LECTURE NOTES OF

CIRCUIT THEORY

3RD SEMESTER ETC

PREPARED BY-

Aishwarya Dash

GOVERNMENT POLYTECHNIC DHENKANAL

DEPARTMENT OF ELCTRONICS AND TELECOMMUNICATION ENGINEERING

Network Elements

In general, network is a combination of element.
Electronic network is a combination of element.

1) Active elements.
2) Passive elements.

1) Active elements :-

Elements which can generate energy or has ability to supply energy are known as active elements.

Ex :- Battery & generators (voltage sources), current sources.. BJTS, FETs, OPAMPs etc.

The behaviour of active elements cannot be described by ohm's low.

2) Passive elements :-

Elements which cannot generate energy but can absorb, consume, store or dissipate it are known as passive elements.

Ex:- Resistors, inductor, capacitors, diodes, thermistors etc.

3) Linear elements :-

Linear elements always obey a straight line low i.e linear elements shows a linear relationship between voltage and current.

A circuit element is linear if the principle of superposition holds and relation between current and voltage involves a constant coefficient e.g. Incase of resistance, inductance & capacitance the relation between voltage and current are given by

$$v = IR \qquad v = L\frac{di}{dt} \qquad v = \frac{1}{c}\int i\,dt$$

4) Nonlinear elements :-

A nonlinear circuit element is one in which the current does not change linearly with the change in applied voltage.

In nonlinear elements the principle of superposition fails.

Ex :- diodes, transitors, temperature dependent resistors (thermistor), voltage ~~depend~~ dependent capacitor (varactor).

5) Unilateral elements :-

When the property or characteristic of elements change with respect to direction of current flow i.e when the polarity of the applied voltage is changed then the element are known as unilateral elements.

Ex :- P-N junction diodes, SCR etc.

6) Bilateral elements :-

When the property or characterstics of elements remains constant with respect to direction of current flow i.e irrespective of polarity of the applied voltage then the elements are known as bilateral elements.

Ex :- Resistors, transmission lines etc

Circuit :-

A circuit is a closed conducting path through which an electric current either flows or is intended to flow.

Circuit consists of active and passive elements in it.

Parameters :-

The various elements of an electric circuit are called parameters like resistance, inductance and capacitance.

The parameters may be lumped or distributed.

Linear circuit :-

A linear circuit is one whoes parameters are constant with time also they do not change with voltage or current and the circuit obeys ohm's law.

Non-linear circuit :-

It is that circuit whoes parameters change with voltage or current.

Bilateral circuit :-

A bilateral circuit is one whoes properties or characterstics are the same in either direction.

The usual transmission line is bilateral, because it can be made to perform its function equally well in either direction.

Unilateral circuit :-

It is that circuit whoes properties or characterstics change with the direction of its operation.

A diode rectifier is a unilateral circuit, because it cannot perform rectification in both directions.

Electric Network :-

A combination of various electric elements, connected in any manner whatsoever, is called an electric Network.

Passive Network :-

It is the one which contains no source of EMF in it.

Active Network :-

It is the One which contains ~~no source~~ one or more than one source of e.m.f along with passive elements.

Node :-

It is a point in a ~~cir~~ circuit where two or more circuit elements are connected together.

Branch :-

It is that part of a Network which lies between two nodes.

Loop :-

It is a closed path in a circuit in which no element or node is ~~encountd~~ encountered more than Once.

Mesh :-

It is a loop that contains no other loop within it.

~~Kirchhoff's point law or current law (KCL) :-~~

Kirchhoff's Laws :-

It is used

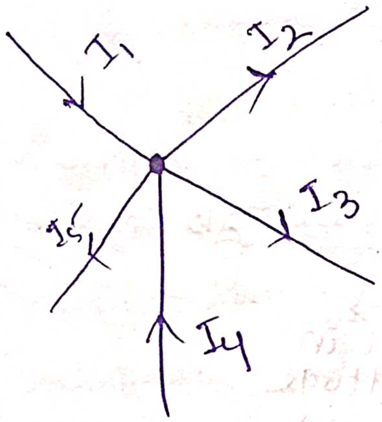
1) in determining the equivalent resistance of a complicated network of conductors.
2) for calculating the current flowing in the various conductors

1) Kirchhoff's point law or current law (KCL) :-

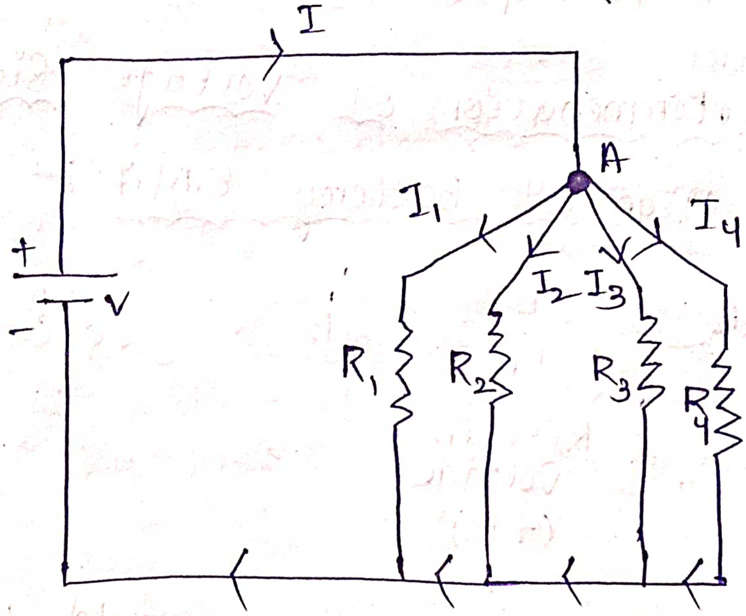
In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.

