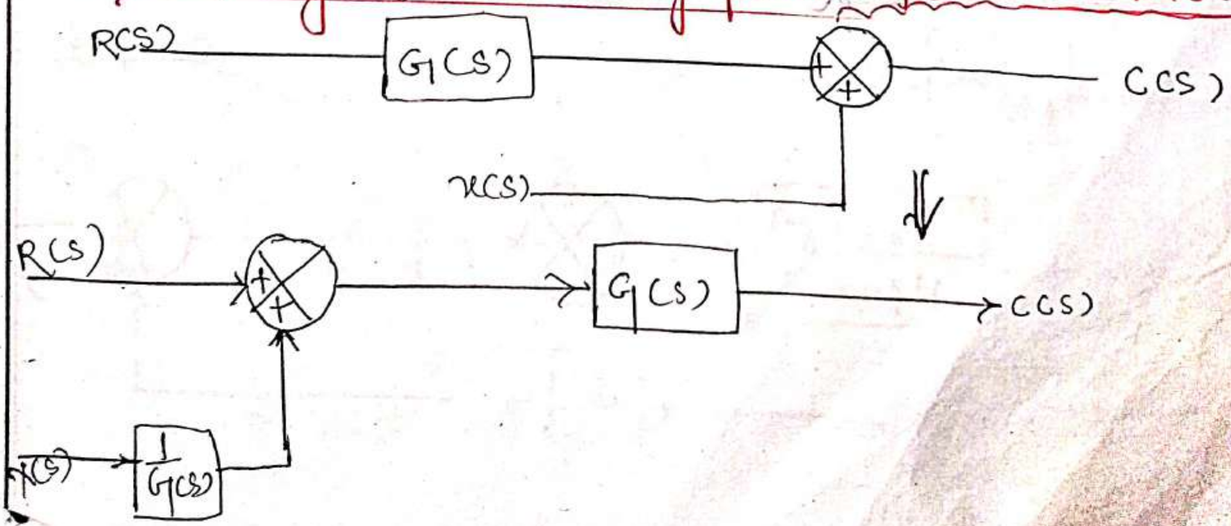
$X(S) = G(S) \times R(S)$

5. Shifting of a summing point before the block :-



$C(S) = G(S)\left[R(S) + X(S)\right]$

(6) Shifting of a summing point after the block :-



Step-1

$$\frac{x(s)}{G(s)}$$

Step-2

$$R(s) + \frac{x(s)}{G(s)}$$

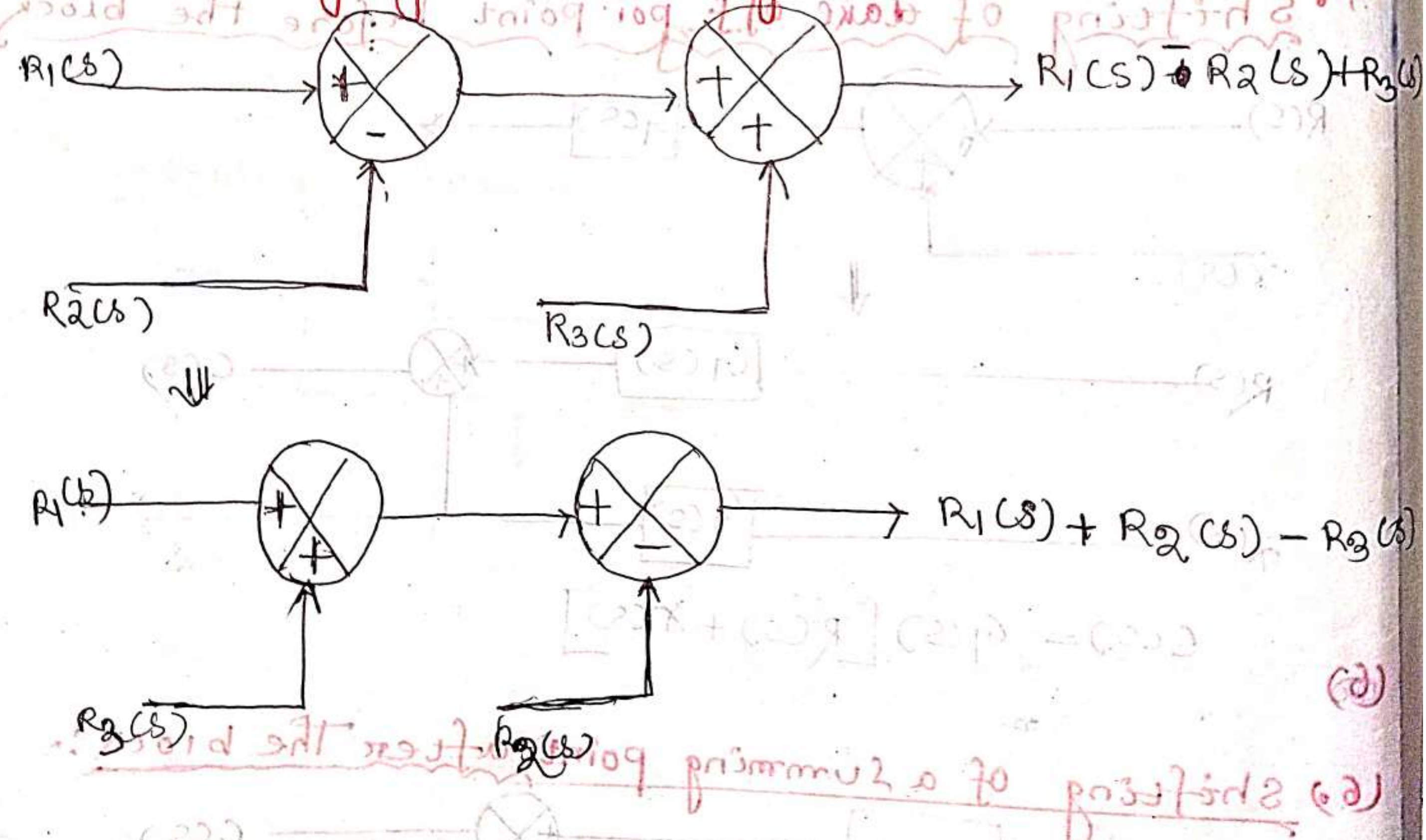
Step-3

$$C(s) = \left[R(s) + \frac{x(s)}{G(s)}\right] G(s)$$

$$= R(s)\cdot G(s) + \frac{x(s)}{\cancel{G(s)}} \times \cancel{G(s)}$$

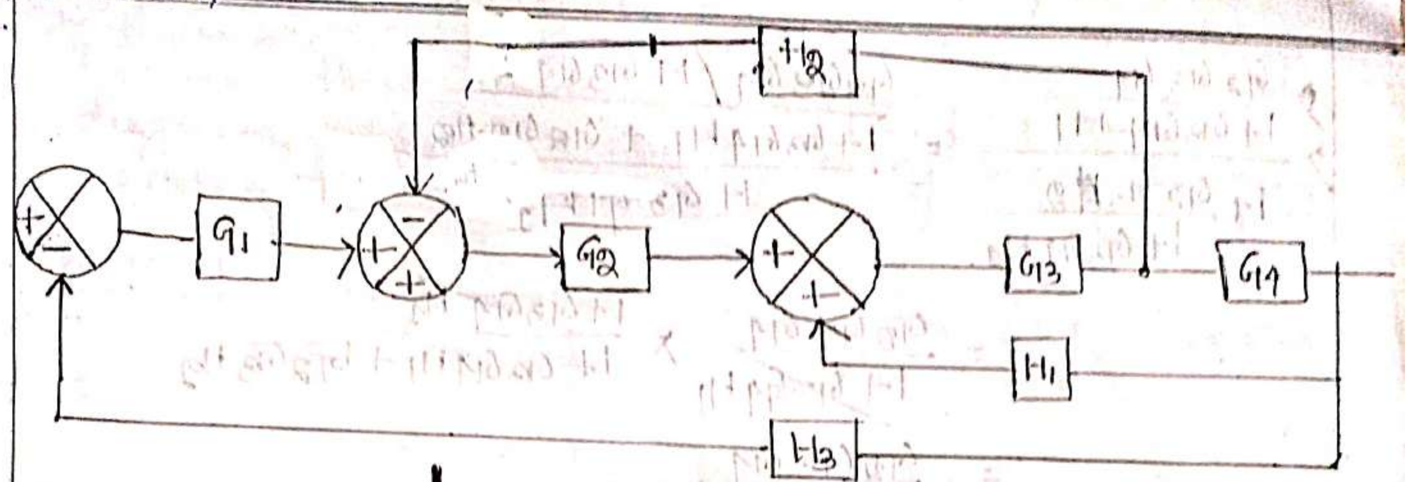
$$= R(s)\cdot G(s) + x(s)$$

07. Interchanging of summing point :→

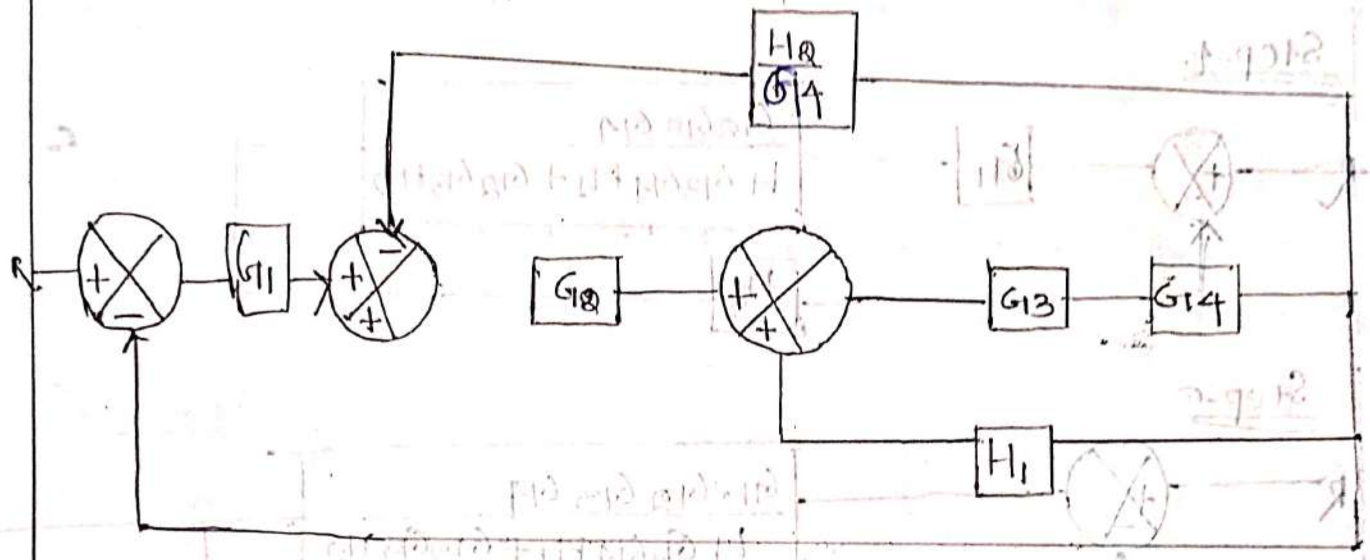


Problem,

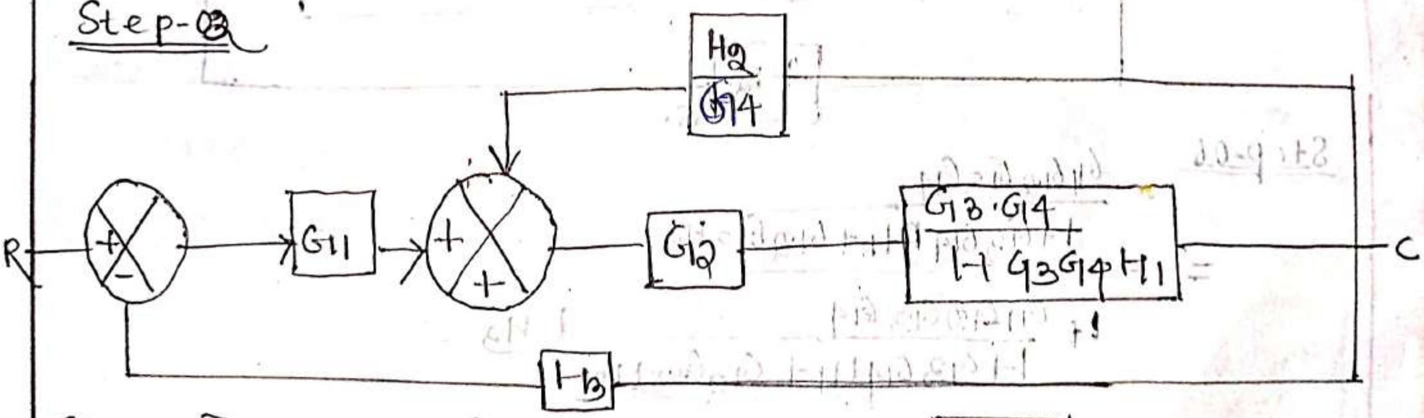
1.



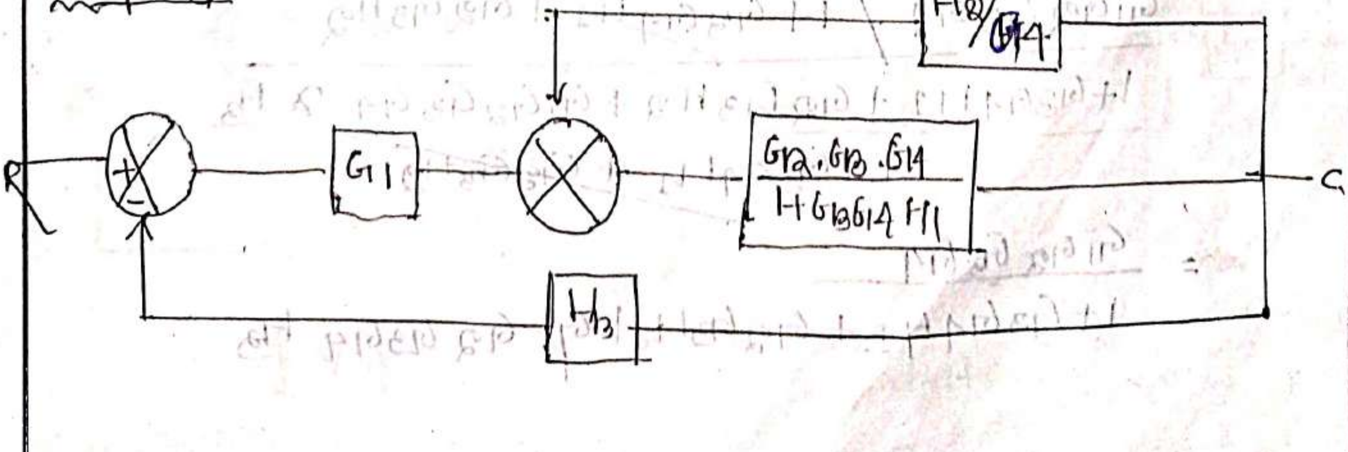
Step-1



Step-02



Step-03

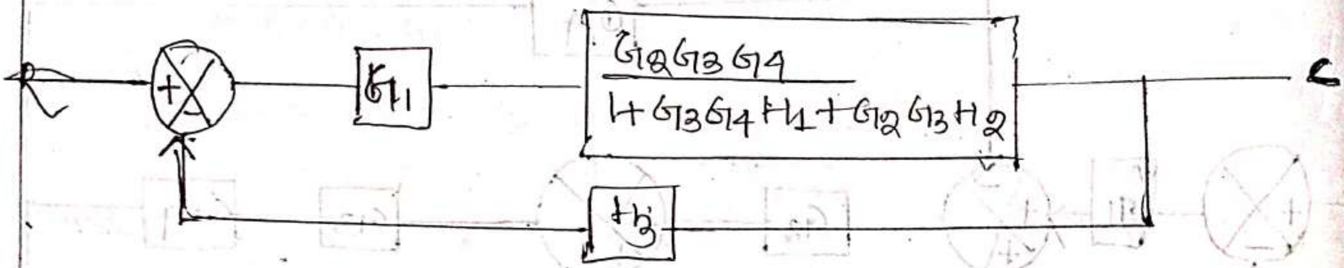


$$\Rightarrow \frac{\frac{G_2G_3G_4}{1+G_3G_4H_1}}{1+\frac{G_2G_3H_2}{1+G_3G_4H_1}} = \frac{G_2G_3G_4 / 1+G_3G_4H_1}{\frac{1+G_3G_4H_1 + G_2G_3H_2}{1+G_3G_4H_1}}$$

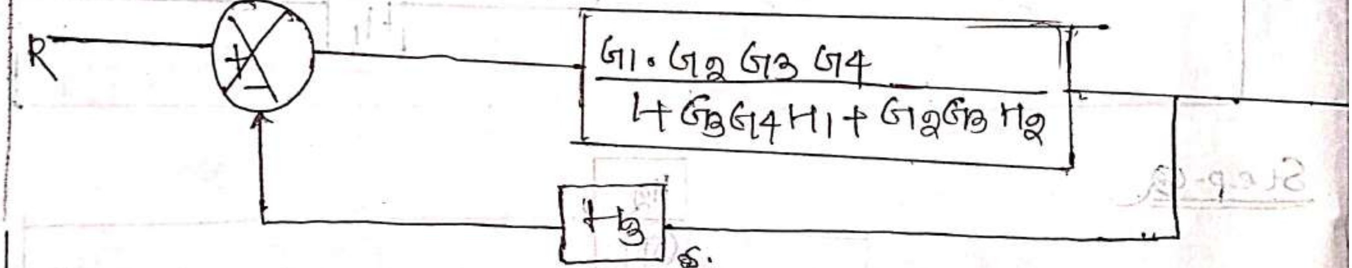
$$= \frac{G_2G_3G_4}{\cancel{1+G_3G_4H_1}} \times \frac{\cancel{1+G_3G_4H_1}}{1+G_3G_4H_1+G_2G_3H_2}$$

$$= \frac{G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2}$$

Step-4



Step-5



Step-06

$$= \frac{\frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2}}{1+\frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2} H_3}$$

$$= \frac{G_1G_2G_3G_4 / \cancel{1+G_3G_4H_1+G_2G_3H_2}}{\frac{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4 \times H_3}{\cancel{1+G_3G_4H_1+G_2G_3H_2}}}$$

$$= \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}$$

Procedure for ~~af~~ Reduction of block diagram

→ Follow these rules for simplifying the block diagram, which is having many blocks, summing points & take-off points.

Rule-1 Check for the blocks connected in series and simplify.

Rule-2 check for the blocks connected in parallel and simplify.

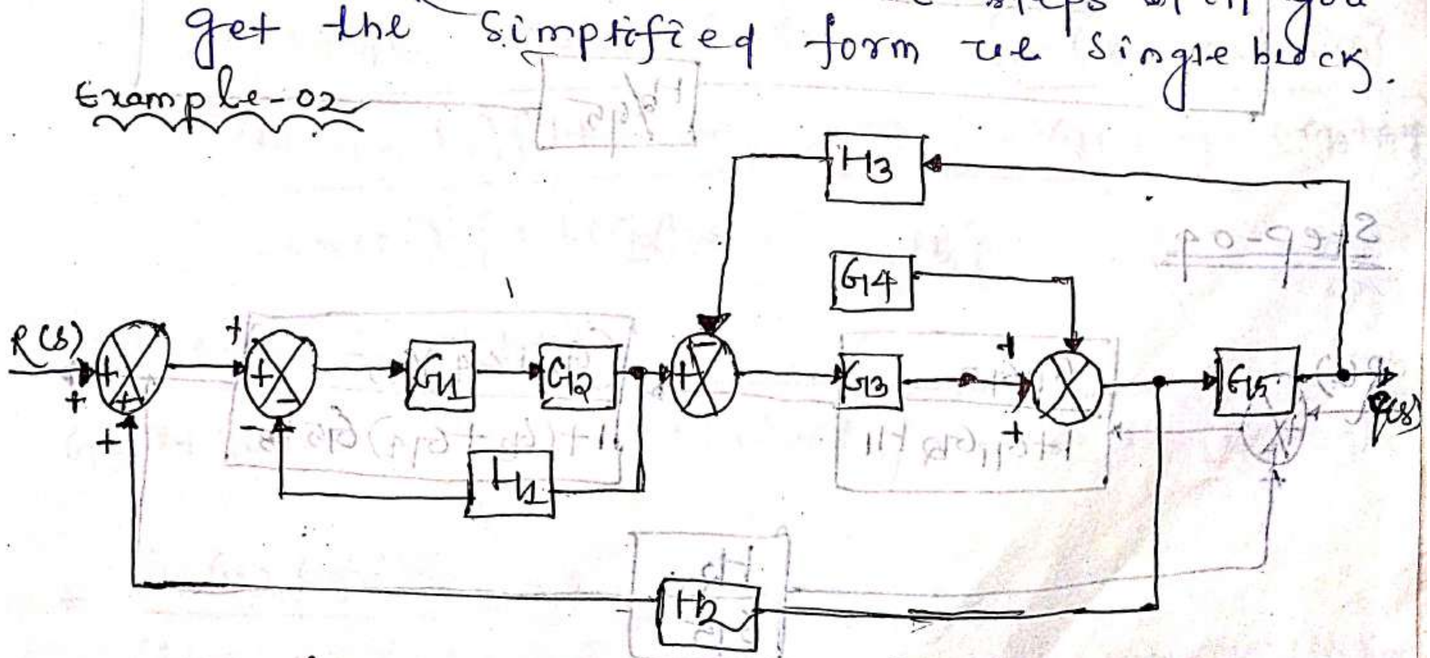
Rule-3 Check for the blocks connected in feedback loop & simplify.

Rule-04 If there is difficulty with take-off point while simplifying, shift it towards right.

Rule-05 If there is difficulty with summing point while simplifying, shift it towards left.

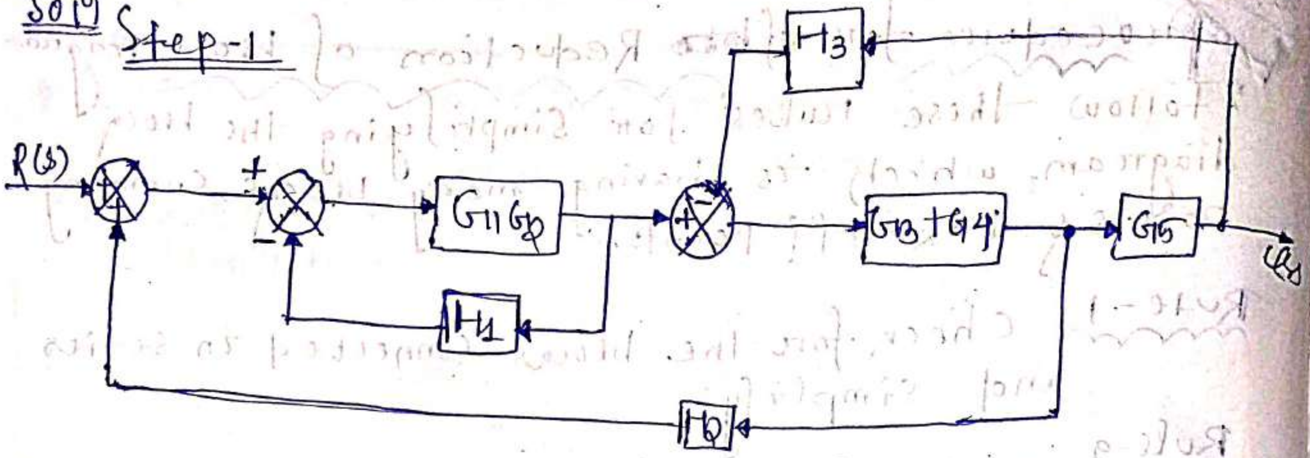
Rule-06 Repeat the above steps till you get the simplified form i.e single block.

Example-02

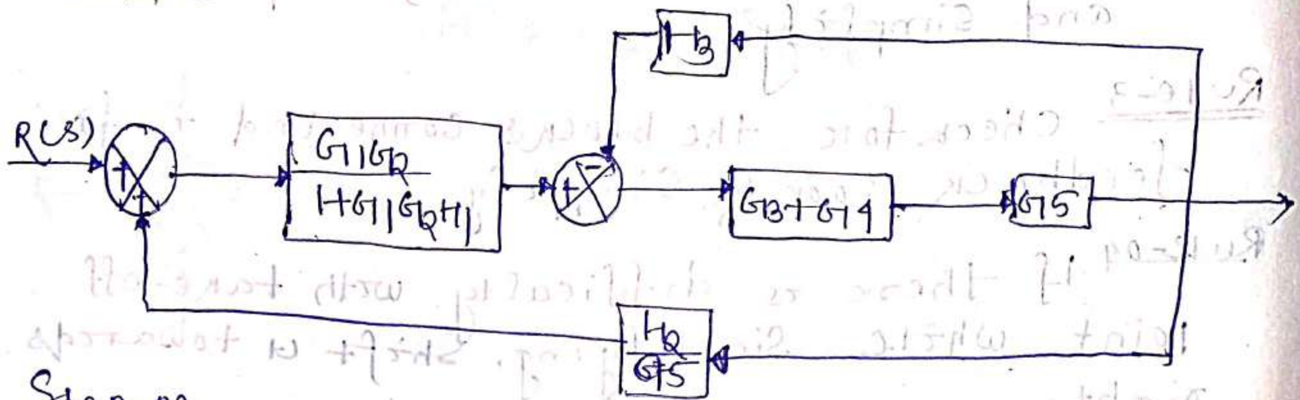


find the Transfer function :→

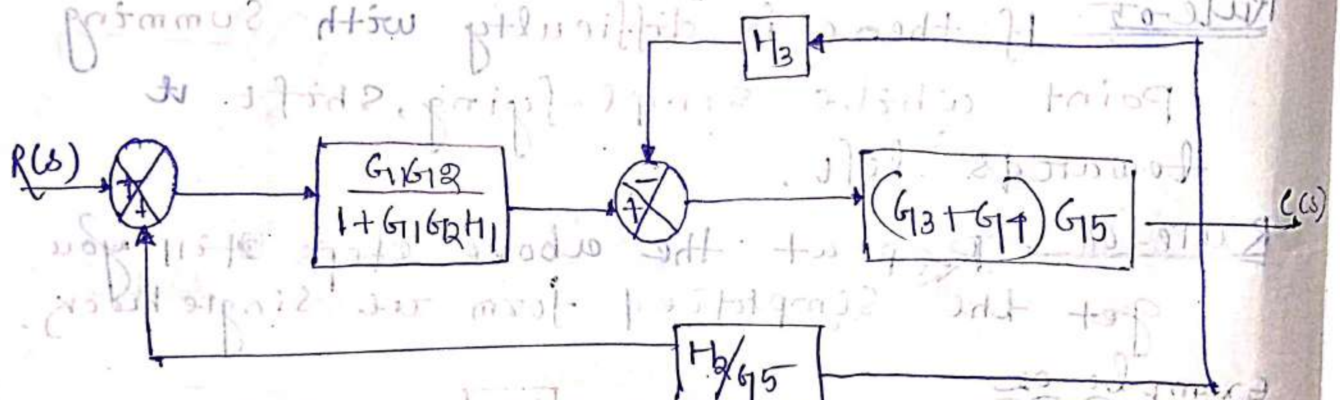
Sol^{n} Step-1:



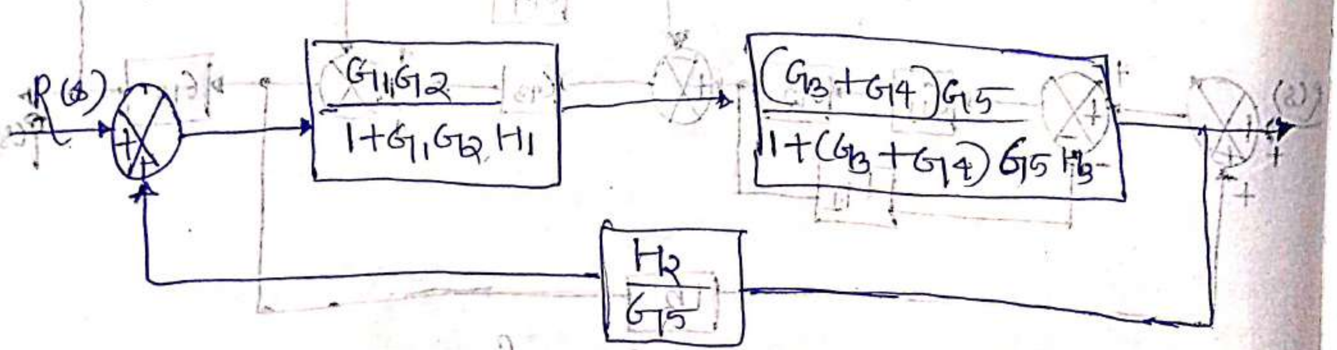
Step-2



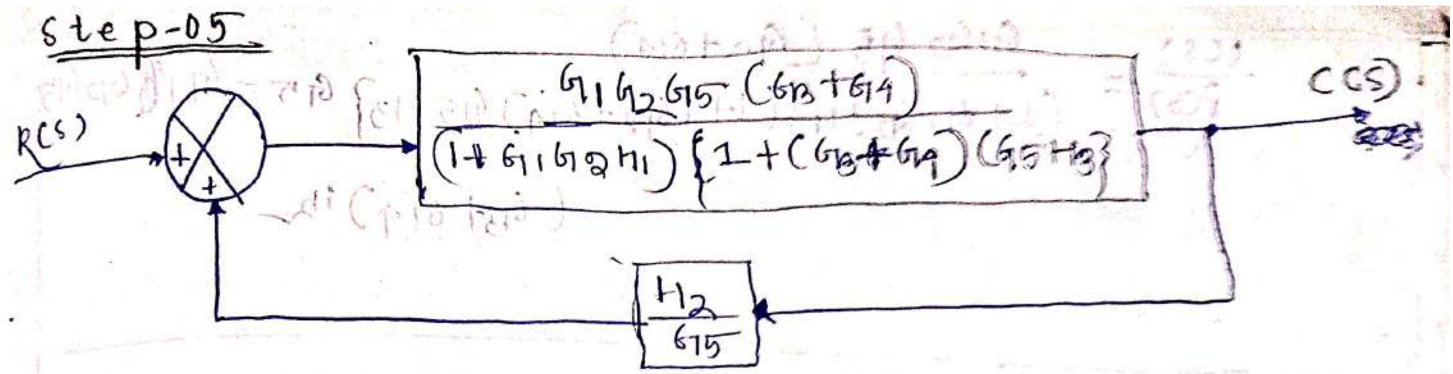
Step-03



Step-04



Step-05



Step-06

$$T.f = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)\cdot H(S)} \quad (+\text{ve feedback})$$

$$\Rightarrow G(S) = \frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5H_3)\}}$$

$$H(S) = \frac{H_2}{G_5}$$

$$\Rightarrow \frac{\dfrac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}}}{1 - \dfrac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}} \times \dfrac{H_2}{G_5}}$$

$$= \frac{G_1G_2G_5(G_3+G_4) \Big/ (1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}}{\dfrac{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5H_3)\} - G_1G_2G_5(G_3+G_4) \times \frac{H_2}{G_5}}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}}}$$

$$= \frac{G_1G_2G_5(G_3+G_4)}{G_5(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5H_3)\} - G_1G_2G_5(G_3+G_4)H_2}$$

$$= \frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5H_3)\}G_5 - G_1G_2G_5(G_3+G_4)H_2}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_5\,(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5 - G_1G_2G_5\,(G_3+G_4)H_2}$$

$$R(s) \rightarrow \boxed{\frac{G_1G_2G_3\,(G_3+G_4)}{(1+G_1G_2H_1)(1+(G_3+G_4)G_5H_3\}G_5 - G_1G_2G_5\,(G_3+G_4)H_2}}$$

Signal Flow Graphs

Node - A node represents a system variable which is equal to the sum of all the incoming signals at the node.

Input Node :- An input node is a node with only outgoing branches. It does not have any incoming branches.

Output Node :- An output node is a node with only incomings branches. It doesnot have any outgoing branches.

Forward path :→
A forward path is a path that starts at an input node and ends at an output node its called as forward path.

Mixed Node :→
A mixed node is a node that has both incoming and outgoing branch.

Loop :→
A loop is a path which originates and terminates at the same node and along which no node is traversed more than once.

Nontouching loop :→
Non Touching loops are loops which do not possess any common node.

Loop gain → The product of the branch gains encountered in traversing the loop is called the loop gain.

Self loop :- A self loop is a loop consisting of a single branch.

Self Node :- A loop that consist of only one Node is called as Self Node.

Mason's gain Formula :→

$$T.f = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta}$$

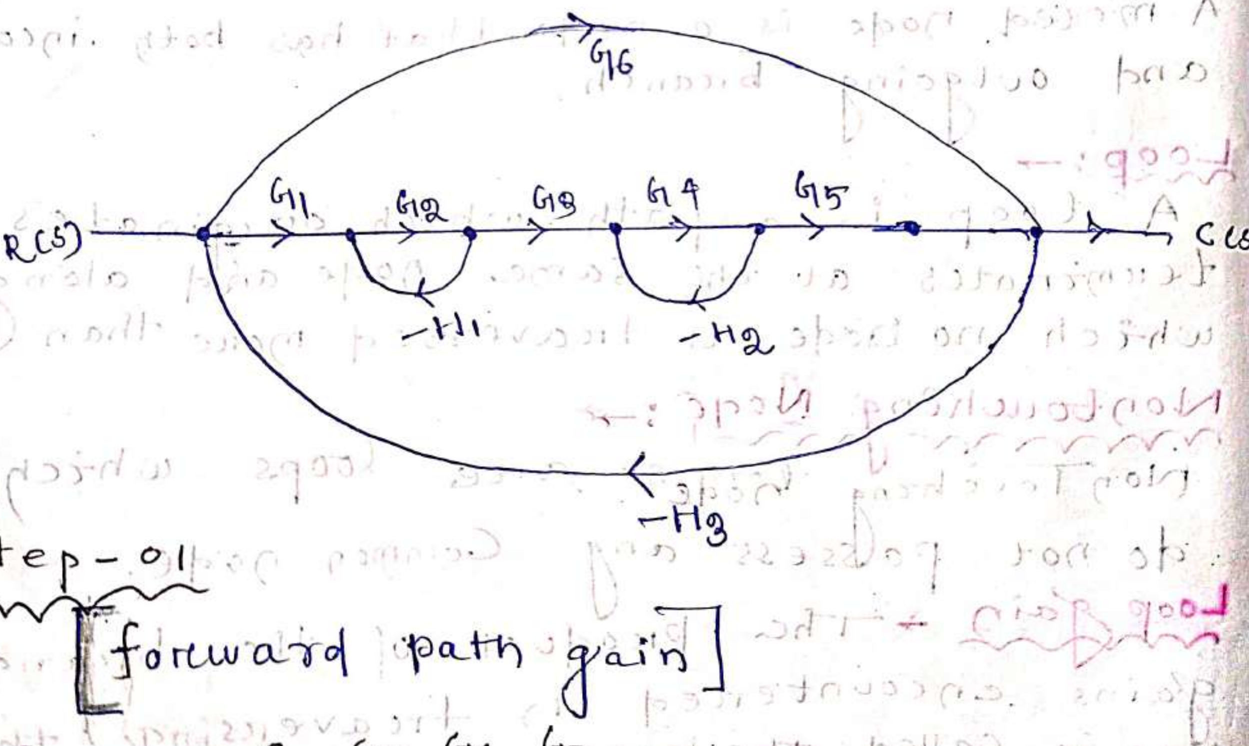
where,

N = Total number of forward path

P_i = gain of forward path.

Δ = 1-(Σ Loop gain) + (Σ gain prod of all possible Combinations of non-touching lo + (Σ gain product of all possible Combinations of Two non-touching loop) + ...

Δ_i = The value of after elementing all loops that touch its forward path.



Step - 01

[forward path gain]

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_6$

Step-02 (Gain of Two non-touching loop)

$L_{12} = L_1 \times L_2 = -G_2 H_1 \times -G_4 H_2$

$= G_2 H_1 G_4 H_2 = G_2 G_4 H_1 H_2$.

$L_{14} = L_1 \times L_4 = -G_2 H_1 \times -G_6 H_3$

$= G_2 G_6 H_1 H_3$

$L_{24} = -G_4 H_2 \cdot -G_6 H_3$

$= G_4 G_6 H_2 H_3$

Step-03 (Loop gain) :

$L_1 = -G_2 H_1$

$L_2 = -G_4 H_2$

$L_3 = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \, G_5 \cdot H_3$

$L_4 = -G_6 H_3$

Step-04 (Gain of three non-touching loop)

$L_{124} = L_1 \times L_2 \times L_4$

$= -G_2 H_1 \times -G_4 H_2 \times -G_6 H_3$

$= -G_2 G_4 G_6 H_1 H_2 H_3$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_{12} + L_{14} + L_{24}) + L_{124}$

$= 1 - (-G_2 H_1 - G_4 H_2 + G_1 G_2 G_3 G_4 G_5 H_3 - G_6 H_3)$

$+ (G_2 G_4 H_1 H_2 + G_2 G_6 H_1 H_3 + G_4 G_6 H_2 H_3)$

$- G_2 G_4 G_6 H_1 H_2 H_3$

$= 1 + G_2 H_1 + G_4 H_2 - G_1 G_2 G_3 G_4 G_5 H_3 + G_6 H_3 +$

$G_2 G_4 H_1 H_2 + G_2 G_6 H_1 H_3 + G_4 G_6 H_2 H_3$

$- G_2 G_4 G_6 H_1 H_2 H_3$

$\Delta_1 = 1$

$\Delta_2 = 1 + (G_2H_1 + G_4H_2)$

$\Delta_2 = 1 - (L_1 + L_2)$

$= 1 - (-G_2H_1 - G_4H_2)$

$= 1 + (G_2H_1 + G_4H_2)$

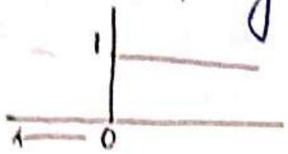
$$T.f = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 \cdot 1 + G_6 + (G_2H_1 + G_4H_2)}{1 - (L_1 + L_2 + L_3 + L_4) + (L_{12} + L_{14} + L_{24}) + L_{12}}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_6 + G_2H_1 + G_4H_2}{1 + G_2H_1 + G_4H_2 - G_1G_2G_3G_4G_5H_3 + G_6H_3 + G_2G_4H_1 + G_2G_6H_1H_3 + G_4G_6H_2H_3 - G_2G_4G_6H_1H_2H_3}$$

(CH.05) (Time Response Analysis)

Unit step signal:–



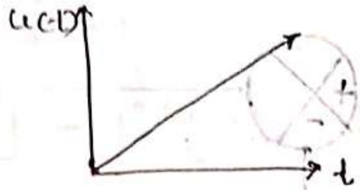
amplitude – 1

$r(t) = u(t)$

Ramp signal:→

→ Ramp is a signal which starts at a value of zero at $t=0$ and increases linearly with time.

$r(t) = At \quad t > 0$

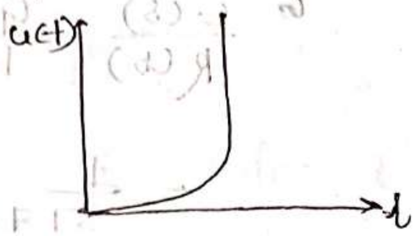


Parabolic Signal

→ The parabolic function represents a signal that is one order faster than the ramp function.

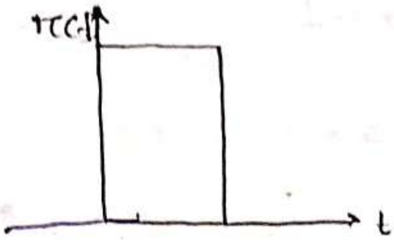
Mathmatically Represent,

$r(t) = At^2/2$



Impulse signal:→

A unit impulse is defined as a signal which has zero value everywhere except at $t=0$ where its magnitude is infinite.



$r(t) = 1$ for $T = 0$

$r(t) = 0$ for $t \neq 0$

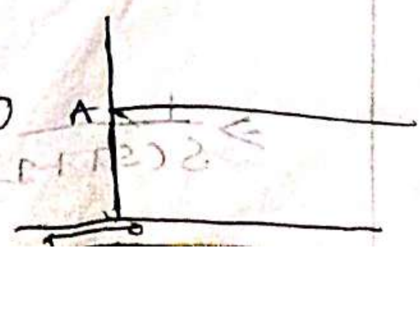
Standard Step Signal:→

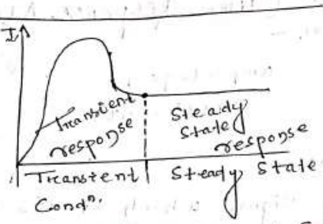
→ The step is a signal whose value changes from one level to another level in zero time.