Total current leaving a junction is equal to the total current entering that junction.

It is obviously true because there is no accumulation of charge at the junction of the network.



(Fig-1)



(Fig-2)

From fig-1

$$I_1 + (-I_2) + (-I_3) + (I_4) + (-I_5) = 0$$

$$\Rightarrow I_1 + I_4 - I_2 - I_3 - I_5 = 0$$

$$\Rightarrow I_1 + I_4 = I_2 + I_3 + I_5$$

Incoming currents = Outgoing currents

From fig-2

For node-A

$$I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0$$

$$\Rightarrow I = I_1 + I_2 + I_3 + I_4$$

$$\boxed{\Sigma I = 0}$$

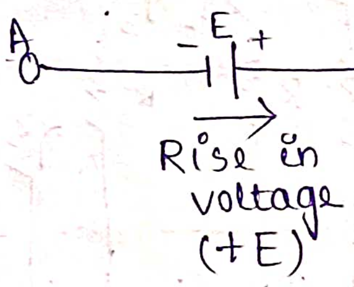
2) **Kirchhoff's Mesh Law or voltage law :-**

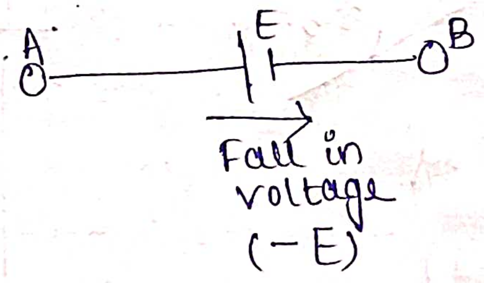
The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of emfs in that ~~part~~ path is zero.

$$\boxed{\sum IR + \sum e.m.f = 0}$$

Determination of voltage sign :-

(a) **Sign of Battery E.M.F :-**

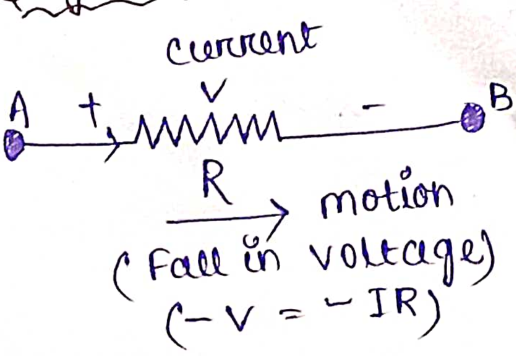


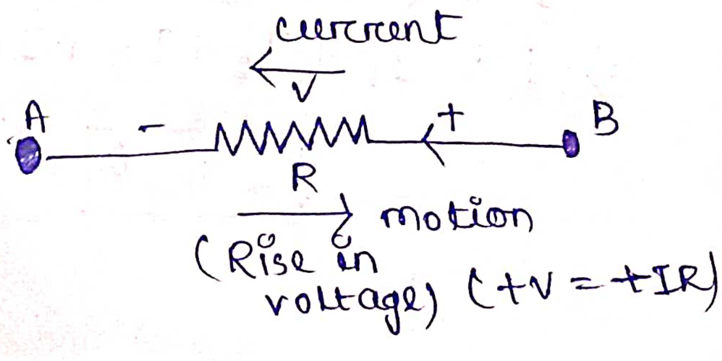


A rise in voltage should be given a +ve sign and a fall in voltage a −ve sign.

* It is clear that as we go from the −ve terminal of a battery to its +ve terminal there is a rise in potential, hence this voltage should be given a +ve sign.
* On the other hand, we go ~~to~~ from +ve terminal to −ve terminal. then there is a fall in potential, hence, this voltage shall be proceded by a − ve ~~segh~~ sign.

(b) **Sign of IR Drop :-**

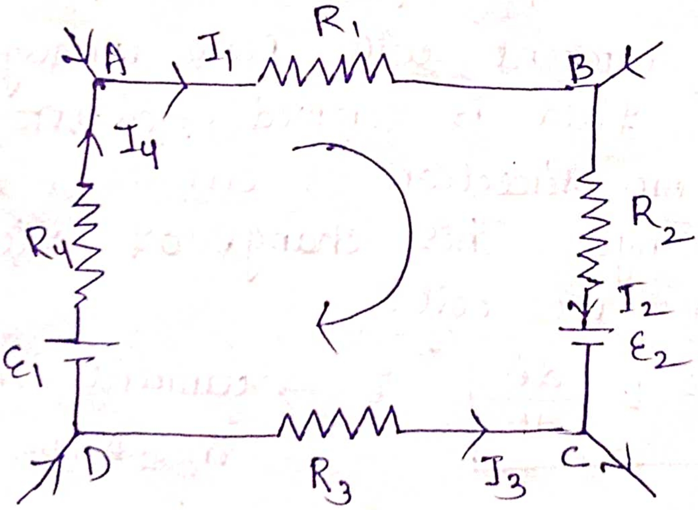




* If we go through a resistor in the same direction as the current, then there is a fall in potential because current flows from higher to a lower potential. Hence, this voltage fall should be taken negative ~~for~~ (-ve).