$r(t) = Au(t)$

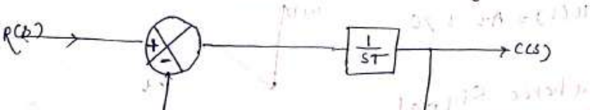
where, $u(t) = A$ for $t \geq 0$

$u(t) = 0$ for $t < 0$





Time response of 1st Order System :→



$G = \frac{1}{ST}$

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{1/ST}{1+\frac{1}{ST}\cdot 1} = \frac{1/ST}{\frac{ST+1}{ST}}$$

$$= \frac{1}{ST+1}$$

Unit step response of first Order System :→

$r(t) = 1$ for $t > 0$

$\Rightarrow L[r(t)] = L(1)$

$\Rightarrow R(s) = \frac{1}{s}$

$T.f = \frac{C(s)}{R(s)} = \frac{1}{ST+1}$

$\Rightarrow C(s) = \frac{1}{ST+1} \cdot R(s)$

$= \frac{1}{ST+1} \cdot \frac{1}{S} = \frac{1}{S(ST+1)}$

$\Rightarrow \frac{1}{S(ST+1)} = \frac{A}{S} + \frac{B}{ST+1}$

$\Rightarrow \frac{A(ST+1)+BS}{S(ST+1)} = \frac{1}{S(ST+1)}$

$\Rightarrow A(ST+1)+BS = 1$ $\qquad A = 1$

$\Rightarrow AST + A + BS = 1$ $\qquad AT + B = 0$

$\Rightarrow S(AT+B) + A = 1$ $\qquad B = -AT = -T$

$C(s) = \frac{1}{S(ST+1)} = \frac{1}{S} - \frac{T}{ST+1}$

$\Rightarrow L^{-1}C(s) = L^{-1}\left\{\frac{1}{S}\right\} - L^{-1}\left[\frac{T}{ST+1}\right]$

$= 1 - L^{-1}\left[\frac{T/T}{ST/T+1}\right]$

$= 1 - L^{-1}\left[\frac{1}{S+\frac{1}{T}}\right]$

$= 1 - e^{-1/T \times t} = 1 - e^{-t/T}$ for $t \geq 0$

★ $r(t) = 1$

$C(T) = 1 - e^{-t/T}$

$t = 0$

$C(t) = 1 - e^{-t/T}$

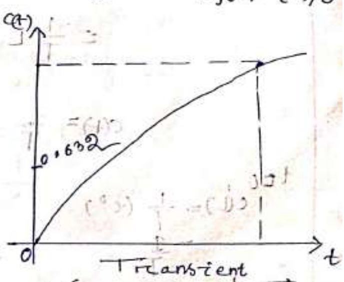
$= 1 - e^{0} = 1 - 1 = 0$

$t = T$

$C(t) = 1 - e^{-1} = 0.632$

★ $t = \infty$ $\quad C(t) = 1 - e^{-\infty}$

$= 1 - 0 = 1$



Error response of the system is :

$= \text{Input} - \text{Output}$

$= r(t) - c(t)$

$= 1 - (1 - e^{-t/T}) = \cancel{1} - \cancel{1} + e^{-t/T} = e^{-t/T}$

(Unit-Impulse Response of 1st Order

$r(t) = 1$

$R(s) = \frac{1}{s}$,

$r(s) = 1$

$r(t) = 0$

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

$$\Rightarrow C(s) = \frac{1}{sT+1} \cdot R(s)$$

$$\Rightarrow C(s) = \frac{1}{sT+1}$$

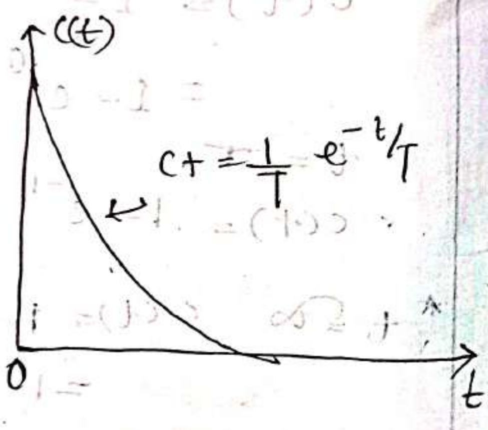
$$L^{-1}[C(s)] = L^{-1}\left[\frac{1}{sT+1}\right]$$

$$c(t) = L^{-1}\left[\frac{1/T}{\frac{sT}{T} + \frac{1}{T}}\right]$$

$$= L^{-1}\left[\frac{1/T}{s+1/T}\right] = \frac{1}{T} L^{-1}\left[\frac{1}{(s+\frac{1}{T})T}\right]$$

$$= \frac{1}{T} L^{-1}\left[\frac{1}{s+\frac{1}{T}a}\right]$$

$$c(t) = \frac{1}{T} \times \left[e^{-t/T}\right]$$



$t=0$,

$$c(t) = \frac{1}{T}(e^0) = \frac{1}{T}$$

$t = T \Rightarrow c(t) = \frac{1}{T}(e^{-1})$

$$= \frac{0.367}{T}$$

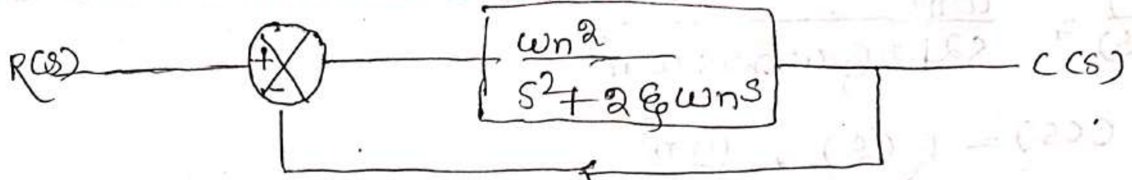
$* \; t = \infty = \frac{1}{T}(e^{-\infty}) = \frac{1}{T} \times 0 = 0$

Error = Input − output

$= r(t) - c(t)$

$= 0 - \frac{1}{T} e^{-t/T}$

Time response of Second order System to the unit step Input :→



$(G) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s} = \frac{G}{1 + G \cdot H}$

$= \frac{\frac{\omega_n^2}{s^2 + 2\xi\omega_n s}}{1 + \frac{\omega_n^2}{s^2 + 2\xi\omega_n s} \cdot 1}$

$= \omega_n^2 / \cancel{s^2 + 2\xi\omega_n s} \times \frac{\cancel{s^2 + 2\xi\omega_n s}}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$= \omega_n^2 / s^2 + 2\xi\omega_n s + \omega_n^2$

Here, ω_n = natural frequency

ξ = Damping ratio.

Characteristics equation $= 1 + G(s) \cdot H(s) = 0$

$= s^2 + 2\omega_n \xi s + \omega_n^2$

$a = 1$, $b = 2\xi\omega_n$, $c = \omega_n^2$

$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4 \cdot 1 \cdot \omega_n^2}}{2}$

$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$

$= \frac{-2\omega\xi_n \pm \sqrt{4\omega_n^2(\xi^2 - 1)}}{2}$

$$\Rightarrow \frac{-\cancel{2}\zeta\omega_n \pm \cancel{2}\omega_n\sqrt{\zeta^2-1}}{\cancel{2}}$$

$$\Rightarrow -\omega\zeta n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$s^2+2\zeta\omega_n s+\omega_n^2 = (s+\zeta\omega_n+j\omega_d)(s+\zeta\omega_n-j\omega_d)$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= 1/s \cdot \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = A/s + \frac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2} \quad \text{---(1)}$$

$$\Rightarrow \frac{\omega_n^2}{\cancel{s(s^2+2\zeta\omega_n s+\omega_n^2)}} = \frac{A(s^2+2\zeta\omega_n s+\omega_n^2)+(Bs+C)s}{\cancel{s(s^2+2\zeta\omega_n s+\omega_n^2)}}$$

$$\Rightarrow \omega_n^2 = A(s^2+2\zeta\omega_n s+\omega_n^2)+(Bs+C)s$$

$$\Rightarrow \omega_n^2 = As^2+2\zeta\omega_n sA+A\omega_n^2+Bs^2+Cs$$

$$\Rightarrow \omega_n^2 = As^2+Bs^2+\cancel{\omega_n^2 A}+2\zeta\omega_n s A+Cs+\omega_n^2 A$$

$$\Rightarrow \omega_n^2 = s^2(A+B)+s(2A\zeta\omega_n+C)+\omega_n^2 A$$

$$s^2(A+B) = \omega_n^2$$

$$A+B=0$$

$$B=-A=-1$$

$$s(2A\zeta\omega_n+C) = \omega_n^2$$

$$2A\zeta\omega_n+C=0$$

$$C=-2A\zeta\omega_n$$

Put the value a, b, c in eq^n (1)

$$\Rightarrow \frac{\omega n^2}{s(s^2+2\xi\omega n s+\omega n^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2\xi\omega n s+\omega n^2}$$

$$= \frac{1}{s} - \frac{s+2\xi\omega n}{s^2+2\xi\omega n s+\omega n^2}$$

$$= \frac{1}{s} - \frac{s+2\xi\omega n}{s^2+2\xi\omega n s+(\xi\omega n)^2-(\xi\omega n)^2+\omega n^2}$$

$$= \frac{1}{s} - \frac{s^2+2\xi\omega n}{(s+\xi\omega n)^2+\omega n^2(1-\xi^2)} \quad \left\{\omega d = \omega n\sqrt{1-\xi^2}\right.$$

$$= \frac{1}{s} - \frac{s^2+2\xi\omega n}{(s+\xi\omega n)^2+\omega d^2}$$

$$= \frac{1}{s} - \frac{(s+\xi\omega n)}{(s+\xi\omega n)^2+\omega d^2} - \frac{\xi\omega n}{(s+\xi\omega n)^2+\omega d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega n}{(s+\xi\omega n)^2+\omega d^2} - \frac{\xi\omega n.\omega d}{\omega d(s+\xi\omega n)^2+\omega d^2}$$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega n}{(s+\xi\omega n)^2+\omega d^2} - \frac{\xi\omega n}{\omega d} \cdot \frac{\omega d}{(s+\xi\omega n)^2+\omega d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega n}{(s+\xi\omega n)^2+\omega d^2} - \frac{\xi}{\sqrt{1-\xi^2}} \cdot \frac{\omega d}{(s+\xi\omega n)^2+\omega d^2}$$

Taking Inverse Laplace Term.

$$\rightarrow L(t) = 1 - e^{-\xi\omega n t}\cos\omega d t - \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega n t}\sin\omega d t$$

$$= 1 - e^{-\xi\omega n t}\left[\cos\omega d t - \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega d t\right]$$

$= 1 - e^{-\xi \omega n t} \cdot \sin(\omega d t + \alpha)$ $\left\{ \begin{array}{l} \text{consider} \\ \sin \omega d t = 1 \\ \cos \omega d t = \dfrac{\xi}{\sqrt{1-\xi^2}} \end{array} \right.$

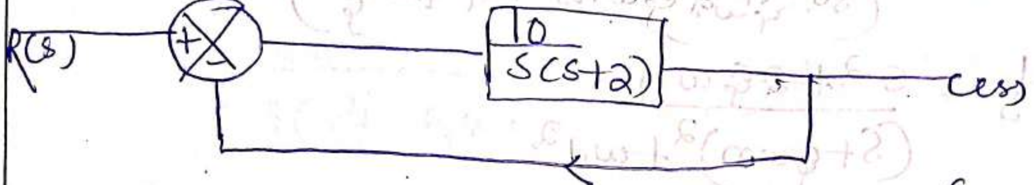
$= \tan \alpha = \dfrac{\sqrt{1-\xi^2}}{\xi}$

$\alpha = \tan^{-1} \dfrac{\sqrt{1-\xi^2}}{\xi}$

Problem A unit feedback Control system has an openloop Transformation ?

$G(s) = \dfrac{10}{s(s+2)}$ find ωn, ωd, & ξ

Solution



$T.f = \dfrac{G}{1+G.H} = \dfrac{10/s(s+2)}{1+\dfrac{10}{s(s+2)}} = \dfrac{10/\cancel{s(s+2)}}{\dfrac{s^2+2s+10}{\cancel{s(s+2)}}}$

$= \dfrac{10}{s^2+2s+10} = \dfrac{10}{s^2+2\xi\omega n s+\omega n^2}$

$\omega n^2 = 10$

$\omega n = \sqrt{10}$ $\qquad 2\xi\omega n s = 2s$

$\Rightarrow 2\xi \omega n = 2$

$\Rightarrow \xi = \dfrac{2}{2\omega n} = \dfrac{\cancel{2}}{\cancel{2}\times\sqrt{10}} = \dfrac{1}{\sqrt{10}}$

$\Rightarrow \omega d = \omega n\sqrt{1-(\xi)^2}$

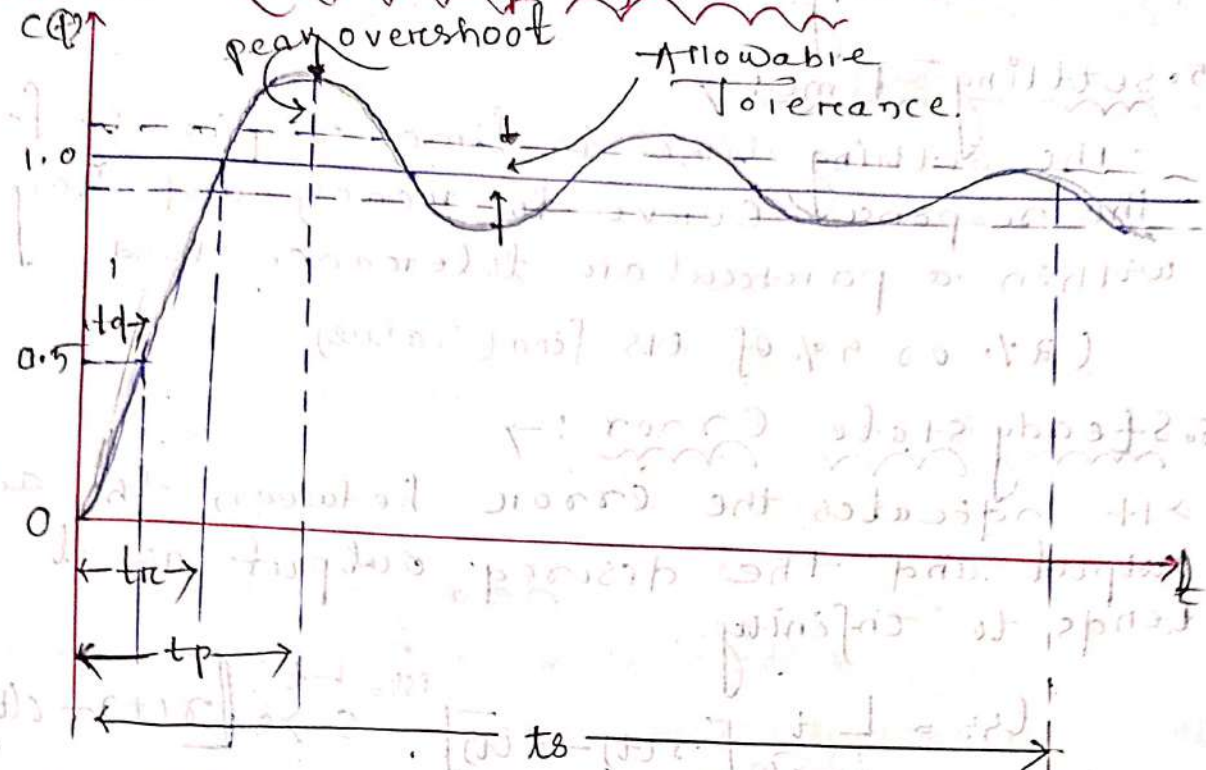
$= \sqrt{10} \times \sqrt{1-\left(\dfrac{1}{\sqrt{10}}\right)^2}$

$= \sqrt{10} \times \sqrt{1-\dfrac{1}{10}} = \sqrt{10} \times \sqrt{\dfrac{9}{10}} = \sqrt{10} \times \dfrac{\sqrt{9}}{\sqrt{10}} = \sqrt{9}$

$= 3$

$\therefore \omega n = \sqrt{10}$, $\omega d = 3$, $\xi = \dfrac{1}{\sqrt{10}}$

Time Response Specifications :→



1. Delay Time :→ (Td) - The delay time required for the response to reach 50% of the final value the very fast time.

2. Rise time :→ (Tr) :→ The Rise time is the time required for the response to rise from 0 to 100% of the final value for Underdamped systems and from 10% to 90% of the final value for overdamped systems.