* However if we go in a direction opposite to that of the current then there is a rise in voltage. Hence this voltage rise should be given a positive sign.

It is clear that the sign of voltage drop ~~across~~ across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f in the circuit under consideration.



I_1R_1 is -ve (fall in potential)

I_2R_2 is -ve (fall in potential)

I_3R_3 is +ve (rise in potential)

I_4R_4 is -ve (fall in potential)

$\mathcal{E}_2$ is -ve (fall in potential)

$\mathcal{E}_1$ is +ve (rise in potential)

using ~~Kir~~ Kirchhoff's voltage law

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - \mathcal{E}_2 + \mathcal{E}_1 = 0$$

$$\Rightarrow I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = \mathcal{E}_1 - \mathcal{E}_2$$

Lenz's law :-

This law states that
Electromagnetically induced current always flow in such direction that the action of the magnetic field set up by ~~direction~~ it tends to oppose the very couse which produces it.

Energy stored in Inductor / Capacitor :-

Inductance :-

Inductance is the property of a material by virtue of which it opposes any change of magnitude or direction of electric current passing ~~tho~~ through the conductor.

As soon as current will flow through the coil, or electromagnetic field is formed. However, with any change of flow or direction of current, the electromagnetic field changes. This change of field induces a voltage (V) across the coil.

$$\boxed{V = L\frac{di}{dt}}$$

$i \rightarrow$ current through inductor in Amp.

The voltage across the inductor would be zero if the current through it remains costant. This means that an inductor behaves as a short circuited coil in steady state, when direct steady current flows through it.

For a minute change in current within zero time ($dt = 0$) gives an infinite voltage across the Inductor which is impossible.

Power absorbed by inductor

$$p = v \times i = Li\frac{di}{dt} \text{ watts}$$

Energy absorbed by the inductor

$$w = \int_0^t P\,dt = \int_0^t Li \frac{di}{dt} = \frac{1}{2} Li^2$$

The inductor can store finite amount of energy, even the voltage across it may be nil.

A pure inductor does not dissipate energy, but - Only stores it.

Capacitance :-

It is the capability of an element to store electric charge within it.

A capacitor stores electric energy in the form of electric field being established by the two ~~pola~~ - polarities of charges on the two electrodes of a capacitor.

Capacitance is a measure of charge per unit - voltage that can be stored in an element.

$$\boxed{C = \frac{q}{V}}$$

$q \rightarrow$ amount of charge
$C \rightarrow$ capacitance
$V \rightarrow$ potential difference.

$$i = C\frac{dv}{dt} \quad \left(\because i = \frac{dq}{dt}\right)$$

$$dv = \frac{1}{C} i\,dt$$

$$\Rightarrow \int_{V_0}^{V_t} dv = \frac{1}{C}\int_0^t i\,dt$$

$$\Rightarrow V_t - V_0 = \frac{1}{C}\int_0^t i\,dt$$

$$\Rightarrow V_t = \frac{1}{C}\int_0^t i\,dt + V_0$$

$V_0 \rightarrow$ initial voltage of capacitor
$V_t \rightarrow$ final voltage of capacitor

The power absorbed by the capacitor is given by

$$P = vi = vc\frac{dv}{dt}$$

Energy stored by the capacitor is

$$w = \int_0^t P\,dt = \int_0^t vc\frac{dv}{dt}\,dt = \frac{1}{2}cv^2$$

$$\boxed{w_c = \frac{1}{2}cv^2}$$

The voltage across the capacitor being constant - current through it is zero.

This means that the capacitor on application of dc voltage and with no initial charge first act as short circuit but as soon as the full charge it retains, the capacitor behaves an open circuit.

⇒ Also a capacitor never dissipates energy it only stores it.

⇒ It can store finite amount of energy, even it the current through it is zero.

Types of Electric power :-

1) Active power (watts or kW)

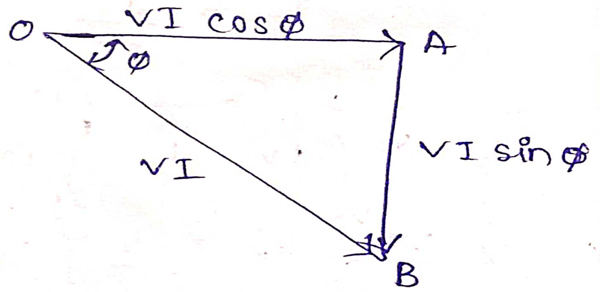
$P = VI\cos\phi$

2) Reactive power (VAR or KVAR)

$P = VI\sin\phi$

3) Apparent power (VA or KVA)

$P = VI$



Power Factor (P.f) = $\cos\phi$ = $\dfrac{\text{active power}}{\text{apparent power}}$

$= \dfrac{KW}{KVA}$

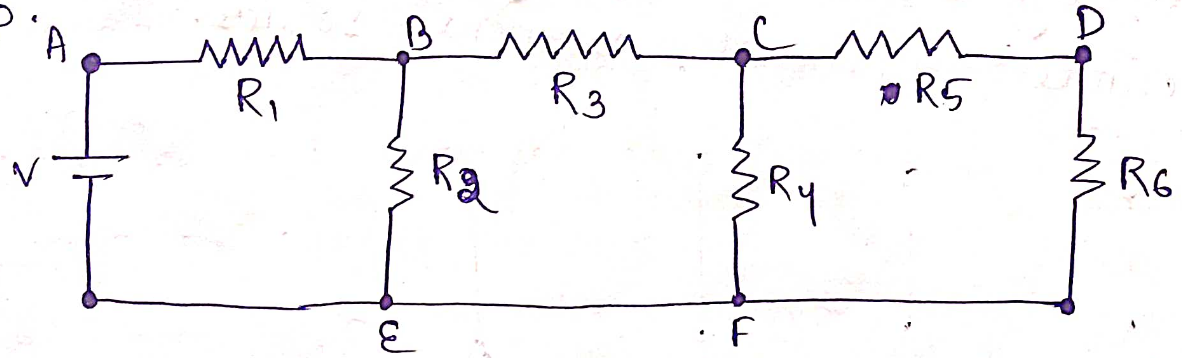
Loop :-

A loop is a closed path in which no elements & ~~nodes~~ nodes are repeated.

There must not be any sub loop.

Mesh :-

Mesh is a loop which consists of more than one loop.



1) Nodes → A, B, C, D, E, F
2) Branch → AB, BC, CD, DF; BE; CF, AE
3) Loop → ABEA, BCFEB, CDFC. (3 No. of loops)
4) Mesh → ACFEA, BDFEB, ADFEA (3 No. of Mesh)

Sources :-

Depending on the type of sources we can classify both voltage & current sources further i.e.

1) Dependent Source
2) Independent Source

1) Dependent Source :-

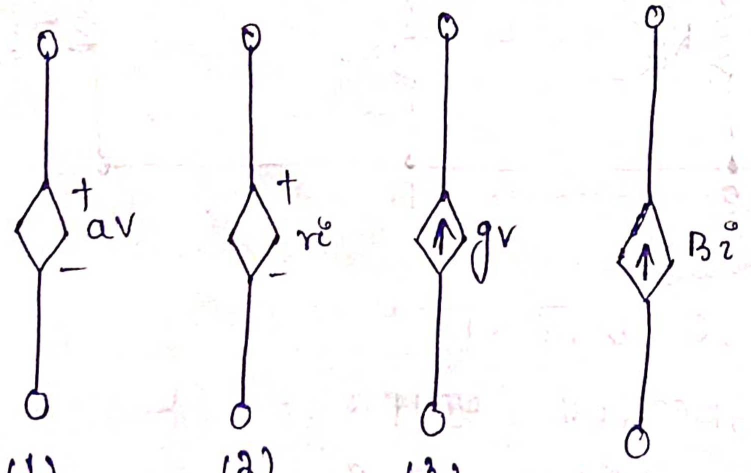
The source that depends on some other circuit-elements {R, L, C} in a given circuit.

2) Independent Source :-

The independent source doesn't depend on any other circuit elements.

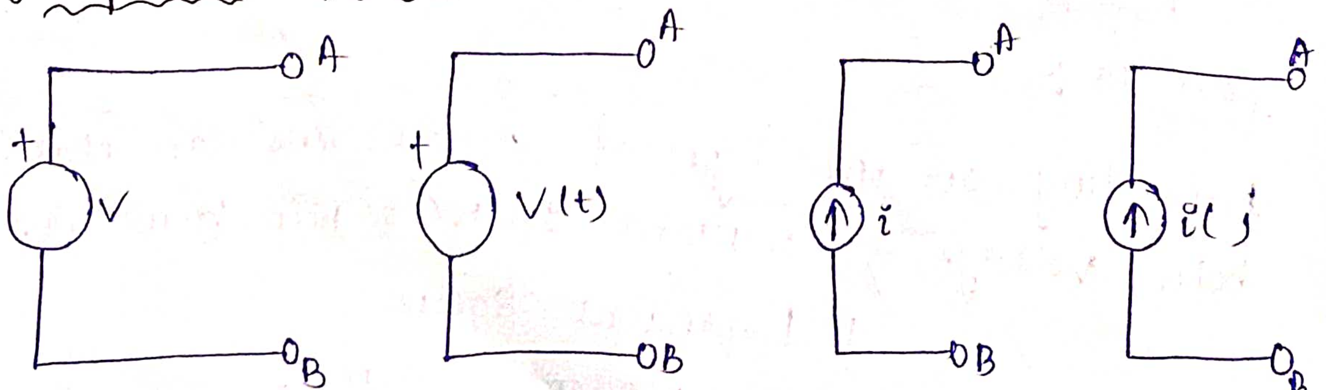
Dependent source is further divided into

1) voltage dependent voltage source.
2) current dependent voltage source
3) voltage dependent ~~voltage~~ current source
4) current dependent current source



(1) (2) (3) (4)

Independent Source :-



Time varying voltage source

Time varying current source

Independent sources actually exist as physical entities such as a battery a.d.c generator and an alternator etc.