3. Peak Time (tp) →
The peak time is the time required for the response to reach the first peak of the overshoot.

4. Maximum Overshoot :→
> It is the normalise difference between peak of the time response & steady output.

Mathmatically, $\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$

where $c(t_p)$ - Maximum value at 1st overshoot

$C(\infty)$ = Steady State value.

5. Settling Time :→

The settling time is time required for the response curve to reach and stay within a particular tolerance band.

(2% or 5% of its final values)

6. Steady State error :→

→ It indicates the error between the actual output and the desired output as t tends to infinity.

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)] \qquad e_{ss} = \lim_{s \to 0} [r(t) - c(t)]$$

Steady state error & error Constants :→

$$T.f = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S) \cdot H(S)}$$

$$C(S) = E(S) \cdot G(S)$$

$$\Rightarrow E(S) = \frac{C(S)}{G(S)}$$

$$E(S) = \frac{C(S)}{G(S)} = \frac{R(S)}{G(S)} = \frac{R(S)}{1 + G(S) \cdot H(S)}$$

1. Static position Error Constant K_p :→

The steady state error of the system for a unit-step input - $r\,t = 1$

$$R(S) = 1/s$$

$$e_{ss} = \lim_{t \to \infty} e t$$

$$e_{ss} = \lim_{t \to \infty} S \cdot E(S)$$

$$= \lim_{t \to \infty} S \cdot \frac{R(S)}{1 + G(S) \cdot H(S)}$$

$$= \underset{t \to \infty}{Lt} \; s \cdot \frac{1/s}{1+G(s) \cdot H(s)}$$

$$= \underset{t \to \infty}{Lt} \frac{1}{1+G(s) H(s)}$$

$$= \frac{1}{1 + \underset{t \to \infty}{Lt} G(s) H(s)}$$

$$K_p = \underset{t \to \infty}{Lt} G(s) H(s)$$

$$E_{ss} = \frac{1}{1+K_p}$$

Static Velocity Error Constant K_v :→

The steady state error of the system for a unit ramp input: $r(t) = 1$

$R(s) = 1/s^2$

$$e_{ss} = \underset{s \to 0}{Lt} \; s E(s)$$

$$= \underset{s \to 0}{Lt} \; s \cdot \frac{R(s)}{1+G(s) \cdot H(s)}$$

$$= \underset{s \to 0}{Lt} \; s \cdot \frac{1/s^2}{1+G(s) \cdot H(s)} = \frac{1}{\underset{s \to 0}{Lt} \; s \, G(s) H(s)} = \frac{1}{\underset{s \to 0}{\lim} s + s G(s) H(s)}$$

$$\frac{1}{0 + s G(s) \cdot H(s)} = K_v = \underset{s \to 0}{Lt} \; s \cdot G(s) H(s)$$

$$= \frac{1}{K_v}$$

Static Acceleration Error Constant K_a:-

The steady-state error of the system for a unit parabolic input $r(t) = t^2/2$, $R(s) = 1/s^3$.

$$e_{ss} = \underset{s \to 0}{Lt} \; s E(s) = \underset{s \to 0}{Lt} \; s \cdot \frac{R(s)}{1+G(s)H(s)} = \underset{s \to 0}{Lt} \frac{s \cdot 1/s^3}{1+G(s) H(s)}$$

$$= \frac{1/s^2}{\underset{s \to 0}{Lt} \; s^2 G(s) H(s)} = \frac{1}{\underset{s \to 0}{\lim} s^2 + s^2 + G(s) \cdot H(s)} = \frac{1}{\underset{s \to 0}{\lim} s^2 G(s) H(s)}$$

where $K_a = \underset{s \to 0}{Lt} \; s^2 G(s)$

Rise Time (T_r): →

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

at $t = t_r$ response 100%, so $c(t_r) = 1$

$c(t_r) = 1$

$$\Rightarrow 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = 0 = \sin\pi$$

$$\Rightarrow \omega_d t_r + \phi = \pi$$

$$\omega_d t_r = \pi - \phi$$

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$$= \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

Peak Time: → (t_p)

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

$c(t_p)$ = Maximum Response.

$$\frac{dc(t_p)}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left[1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \phi)\right] = 0$$

$$\Rightarrow 0 - \frac{d}{dt}\left[\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \phi)\right] = 0$$

$\frac{d}{dt}(ab) = a\frac{d}{dt}b + b\frac{d}{dt}a$

$$\Rightarrow \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \cos(\omega_d t_p + \phi) \cdot \omega_d + \sin(\omega_d t_p + \phi) \cdot \frac{e^{-\xi\omega_n t}}{1-\xi^2}(-\xi\omega_n) = 0$$

$\Rightarrow \omega_d \cos(\omega_d t_p + \phi) - \xi\omega_n \sin(\omega_d t_p + \phi) = 0$

$\frac{d}{dx} e^{ax} = a e^{ax}$

$\Rightarrow \omega_n\sqrt{1-\xi^2} \cos(\omega_d t_p + \phi) - \xi\omega_n \sin(\omega_d t_p + \phi) = 0$

$\frac{d}{dx} \sin ax = a\cos ax$

$\Rightarrow \sqrt{1-\xi^2} \cos(\omega_d t_p + \phi) - \xi \sin(\omega_d t_p + \phi) = 0$

$\Rightarrow \sin\phi \cdot \cos(\omega_d t_p + \phi) - \cos\phi \cdot \sin(\omega_d t_p + \phi) = 0$

$\Rightarrow \sin(\omega_d t_p + \phi - \phi) = 0 = \sin\pi$

$\Rightarrow \omega_d t_p = \pi$

$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

(Mp) Maximum Peak Overshoot: $\rightarrow$ $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$

$M_p = c(t_p) - 1$

$= \cancel{1} - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \phi) - \cancel{1}$

$= -\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \phi)$

put the value t_p

$= -\frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\cancel{\omega_n\sqrt{1-\xi^2}} \times \frac{\pi}{\cancel{\omega_n\sqrt{1-\xi^2}}} + \phi\right)$

$= -\frac{e^{-\xi\omega_n \pi/\omega_n\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sin(\pi + \phi)$

$= -\frac{e^{-\xi\cancel{\omega_n} \frac{\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot -\sin\phi$

$\Rightarrow -\frac{e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \cdot (-\sin\phi)$

$$= \frac{e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}}{\cancel{\sqrt{1-\xi^2}}} \cdot \cancel{\sqrt{1-\xi^2}} \qquad \left\{ \because \sqrt{1-\xi^2} = \sin\phi \right.$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}}$$

Therefore, the peak percent overshoot is

$$= 100 \times e^{-\pi\xi/\sqrt{1-\xi^2}} \ \%$$

Settling Time (t_s) :→

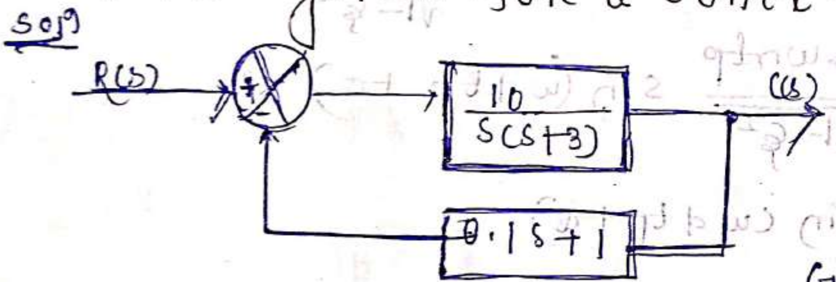
$$t_s = 4T = \frac{4}{\xi\omega_n} \quad (2\% \text{ band})$$

$$t_s = 3T = \frac{3}{\xi\omega_n} \quad (5\% \text{ band})$$

Ex :- 1

A positional control system with velocity feedback is shown in fig, what is response of the system for a unit step input?

Soln



$$G(s) = \frac{10}{s(s+3)}$$

$$H(s) = 0.1s + 1$$

T.f of the system

$$\frac{C(s)}{R(s)} = \frac{10/s(s+3)}{1 + \frac{10}{s(s+3)} \times (0.1s+1)}$$

$$= \frac{10/s(s+3)}{s^2+4s+10}$$

for a unit step input $r(t) = 1$

$r(s) = 1/s$

$$\Rightarrow C(s) = R(s) \cdot \frac{10}{s^2+4s+10}$$

$$= \frac{1}{s} \cdot \frac{10}{s^2+4s+10}$$

$$\Rightarrow \frac{10}{s(s^2+4s+10)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+10} \quad \text{—— (i)}$$

$$= \frac{A(s^2+4s+10) + (Bs+C)s}{s(s^2+4s+10)}$$

$$= As^2 + 4As + 10A + Bs^2 + Cs$$

$$= (A+B)s^2 + s(4A+C) + 10A$$

$(A+B)\cancel{s} = 10$ $\qquad$ $10A = 10$

$\Rightarrow A + B = 0$ $\qquad$ $A = \frac{10}{10} = 1$

$B = -A$ —— (ii) $\qquad$ $\boxed{A=1}$

$4A + C = 0$

$4 \times 1 + C = 0$

$\boxed{C = -4}$ $\qquad$ $\boxed{B = -1}$

Put the value A, B, & C in eqn (1)

$$\Rightarrow C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+4s+10}$$

$$= \frac{1}{s} - \frac{s+4}{s^2+2\cdot 2s+4+10-4}$$

$$= \frac{1}{s} - \frac{s+4}{(s+2)^2+(\sqrt{6})^2}$$

$$= \frac{1}{s} - \frac{s+2}{(s+2)^2+(\sqrt{6})^2} - \frac{2}{(s+2)^2+(\sqrt{6})^2}$$

$$= \frac{1}{s} - \frac{s+2}{(s+2)^2+(\sqrt{6})^2} - \frac{1}{\sqrt{6}} \times \frac{2\sqrt{6}}{(s+2)^2+(\sqrt{6})^2}$$

$$= \frac{1}{s} - \frac{s+2}{(s+2)^2+(\sqrt{6})^2} - \frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{(s+2)^2+(\sqrt{6})^2}$$

Taking Inverse Laplace transform, the response is

$$c(t) \Rightarrow L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{s+2}{(s+2)^2+(\sqrt{6})^2}\right\} - \frac{2}{\sqrt{6}} L^{-1}\left\{\frac{2\sqrt{6}}{(s+2)^2+(\sqrt{6})^2}\right\}$$

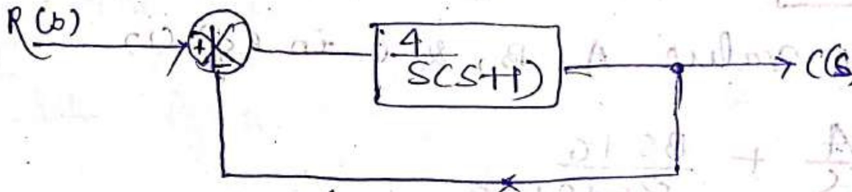
$$= 1 - e^{-2t}\cos\sqrt{6}t - \frac{2}{\sqrt{6}} e^{-2t}\sin\sqrt{6}t \quad \text{(Ans)}$$

Ex-02 The open loop transfer function of a unity feedback system is

$$G(s) = \frac{4}{s(s+1)}$$

Determine the nature of response of the closed loop system for a unit-step input. Also determine the rise time, peak time, peak Overshoot and settling time.