But dependent sources are parts of models that are used to represent electrical properties of electronic devices such as operational amplifier & transistors etc.

Circuit Elements & Energy Sources

Circuit Elements :-

1) Resistance :-

Electrical resistance is the property of a material by virtue of which it opposes the flow of electrons - through the material.

Thus resistance restricts the flow of electrons - through the material.

S.I unit = ohm (Ω)

$$\boxed{R = \frac{V}{I}}$$

V → Potential difference across the material.

I → current in ampere.

When an electric current flows through any - conductor, heat is generated due to collision of free electrons with atoms.

If I = strength of current in Amp.
V = Potential difference in volts across the conductor.

The power absorbed by the resistor.

$$\boxed{P = VI = I^2R} \text{ watts.}$$

The energy lost in the resistance in form of heat is then expressed as.

$$W = \int_0^t P.dt = Pt = I^2Rt = \frac{V^2}{Rt}$$

2) Inductance:-

Inductance is the property of a material by virtue of which it opposes any changes of magnitude or direction of electric current passing through the conductor.

S.I unit of inductance = Henry.

It is given by faraday's laws of Electromagnetic - Induction. Inductance is said to be one henry when current through a coil of conductor changes at the rate of one ampere per second inducing one volt across the coil.

A wire of finite length, when twisted into a coil, it becomes a simplest inductor.

As soon as current will flow through the coil, an electromagnetic field is formed. However, with any change of flow or direction of current the electromagnetic field changes. This change of field induces a voltage(v) across the coil given by.

$$V = L\frac{di}{dt} \quad \text{——} \quad (1)$$

Where $i \rightarrow$ is the current through the inductor in ampere.

It may be noted from above equation that voltage across the inductor would be zero if the current through it remains constant. This means that an inductor behaves as a short circuited coil in steady state, when direct steady current flows through it.

However for any small change in current strength or change in direction, inductance will appear.

Thus an inductor behaves as open circuit just after ~~so~~ switching across d.c voltage, but as short circuit at steady state.

The power absorbed by the inductor ~~will thus~~ is given by

$$P = V \times i = Li \frac{di}{dt} \text{ watts} \quad \text{——— (2)}$$

Energy absorbed by the inductor will thus be given by

$$W = \int_0^t P dt = \int_0^t Li \frac{di}{dt} dt = \frac{1}{2} Li^2 \quad \text{——— (3)}$$

Thus from eqn - (2) & (3) it is evident that the inductor can store finite amount of energy, even the voltage across it may be nil.

A pure inductor does not dissipate energy but only stores it.

3) Capacitance :-

It is the capability of an element to store electric charge within it.

A capacitor stores electric energy in the form of ~~electric~~ field being established by the two polarities of charges on the two electrodes of a capacitor.

Quantitatively capacitance is a measure of charge per unit voltage that can be stored in an element.
unit of capacitance = Farad (F)

$$\boxed{C = \frac{q}{V}}$$

C → capacitance of the capacitor
q → amount of charge that can be stored.
V → Potential difference

i.e $i = C\frac{dv}{dt}$ $\left[\because i = \frac{dq}{dt}\right]$

$dv = \frac{1}{C} i\,dt$

$\Rightarrow \int_{V_0}^{V_t} dv = \frac{1}{C}\int_0^t i\,dt$

$\Rightarrow V_t - V_0 = \frac{1}{C}\int_0^t i\,dt$

$\Rightarrow V_t = \frac{1}{C}\int_0^t i\,dt + V_0$

The power absorbed by the capacitor is

$$P = vi = vC\frac{dv}{dt}$$

The energy stored by the capacitor

$$w = \int_0^t P\,dt = \int_0^t vC\frac{dv}{dt}\,dt = \frac{1}{2}Cv^2$$

Thus we observe that the voltage across the capacitor being constant, current through it is zero. This means that the capacitor, on application of d.c voltage and with no initial charge first acts as short circuit but as soon as the full charge it retains, the capacitor behaves an open circuit.

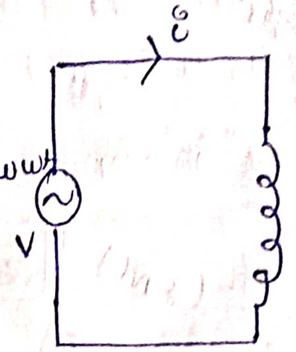
Also a capacitor never dissipates energy and only stores it. It can store finite amount of energy even if the current through it is zero.

Circuit Elements

1. Resistor — R — (Energy dissipating element)
2. Inductor — L — (Energy storage element)
   ↳ Inductance

$X_L = 2\pi f L$
↓
Inductive Reactance

$i = I_0 \sin \omega t$



emf = electro motive force

$V = IR$

$\mathcal{E} = I(R+r)$

$\mathcal{E}$ = self induced emf

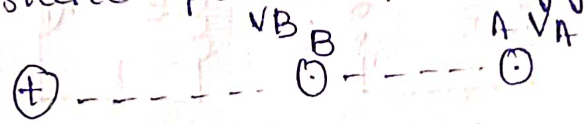
$\mathcal{E} = -L \frac{di}{dt}$

(energy stored in magnitude in magnetive field)

3. Capacitor — C

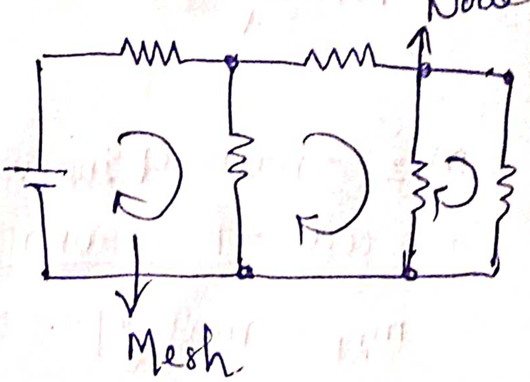
capacitance → capacity

* Energy is stored in electric field in the form of electrostatic potential energy.



$PD = VB - VA$

$E = \frac{1}{2} CV^2$ $\quad \mathcal{E} = I^2 R t$



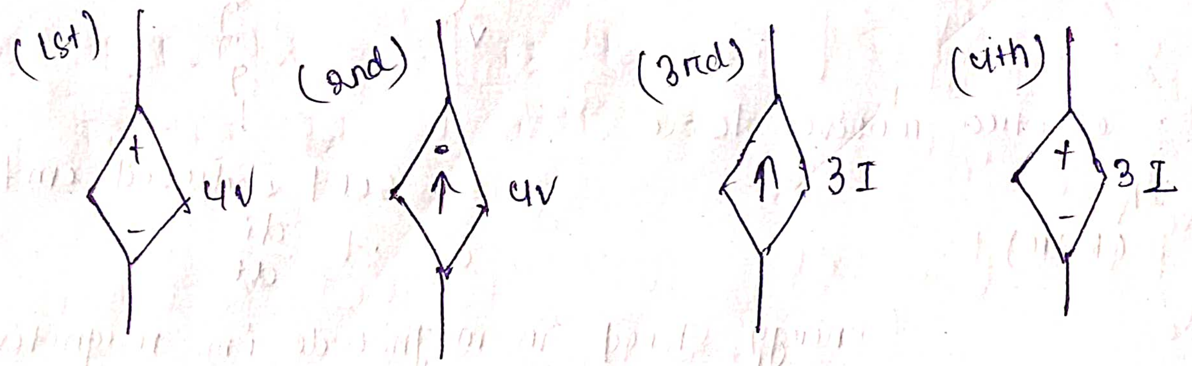
Branch - connection of two node

loop - All meshes are loop but
All loops are not meshes.

Source
- Independent source
  - A C source
  - Dc source
- Dependent source
  - current source
  - voltage source

Dependent source :-

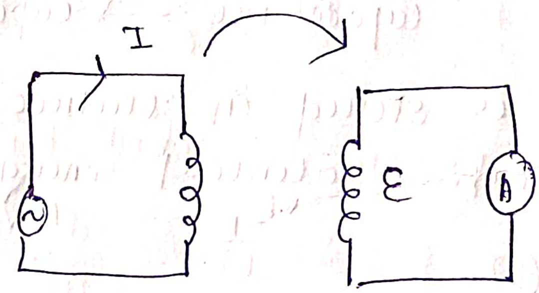
① voltage dependent voltage source
② " " current "
③ current " " "
④ " " voltage "



ELECTROMAGNETIC INDUCTION

Self Induction :-

Linking of flux →



Self Induced emf
↓
Self Inductance

$\mathcal{E} = -L \frac{dI}{dt}$

M_{12} - mutual Inductance
coil - 1 WRT coil - 2

M_{21} = mutual

$\boxed{L = \mu_0 n^2 IA}$

n = turns per unit length.
N = Total no. of turns

μ_0 - Permeability of free space.

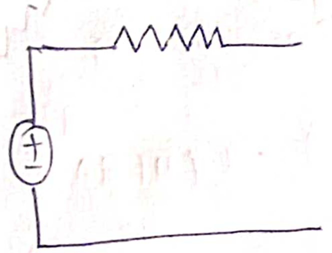
$L = \mu_0 N^2 A$

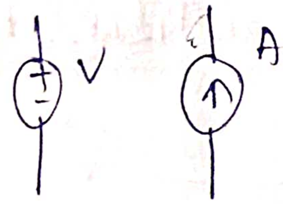
$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