Solution $G(s) = \frac{4}{s(s+1)}$



$$T.f = \frac{4/s(s+1)}{1+\frac{4}{s(s+1)}} = \frac{4}{s^2+s+4}$$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2+s+4} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\Rightarrow \omega_n^2 = 4$

$\Rightarrow \omega_n = \sqrt{4} = 2$

$\Rightarrow 2\xi\omega_n = 1$

$\Rightarrow 2\xi 2 = 1$

$\Rightarrow \xi = \frac{1}{4} = 0.25$

since $\xi < 1$, the system is an underdamped

$\omega_n = 2$ and $\xi = 0.25$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 2\times\sqrt{1-(0.25)^2} = 1.936 \text{ rad/s}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-(0.25)^2}}{0.25} = 1.310 \text{ rad}$$

(1) The rise time $t_r = \frac{\pi-\theta}{\omega_d} = \frac{3.141 - 1.310}{1.936}$

$= 0.945 s.$

(2) The peak time. $t_p = \frac{\pi}{\omega_d} = \frac{3.141}{1.936} = 1.622 s.$

(3) The peak Overshoot $=$

$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$= 0.4326$

% of peak over shoot

$M_p \times 100\% = 0.4326 \times 100\%$

$= 43.26$

(4) The settling time for 5% error $=$

$t_s = \frac{3}{\xi\omega_n} = \frac{3}{0.25\times 2} = 6 s$

for 2% error $t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.25\times 2} = 8 s$

Dt :- 22-04-22

(5.5) Types of Control System [Steady state errors in Type-0, Type-1, Type-2 System]

Pole-Zero form

$$G(s)\cdot H(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)+\cdots}{s^n(s+p_1)(s+p_2)(s+p_3)+\cdots}$$

Type-0 system = $n=0$
Type-1 system = $n=1$
Type-2 system = $n=2$

Steady state error: Type 0 system :→

$$G(s)\cdot H(s) = \frac{K(s+z_1)(s+z_2)+\cdots}{s^N(s+p_1)(s+p_2)+\cdots}$$

- $K_p = \lim_{s\to 0} G(s)H(s)$

$$= \lim_{s\to 0} \frac{K(s+z_1)(s+z_2)+\cdots}{s^N(s+p_1)(s+p_2)+\cdots}$$

$$= \frac{K(z_1)(z_2)(z_3)\cdots}{p_1\times p_2\times p_3\cdots} = \text{Constant}.$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\text{constant}} = \text{Constant}.$$

- $K_V = \lim_{s\to 0} s\, G(s)\cdot H(s)$

$$= \lim_{s\to 0} \frac{s\cdot K(s+z_1)(s+z_2)+\cdots}{(s+p_1)(s+p_2)+\cdots}$$

$s=0$ $= 0$

$$e_{ss} = \frac{1}{K_V} = \frac{1}{0} = \infty$$

• $K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)$

$= \lim_{s \to 0} \left[s^2 \times \frac{K(s+z_1)(s+z_2)(s+z_3)}{(s+p_1)(s+p_2)(s+p_3)} \right]$

$s^2 = 0 \Rightarrow 0$

$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$

Type-0 Control system :→

$K_p = \text{Constant}$. $\quad$ $e_{ss}(\text{position}) = \text{Constant}$

$K_v = 0$ $\quad$ $e_{ss}(\text{velocity}) = \infty$

$K_a = 0$ $\quad$ $e_{ss}(\text{acceleration}) = \infty$

Steady state error. Type-1 system :-

$G(s) \cdot H(s) = \frac{K(s+z_1)(s+z_2)+ \ldots}{s(s+p_1)(s+p_2)+ \ldots}$ $\Big\}$ Type-1 system $\eta = 1$ so $s^{\eta} = s$

• $K_p = \lim_{s \to 0} G(s) \cdot H(s)$

$= \lim_{s \to 0} \frac{K(s+z_1)(s+z_2)+ \ldots}{s(s+p_1)(s+p_2)+ \ldots}$

$= \lim_{s \to 0} \frac{K(s+z_1)(s+z_2)}{0 \times p_1 \times p_2} =$

$= \frac{K(s+z_1)(s+z_2)}{0} = \infty$

e_{ss} (steady state error) =

$\frac{1}{1+K_p} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

• $K_V = \lim_{s \to 0} s\, G(s) \cdot H(s)$

$$= \lim_{s \to 0} \cancel{s} \cdot \frac{K(s+z_1)(s+z_2)+\cdots}{\cancel{s}(s+p_1)(s+p_2)+\cdots}$$

$$= \lim_{s \to 0} \frac{K(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$$

$$= \frac{K z_1 z_2}{p_1 p_2} = \text{Constant}$$

ess (steady state error) $= \frac{1}{K_V} = \frac{1}{\text{Constant}} = \text{Constant}$

• $K_a = \lim_{s \to 0} s^2\, G(s) \cdot H(s)$

$$= \lim_{s \to 0} \cancel{s^2}\, \frac{K(s+z_1)(s+z_2)+\cdots}{\cancel{s}(s+p_1)(s+p_2)+\cdots}$$

$$= \lim_{s \to 0} s\, \frac{K(s+z_1)(s+z_2)+\cdots}{(s+p_1)(s+p_2)+\cdots}$$

$s = 0 \Rightarrow 0$

$K_a = 0$ ess (steady state error) $= \frac{1}{K_a} = \frac{1}{0} = \infty$

Type = 1 Control system :-

$K_p = \infty$ ess (position) $= 0$

$K_v = \text{Constant}$ ess (Velocity) $= \text{Constant}$

$K_a = 0$ ess (acceleration) $= \infty$

Steady state error :- Type-2 system :-

$$G(s)\cdot H(s) = \frac{K(s+z_1)(s+z_2)+\cdots}{s^2(s+p_1)(s+p_2)+\cdots}$$

- $K_p = \lim_{s\to 0} G(s)\cdot H(s) = \lim_{s\to 0} \frac{K(s+z_1)(s+z_2)}{s^2(s+p_1)(s+p_2)}$

$$= \lim_{s\to 0} \frac{K\cdot z_1 z_2}{s^2 p_1 p_2}$$

$$s=0 = \frac{K z_1 z_2}{s^2 \times p_1 p_2} = \frac{\text{Constant}}{0} = \infty$$

ess (steady state error) $= \frac{1}{1+K_p} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

- $K_v = \lim_{s\to 0} s\cdot G(s)\cdot H(s)$

$$= \lim_{s\to 0} \cancel{s}\cdot \frac{K(s+z_1)(s+z_2)+\cdots}{s^{\cancel{2}}(s+p_1)(s+p_2)+\cdots}$$

$$= \lim_{s\to 0} \frac{K(s+z_1)(s+z_2)+\cdots}{s(s+p_1)(s+p_2)+\cdots}$$

$$s=0 \Rightarrow \frac{K(s+z_1)(s+z_2)+\cdots}{0(s+p_1)(s+p_2)+\cdots}$$

$$= \frac{K(s+z_1)(s+z_2)}{0} = \frac{\text{Constant}}{0} = \infty$$

$$ess = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

- $K_a = \lim_{s\to 0} s^2\, G(s)\cdot H(s)$

$$= \lim_{s\to 0} \cancel{s^2}\cdot \frac{K(s+z_1)(s+z_2)+\cdots}{\cancel{s^2}(s+p_1)(s+p_2)+\cdots}$$

$$= \frac{K(s+z_1)(s+z_2)+\cdots}{(s+p_1)(s+p_2)+\cdots}$$

= Constant

ess (steady state error) = $\frac{1}{Ka} = \frac{1}{Constant}$ = Constan

Type-2 Control System:

$Kp = \infty$ ess (position) = 0

$Kv = \infty$ ess (velocity) = 0

Ka = Constant ess (acceleration) = Constant

CH·06 : Analysis of Stability By Root Locus Technique :→

Rules for Construction of the root Locous.

Rule-01

- Root locous should be symmetrical about Real axis.

Rule-02

- At the open loop poles the Value of K equal to Zero. at the open loop Zeros. the Value of K equal to infinite.

Rule-03

- Segment of the Real axis having an odd no of Real axis open loop poles and Zeros.
- To their right are parts of root locus.

Rule-04

The (p-Z) branches of the root locus which go to '∞' travel along straight line asymptote. whose angle are given by. $Q = \frac{(2q+1)\pi}{p-Z}$

∵ n = no of poles.

m = no of Zeros.

q = 0, 1, 2, 3, 4 . . . . (n-m-1)

Rule-05

→ The Asymptotes thus the real axis at a point Known as Centroid determined by the relationship

$$\text{Centroid } \sigma_c = \frac{\text{Sum of part on Real axis} - \text{Sum of part on imaginary axis}}{\text{No of poles} - \text{No of Zeros}}$$

Rule-06 Break away points & break in points

→The breakaway points and break in points of the root locus at the solution of $\frac{dk}{ds} = 0$

Rule-07

The angle of departure from an openloop pole given by. $\theta^\circ = (2q+1)\pi + \theta$

where, θ = net angle contribution at this open loop pole of all other zeros.

θ = Diprachers angle.

Rule-08

→The point of intersection on of the root locus branches with the imaginary axis and the critical value of K can be determined by use of the "routh Hurwitz Criterian"

Q. Construct root Locus of a close loop Control system having $G(s).H(s) = \frac{K}{s(s+2)(s+4)}$

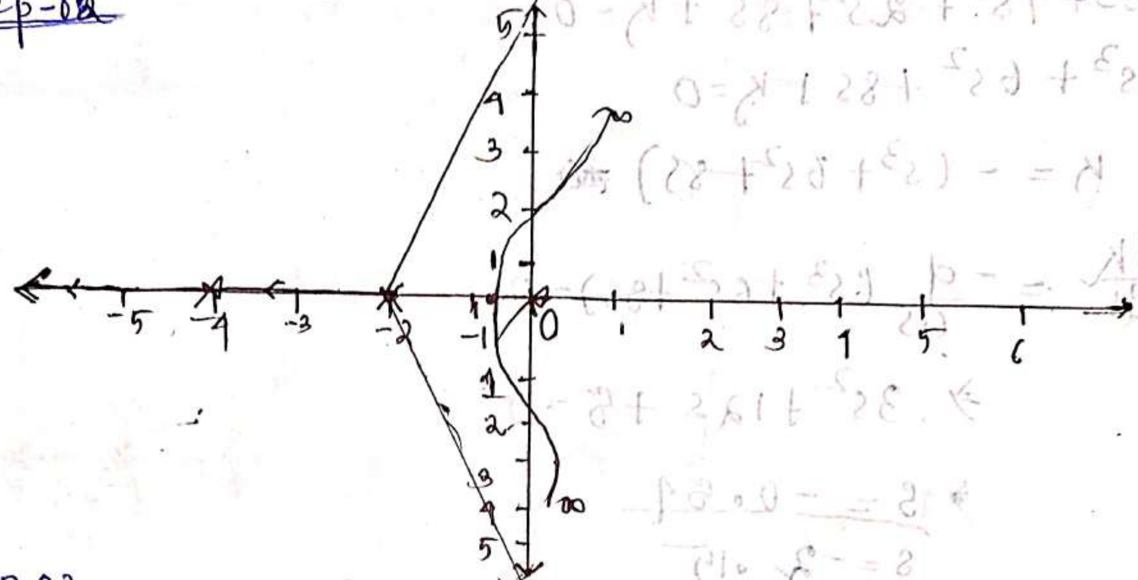
Step-01 * Find poles and zeros.

Pole → $0, -2, -4$ $\quad P = 3$

Zeros → No zeros $\quad Z = 0$

No of root locus = $P - Z = 3 - 0 = 3$

Step-02



Step-03 Angle of asymptotes.

$\theta_q = \frac{(2q+1)\pi}{P-Z}$ $\qquad q = P - Z - 1 = 3 - 1 = 2$

$q = 0, 1, 2$

$\theta_0 = \frac{(2\times0+1)\pi}{3} = \frac{\pi}{3} = 60°$

$\theta_1 = \frac{(2\times1+1)\pi}{3} = \frac{3\pi}{3} = 180°$

$\theta_2 = \frac{(2\times2+1)\pi}{3} = \frac{5\pi}{3} = 300°$

Step-04 find the Centroid point.

Centroid = $\frac{\text{sum of poles} - \text{Sum of Zeros}}{P-Z}$

$\sigma_c = \frac{0-2-4}{3-0} = \frac{-6}{3} = -2$

Step-05 Break away points :-

$$1 + G(s) \cdot H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\Rightarrow s(s+2)(s+4) + K = 0$$

$$\Rightarrow (s^2+2s)(s+4) + K = 0$$

$$\Rightarrow s^3 + 4s^2 + 2s^2 + 8s + K = 0$$

$$\Rightarrow s^3 + 6s^2 + 8s + K = 0$$

$$K = -(s^3 + 6s^2 + 8s)$$

$$\frac{dK}{ds} = -\frac{d}{ds}(s^3 + 6s^2 + 8s) = 0$$

$$\Rightarrow 3s^2 + 12s + 8 = 0$$

$$\Rightarrow \underline{s = -0.84}$$

$$s = -3.15$$

Step-06

Intersection point of an imaginary axis

Routh-Hurwitz Criterion

$$s^3 + 6s^2 + 8s + K = 0$$

s^3	1	8	
s^2	6	K	
s^1	$\frac{48-K}{6}$		0
s^0	K		

$$\Rightarrow \frac{48-K}{6} = 0$$

$$\Rightarrow 48 = K$$

Aux. eqn $\Rightarrow 6s^2 + K = 0$

$$\Rightarrow 6s^2 = -48$$

$$\Rightarrow s^2 = -\frac{48}{6} = -8$$

$$s = \sqrt{-8} = \pm j2.82$$

② Draw the root locus of the system having

$$G(s) \cdot H(s) = \frac{K}{s(s+4)(s+5)}$$

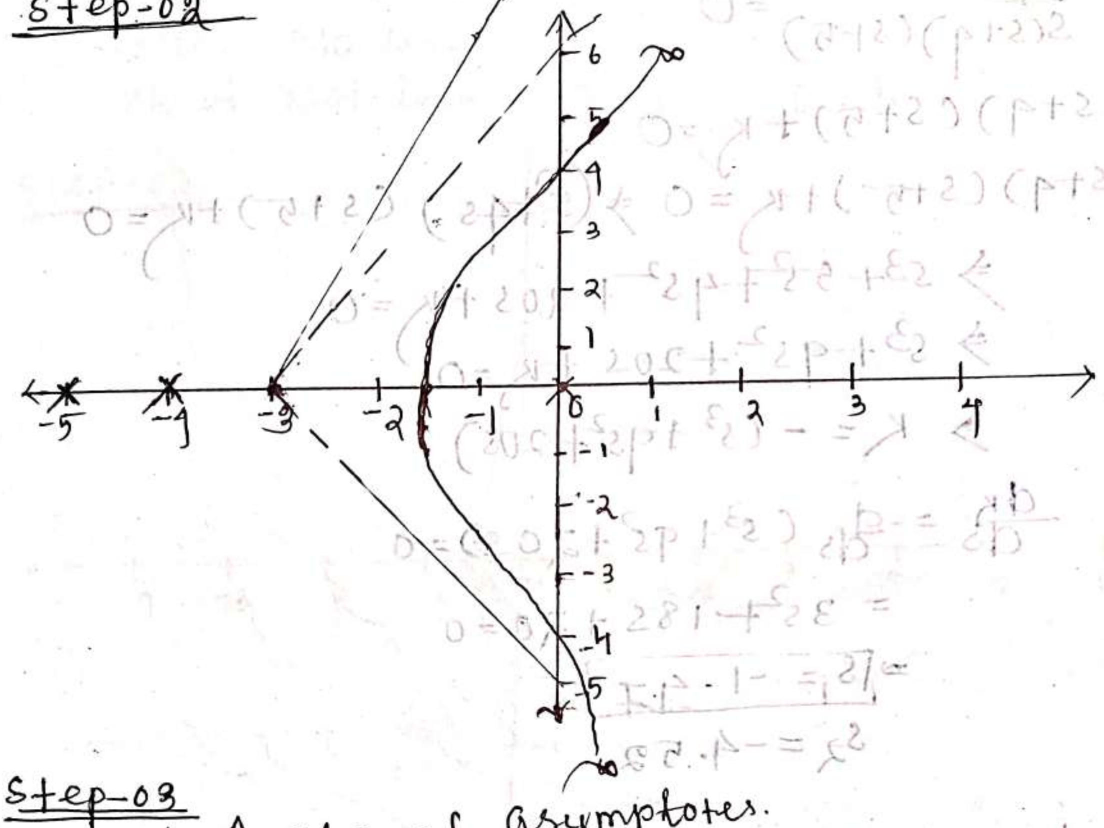
Step-01 Find poles and zeros.

pole = 0, −4, −5 $\quad$ $P = 3$.

zero = No zeros. $\quad$ $Z = 0$

No of root Locus = $P - Z = 3 - 0 = 3$.

Step-02



Step-03

Angle of asymptotes.

$q = P - Z - 1 = 3 - 1 = 2$

$q = 0, 1, 2$

$$Q_q = \frac{(2q+1)\pi}{P-Z}$$

$$Q_0 = \frac{(2\times 0+1)\pi}{3} = \frac{\pi}{3} = 60$$

$$Q_1 = \frac{(2\times 1+1)\pi}{3} = \frac{3\pi}{3} = 180$$

$$Q_2 = \frac{(2\times 2+1)\pi}{3} = \frac{5\pi}{3} = 300$$

Step-04 Find the centroid point.

$$\text{Centroid } \sigma_c = \frac{\text{Sum of poles} - \text{Sum of zeros}}{P-Z}$$

$$= \frac{0-4-5}{3} = -\frac{9}{3} = -3.$$

Step-05 Breakaway point :→

$1+G(s)\cdot H(s)=0$

$\Rightarrow 1+\frac{K}{s(s+4)(s+5)}=0$

$\Rightarrow s(s+4)(s+5)+K=0$

$\Rightarrow s(s+4)(s+5)+K=0 \Rightarrow (s^2+4s)(s+5)+K=0$

$\Rightarrow s^3+5s^2+4s^2+20s+K=0$

$\Rightarrow s^3+9s^2+20s+K=0$

$\Rightarrow K=-(s^3+9s^2+20s)$

$\frac{dK}{ds}=-\frac{d}{ds}(s^3+9s^2+20s)=0$

$=3s^2+18s+20=0$

$\Rightarrow \boxed{s_1=-1.47}$

$s_2=-4.52$

Step-06 Intersection point of imaginary axis :→

$s^3+9s^2+20s+K=0$

$\Rightarrow$ Routh Hurwitz Criterian.

s^3	1 20
s^2	9 K
s^1	$180-K/9$
s^0	K

$\frac{180-K}{9}=0$

$\Rightarrow 180-K=0$

$\Rightarrow K=180$

Auxillary eq^n $=9s^2+K=0$

$\Rightarrow 9s^2+180=0$

$\Rightarrow 9s^2=-180$

$s^2=-180/9=-20$

$s = \sqrt{-20} = (2\sqrt{5})^2 = \pm j4.47$

→ Draw the root Locus. of the system having -

$$G(s)\cdot H(s) = \frac{K}{s^2(s+2)} = \frac{K}{s \times s \times (s+2)}$$

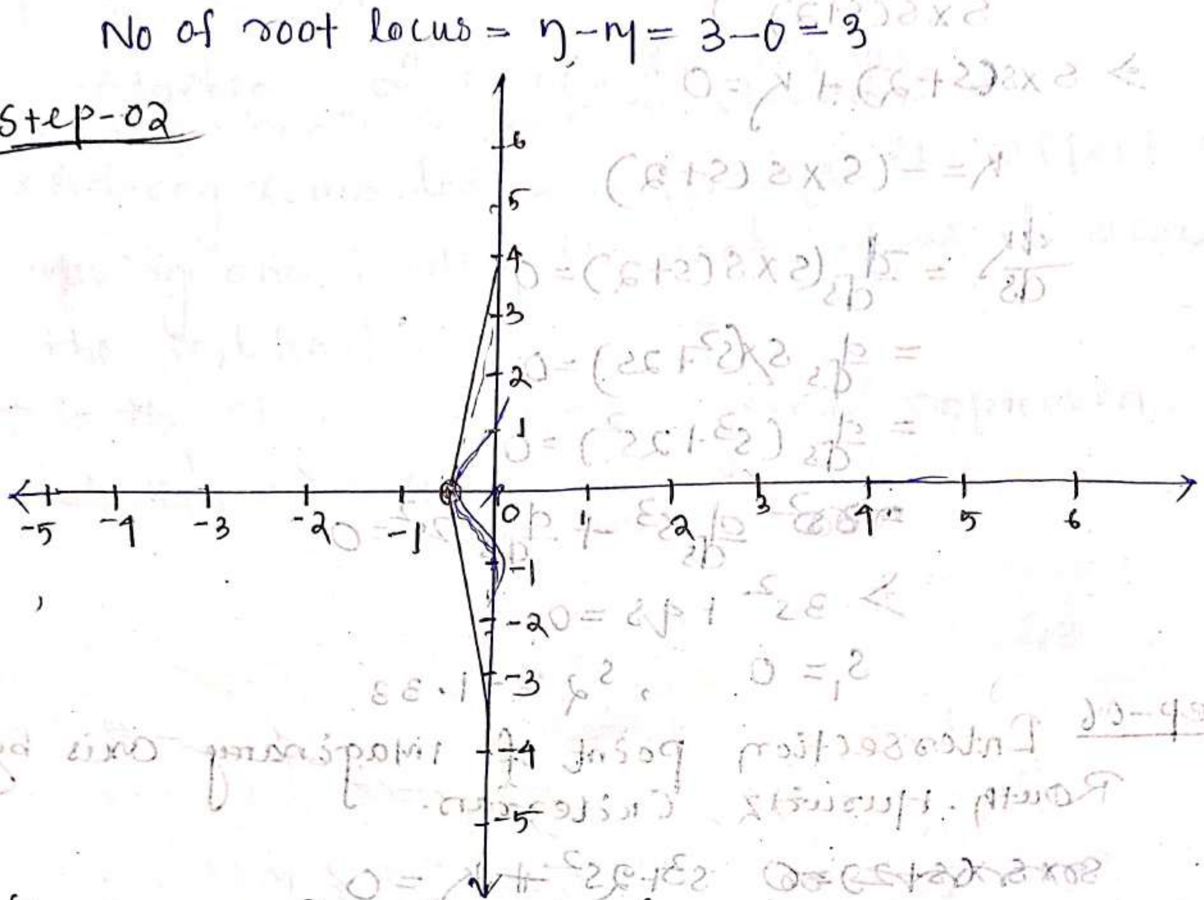
Soln step-01 find poles and zeros.

pole - 0, 0, -2

Zeros = No zeros.

No of root locus = $n - m = 3 - 0 = 3$

Step-02



Step-03 Angle of asymptotes.

$q = p - z - 1 = 3 - 1 = 2$

$q = 0, 1, 2$

$\alpha_q = \left(\frac{2q+1}{p-z}\right)\pi$

$\alpha_1 = \frac{0+1}{3}\pi = \frac{\pi}{3} = 60$

$\alpha_2 = \left(\frac{4+1}{3}\right)\pi = \frac{5\pi}{3} = 300$

Step-04 Centroid point

$$\text{Centroid } \sigma_c = \frac{\text{sum of poles} - \text{sum of zeros}}{P-Z}$$

$$= \frac{0+0-2}{3} = \frac{-2}{3} = -0.6.$$

Step-05 Breakaway point

$1 + G(s) \cdot H(s) = 0$

$\Rightarrow 1 + \frac{K}{s \times s(s+2)} = 0$

$\Rightarrow \frac{s \times s(s+2) + K}{s \times s(s+2)} = 0$

$\Rightarrow s \times s(s+2) + K = 0$

$K = -(s \times s(s+2))$

$\frac{dK}{ds} = \frac{d}{ds}(s \times s(s+2)) = 0$

$= \frac{d}{ds} s \times (s^2+2s) = 0$

$= \frac{d}{ds}(s^3+2s^2) = 0$

$\frac{d}{ds}s^3 + \frac{d}{ds}2s^2 = 0$

$\Rightarrow 3s^2 + 4s = 0$

$s_1 = 0 \quad , s_2 = -1.33$

Step-06 Intersection point of imaginary axis by Routh-Hurwitz Criterion.

$s^3 + 2s^2 + K = 0$

s^3	1	0
s^2	2	K
s^1	$0 - \frac{K}{2}$	
s^0	K	

$-\frac{K}{2} = 0$

$-K = -2$

$K = 2$

Auxilary eqn $= 2s^2 + K = 0$

$= 2s^2 + 2 = 0$

$\Rightarrow 2s^2 = -2$

$s^2 = -2/2 = -1$

$s = \sqrt{-1} = \pm j1$

Effect of adding poles and Zeros to G(s) and H(s)

Addition of Poles:→

↳ Adding a pole G(s).H(s) has the effect of pushing the root loci towards the right half.

↳ The complex path of the root loci bends to the right.

↳ The angle of asymptotes reduces and the Centroid is shifted to the left. and the system stability will be reduced.

Addition of Zeros to G(s).H(s)

↳ Adding zeros the G(s). H(s). Has the effect of Moving and bending the root locus. towards. the left half.

↳ So the stability of the system improved by the addition of a zero.

Frequency Response Analysis: CH- 07

- The Response of the system can be patition into both Transient position and Steadystate Response.
- The steady state response of the system for an input sinusodial signal is known as frequency Response.
- If a sinusodial signal is applied as an input to a linear time invarient system.
- Then it produces a steadystate output which is also a sinusodial signal.
- The input & the output at the same frequency.

$$v = v_m \sin \omega t$$

$$x(t) = A_0 \sin \omega_0 t$$

- Under steadystate the system output as wellas the signals at all other points of system are sinusodial.
- The steady state output may be written as.

$$c(t) = B \sin(\omega_0 t + \phi) \qquad B = \text{Magnitude}$$

$$r(t) = A \sin \omega t \angle 0 \qquad A = \text{phase angle.}$$

Co-Relation between Time and Frequency Response:-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Standard form of T.f of a Second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$R(j\omega)$

Replace $s = j(\omega)$

$$T(j\omega) = \frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{j^2\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

Dividing ω_n^2 in both numerator & denominator.

$$T(j\omega) = \frac{\omega_n^2/\omega_n^2}{\frac{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2}{\omega_n^2}}$$

$$= \frac{\omega_n^2/\omega_n^2}{\frac{-\omega^2}{\omega_n^2} + \frac{2\xi\omega_n j\omega}{\omega_n^2} + \frac{\omega_n^2}{\omega_n^2}}$$

$$= \frac{1}{\frac{-\omega^2}{\omega_n^2} + \frac{2\xi j\omega}{\omega_n} + 1}$$

$$= \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\frac{2\xi\omega}{\omega_n}}.$$

Putting $(\omega/\omega_n) = u$ where u is normalized driving signal frequency.

$$T(j\omega) = \frac{1}{1-u^2 + j2\xi u}$$

$$|T(j\omega)| = \frac{1}{\sqrt{(\text{Real part})^2 + (\text{imaginary part})^2}}$$

$$= \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\frac{2\xi u}{1-u^2}$$

~~when u = 0~~

$$\angle T(j\omega) = -\tan^{-1}\frac{2\xi u}{1-u^2}$$

$$Q = -\tan^{-1}\frac{b}{a}$$

$$|Tj(\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$$

$$M = \frac{1}{\sqrt{(1-0)^2 + 0}} = 1$$

$$Q = -\tan^{-1}(0)$$

$$= 0^\circ$$

$u = 1$ $\qquad\qquad\qquad\qquad$ $u = \infty$

$M = \frac{1}{2\xi}$ $\qquad\qquad\qquad\qquad$ $M = 0$

$\phi = -\tan^{-1}(\infty)$ $\qquad\qquad$ $\phi = -\tan^{-1}(0)$

$= -90^\circ$ $\qquad\qquad\qquad\qquad$ $= 0^\circ$ or -180°

$c(t) = B \sin(\omega t - \phi)$ $\qquad$ $= \sin\left(\omega t + \tan^{-1}\frac{2\xi u}{1-u^2}\right)$

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}$$

The frequency where, M has a peak value is known as the resonant frequency.

> At this frequency the slope of the magnitude curve be zero.