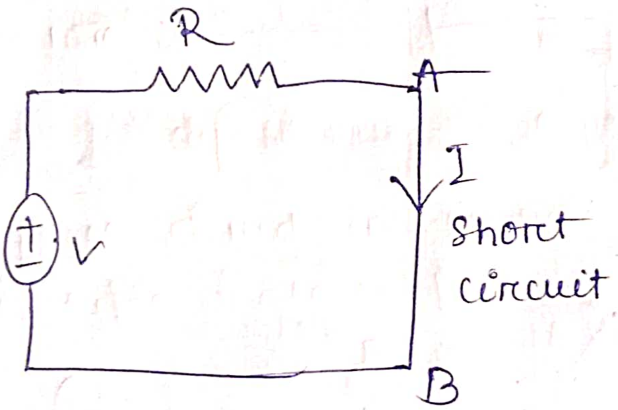
$= \mu_0 n_1 n_2 A l$

$= \mu_0 n_1 n_2 \pi r^2 l$

Source conversion :-

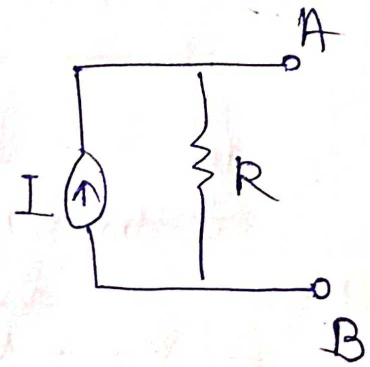






$V = IR$

$I = \frac{V}{R}$



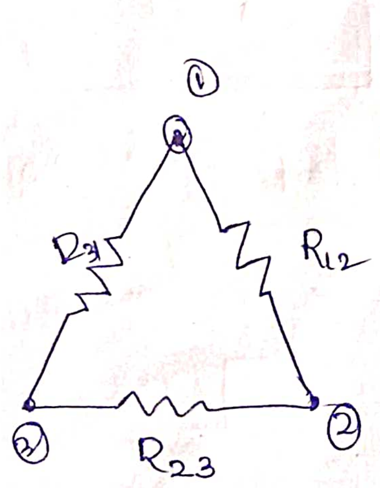
Stare Delta Transformation :-

Y to Δ :-

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$



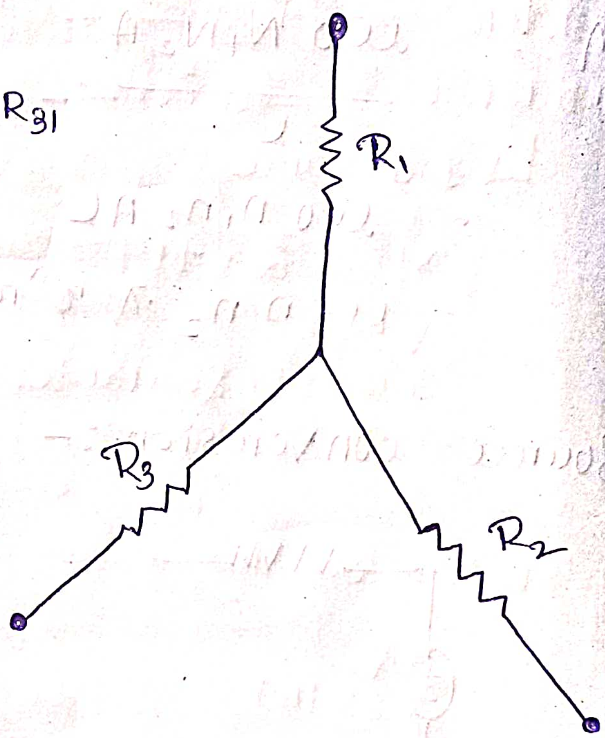
Δ → Y (Delta to star)

$\Delta \to Y$ (Delta to Star)

$$R_1 = \frac{R_1 R_2 \times R_3 R_1}{R_1R_2 + R_2R_3 + R_3R_1} = \frac{R_{12} \times R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_2 R_3 \times R_1 R_2}{R_1R_2 + R_2R_3 + R_3R_1} = \frac{R_{23} \times R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_2 R_3 \times R_3 R_1}{R_1R_2 + R_2R_3 + R_3R_1} = \frac{R_{23} \times R_{31}}{R_{12} + R_{23} + R_{31}}$$