Let, ω = Resonant frequency

u = are normalized resonant frequency

$$u_r = \frac{\omega_r}{\omega_n}$$

$$\left.\frac{dM}{du}\right|_{u_r} = \frac{d}{du}\left[\frac{1}{\sqrt{(1-u_r^2)^2 + (2\xi u_r)^2}}\right]$$

$$= \frac{0 - 1 \cdot \frac{d}{du}\left[\sqrt{(1-u_r^2)^2 + (2\xi u_r)^2}\right]}{(1-u_r^2)^2 + (2\xi u_r)^2}$$

$$= \frac{\frac{1}{2}\left[\frac{d}{du_r}(1-u_r^2)^2 + \frac{d}{du_r}(2\xi u_r)^2\right]}{(1-u_r^2)^2 + (2\xi u_r)^2}$$

$$= -\frac{1}{2}\,\frac{2(1-u_r^2)\cdot\frac{d}{du_r}(1-u_r^2) + \frac{d}{du_r}(4\xi^2 u_r^2)}{(1-u_r^2)^2 + (2\xi u_r)^2}$$

$$= -\frac{1}{2} \frac{2(1-u_r^2)(-2u_r) + 8\xi^2 u_r}{(1-u_r)^2 + (2\xi u_r)^2}$$

Equating, $\frac{dM}{du_r} = 0$

$$= -\frac{1}{2} \times \frac{2(1-u_r^2)(-2u_r) + 8\xi^2 u_r}{(1-u_r)^2 + (2\xi u_r)^2} = 0$$

$$= 2(1-u_r^2)(-2u_r) + 8\xi^2 u_r = 0$$

$$\Rightarrow -4(1-u_r^2)u_r + 8\xi^2 u_r = 0$$

$$\Rightarrow 4\left[-(1-u_r^2)u_r + 2\xi^2 u_r\right] = 0$$

$$\Rightarrow \cancel{4\left[-(1-u_r^2)u_r +\right.}$$

$$\Rightarrow 2\xi^2 u_r - (1-u_r^2)u_r = 0$$

$$\Rightarrow u_r(2\xi^2 - (1-u_r^2)) = 0$$

$$\Rightarrow 2\xi^2 - 1 + u_r^2 = 0$$

$$\Rightarrow u_r^2 = 1 - 2\xi^2$$

$$\boxed{u_r = \sqrt{1-2\xi^2}}$$

$$\frac{d}{du_r}(1-u_r^2)^2$$

$$= 2(1-u_r^2)\frac{d}{du_r}(1-u_r^2)$$

$$= 2(1-u_r^2)(-2u_r)$$

$$\frac{d}{du_r}(4\xi^2 u_r^2)$$

$$= 4\xi^2 \times 2u_r$$

$$= 8\xi^2 2u_r$$

$$u_r = \frac{\omega_r}{\omega_n}$$

$$\Rightarrow \omega_r = u_r \omega_n$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$$

↳ Substituting these value of (u_r) or u in M the maximum value of magnitude known as resonant peak.

$$M = \frac{1}{\sqrt{(1-u_r)^2 + (2\xi u_r)^2}}$$

$$M_r = \frac{1}{\sqrt{1-(1-2\xi^2)^2 + (2\xi\sqrt{1-2\xi^2})^2}} = \boxed{\frac{1}{2\xi\sqrt{1-\xi^2}}}$$

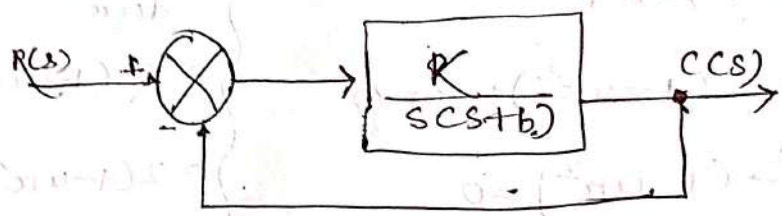
$$\phi_r = -\tan^{-1}\frac{2\xi u_r}{1-u^2} = -\tan^{-1}\frac{2\xi\sqrt{1-2\xi^2}}{\cancel{1}-\cancel{1}+2\xi^2}$$

$$\boxed{\phi_r = -\tan^{-1}\frac{\sqrt{1-2\xi^2}}{\xi}}$$

02-06-22

> Find the value of K and b to satisfy the following frequency-domain specifications. $M_r = 1.1$ $\omega_r = 12$ rad/sec. For the values of (K) and (b) determined in part

Soln



Soln Given ⇒ $M_r = 1.1$

$\omega_r = 12$ rad/sec

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$= \frac{K/s(s+b)}{1+K/s(s+b)\cdot 1} = \frac{K/s(s+b)}{\frac{s(s+b)+K}{s(s+b)}\cdot 1}$$

$$= \frac{K}{s(s+b)+K} = \frac{K}{s^2+bs+K}$$

Standard 2nd order system

$$\frac{K}{s^2+sb+K} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = K$ $\quad \omega_n = \sqrt{K}$

$$2\xi\omega n = b$$

$$2\xi\sqrt{k} = b$$

$$\xi = \frac{b}{2\sqrt{k}}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$= \frac{1}{\cancel{2}\times\frac{b}{\cancel{2}\sqrt{k}}\sqrt{1-\left(\frac{b}{2\sqrt{k}}\right)^2}} = 1.1$$

$$\Rightarrow 1.1 = \frac{1}{\frac{b}{\sqrt{k}}\sqrt{1-\frac{b^2}{4k}}}$$

$$\Rightarrow (1.1)^2 = \left(\frac{1}{\frac{b}{\sqrt{k}}\sqrt{1-\frac{b^2}{4k}}}\right)^2$$

$$\Rightarrow \frac{1}{\frac{b^2}{k}\left(1-\frac{b^2}{4k}\right)} = 1.21$$

$$\Rightarrow \frac{1}{\frac{b^2}{k}-\frac{b^4}{4k^2}} = 1.21 \Rightarrow \frac{1}{\frac{b^2 4k^2 - k b^4}{4k^2}} =$$

$$\Rightarrow \frac{b^2 4k^2 - k b^4}{4k^2} = 1.21$$

$$\omega r = \omega n\sqrt{1-2\xi^2}$$

$$\Rightarrow 12 = \sqrt{k}\cdot\sqrt{1-2\left(\frac{b}{2\sqrt{k}}\right)^2}$$

$$\Rightarrow 12 = \sqrt{k}\cdot\sqrt{1-2\times\frac{b^2}{4k}}$$

$$\Rightarrow (12)^2 = \left[\sqrt{k}\cdot\sqrt{\left(1-\cancel{2}\times\frac{b^2}{\cancel{4}_{2}k}\right)}\right]^2$$

$$\Rightarrow 144 = \cancel{k}\cdot\left(\frac{1-b^2}{2k}\right) \Rightarrow 144 = k - \frac{\cancel{k}b^2}{2\cancel{k}}$$

$$\Rightarrow k - \frac{b^2}{2} = 144$$

$$k = 144 + \frac{b^2}{2}$$

$$b^2 = 2k - 288$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$= \frac{1}{2 \times \frac{b}{\cancel{2}\sqrt{k}} \sqrt{1 - \left(\frac{b}{2\sqrt{k}}\right)^2}} = 1.1$$

$$\Rightarrow \left(\frac{1}{\frac{b}{\sqrt{k}}\sqrt{1 - \frac{b^2}{4k}}}\right)^2 = (1.1)^2$$

$$\Rightarrow \frac{1}{\frac{b^2}{k} \times \left(1 - \frac{b^2}{4k}\right)} = 1.21 \quad \text{--- (1)}$$

Put the value (b^2) in eq^n 1

$$\Rightarrow \frac{1}{\frac{2k-288}{k} \times \left(1 - \frac{2k-288}{4k}\right)} = 1.21$$

$$\Rightarrow \frac{2k-288}{k} \times \left(1 - \left(\frac{2k-288}{4k}\right)\right) = \frac{1}{1.21}$$

$$\Rightarrow \left(\frac{2\cancel{k}}{\cancel{k}} - \frac{288}{k}\right)\left(1 - \frac{1}{2} + \frac{72}{k}\right) = \frac{1}{1.21}$$

$$\Rightarrow \left(2 - \frac{288}{k}\right)\left(\frac{1}{2} + \frac{72}{k}\right) = \frac{1}{1.21}$$

$$\Rightarrow 2\left(\frac{1}{2} + \frac{72}{k}\right) - \frac{288}{k}\left(\frac{1}{2} + \frac{72}{k}\right) = \frac{1}{1.21}$$

$$\Rightarrow 1 + \cancel{\frac{144}{k}} - \cancel{\frac{144}{k}} - \frac{20736}{k^2} = \frac{1}{1.21}$$

$$\Rightarrow 1 - \frac{20736}{k^2} = \frac{1}{1.21}$$

$$\Rightarrow \frac{K^2 - 20736}{K^2} = \frac{1}{1.21}$$

$$\Rightarrow (K^2 - 20736) 1.21 = K^2$$

$$\Rightarrow 1.21K^2 - 25090.56 = K^2$$

$$\Rightarrow 1.21K^2 - K^2 = 25090.56$$

$$\Rightarrow 0.21K^2 = 25090.56$$

$$K = \sqrt{\frac{25090.56}{0.21}}$$

$$= 345.65$$

$$b^2 = 2K - 288$$

$$= 2 \times (345.65) - 288$$

$$= 403.3$$

$$b = \sqrt{403.3} = 20.08.$$

POLAR PLOT :-

- The plot which is drawn between the Magnitude and phase angle of $G(j\omega) \times H(j\omega)$ by varying the Value of ω from '0' to '∞'

Rules for Drawing Polar plot :→

1. Substitute $S = j\omega$ in the openloop Transfer function in Open Loop T.f.
2. Write the expression For Magnitude & Phaseangle of $G(j\omega) \times H(j\omega)$
3. Find the starting Magnitude and the phase of $G(j\omega) \times H(j\omega)$ by Substituting $\omega = 0$ The polar plot starts with magnitude & Phase angle.

Find the ending Magnitude and the phase of $G(j\omega) \times H(j\omega)$ by Substituting $\omega = \infty$

The polar plot ends with this magnitude & Phase angle.

5. Check wheather the polar plot Intersect the Real axis by making the imaginary part $G(j\omega) \times H(j\omega) = 0$

6. Check wheather the polar plot intercept the imaginary axis making the Real term. $G(j\omega) \cdot H(j\omega) = 0$ Find the value in ω.

7. Drawing the polar plot purely find the magnitude & phase of $G(j\omega) \cdot H(j\omega)$ by Considering other values of ω.

Dt - 03-05-2020

Q.1 Sketch the polar plot of the Transfer function given below

$$G(s) = \frac{1}{(s+1)(2s+1)}$$

Put $s = j\omega$

$$G(s) = \frac{1}{(j\omega+1)(2j\omega+1)}$$

$$= \frac{1}{j\omega(2j\omega+1) + 1(2j\omega+1)}$$

$$= \frac{1}{2j^2\omega^2 + j\omega + 2j\omega + 1}$$

$$= \frac{1}{-2j^2\omega^2 + 3j\omega + 1}$$

$j^2 = -1$

$$|G(j\omega)| = \frac{1}{\sqrt{(\underset{a}{1-2\omega^2})^2 + (\underset{b}{3\omega})^2}}$$

$$= \frac{1}{\sqrt{1+4\omega^4 - 4\omega^2 + 9\omega^2}} = \frac{1}{\sqrt{4\omega^4 + 5\omega^2 + 1}}$$

$$\theta = -\tan^{-1}\frac{b}{a}$$

$$= -\tan^{-1}\frac{3\omega}{1-2\omega^2}$$

put $\omega = 0$ $\quad M = \frac{1}{\sqrt{0+0+1}} = \frac{1}{1} = 1$

$$\theta \Rightarrow \tan^{-1}(0) = 0$$

$\omega = 1 \rightarrow \frac{1}{\sqrt{4\omega^4+5\omega^2+1}} = \frac{1}{\sqrt{4+5+1}} = \frac{1}{\sqrt{10}} = 0.31$

$$\theta = \tan^{-1}\frac{3\omega}{1-2\omega^2} \Rightarrow -\tan^{-1}\frac{3}{-1} = \boxed{71.5}$$

$\omega = 2 \quad \frac{1}{\sqrt{4\omega^4+5\omega^2+1}} = \frac{1}{\sqrt{4(2)^4+5(2)^2+1}}$

$$= \frac{1}{\sqrt{85}} = \boxed{0.108}$$

$$\cancel{\omega = \infty \quad \frac{1}{\sqrt{4(\infty)^4+5(\infty)^2+1}} = \frac{1}{\infty} = 0}$$

$$\theta = \tan^{-1}\frac{3\omega}{1-2\omega^2}$$

$$\Rightarrow \tan^{-1}\frac{6}{-7} = \boxed{-40.60}$$

$\omega = \infty \quad \frac{1}{\sqrt{4\times(\infty)^4 + 5\times(\infty)^2+1}} = \frac{1}{\infty} = 0$

$$= \tan^{-1}\frac{3\infty}{1-2(\infty)^2} = \boxed{0}$$

ω	M	α
$\omega = 0$	1	0
$\omega = 1$	0.31	-71.5
$\omega = 2$	0.108	-40.60
$\omega = \infty$	0	0

$$G(j\omega) = \frac{1}{(1-2\omega^2)+j3\omega}$$

$$= \frac{1-2\omega^2-j3\omega}{(1-2\omega^2+j3\omega)(1-2\omega^2-j3\omega)}$$

$$= \frac{1-2\omega^2-j3\omega}{(1-2\omega^2)^2-(j3\omega)^2}$$

$$= \frac{1-2\omega^2-j3\omega}{1+4\omega^4-4\omega^2+9\omega^2} = \frac{1-2\omega^2-j3\omega}{4\omega^4+5\omega^2+1}$$

$$= \frac{1-2\omega^2}{4\omega^4+5\omega^2+1} - j\frac{3\omega}{4\omega^4+5\omega^2+1}$$

Step-05

$$\frac{3\omega}{4\omega^4+5\omega^2+1} = 0$$

$\Rightarrow 3\omega = 0$

$\Rightarrow \omega = 0$

$\Rightarrow 1-2\omega^2 = 0$

$\Rightarrow 1-2\omega^2 = 0$

$\Rightarrow 2\omega^2 = 1$

$\Rightarrow \omega^2 = \frac{1}{2} \Rightarrow \omega = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$$\omega = \frac{1}{\sqrt{2}} \quad M = \frac{1}{\sqrt{4\times\left(\frac{1}{\sqrt{2}}\right)^4 + 5\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 1}} = -0.47$$

$$\varphi = -\tan^{-1} \frac{3\times 1/\sqrt{2}}{1 - 2\times\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= -\tan^{-1} \frac{3/\sqrt{2}}{0}$$

$$= -\tan^{-1}\infty = 90^\circ$$