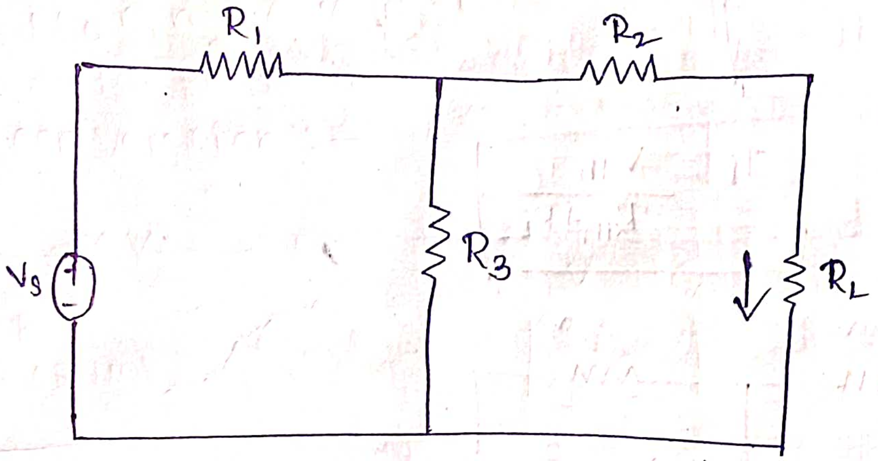


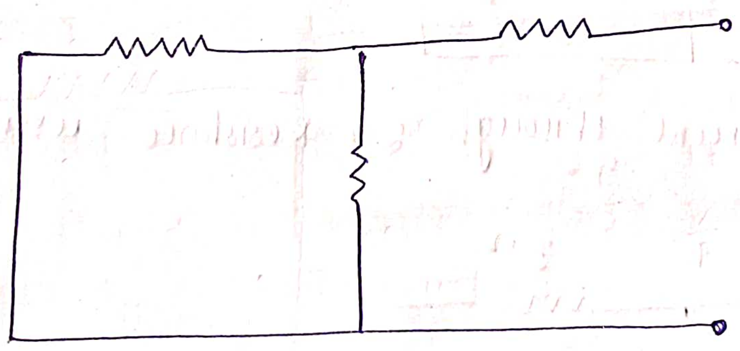
Date- 28. 10. 2022

Thevenin's Theorem :-

Statement :-

Any two terminal bilatral lineare DC circuit can be - replaced by an equivalent circuit consisting of a voltage Source and a series resistor.

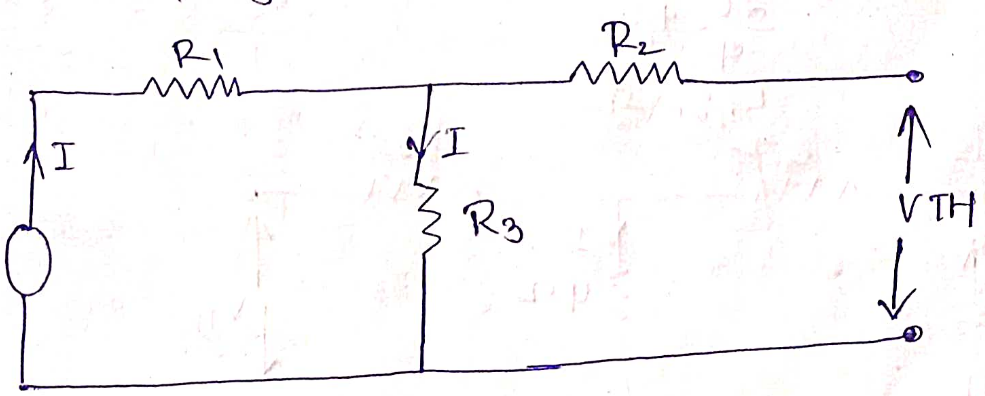




R_{TH} = Equivalent Thevenins Resistance.

$$R_{TH} = \left[R_1 \parallel R_3\right] + R_2$$

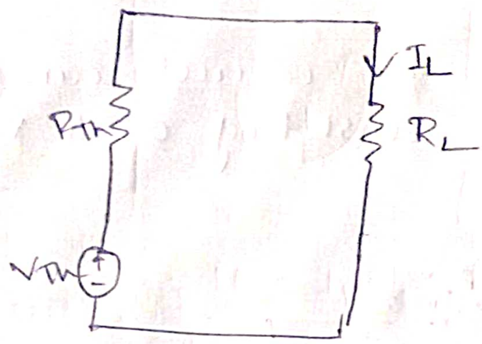
$$= \frac{R_1 R_3}{R_1 + R_3} + R_2$$



$$V_{OC} = V_{Th} = I \times R_3$$

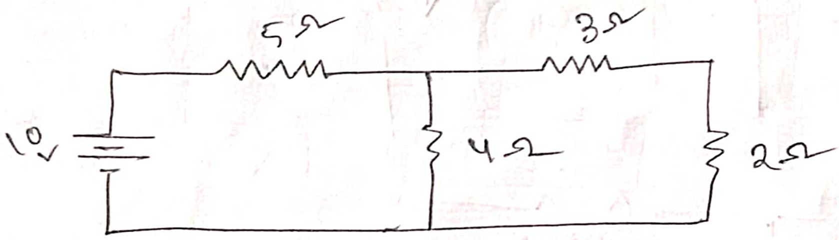
$$= \frac{V_S}{R_1 + R_3} \times R_3$$

Equivalent circuit



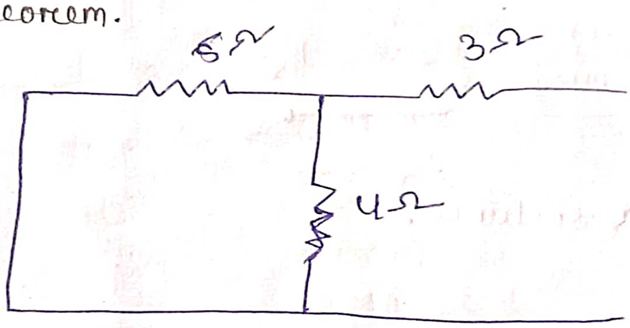
$$\boxed{I_L = \frac{V_{Th}}{R_{Th}+R_L}}$$

Q.

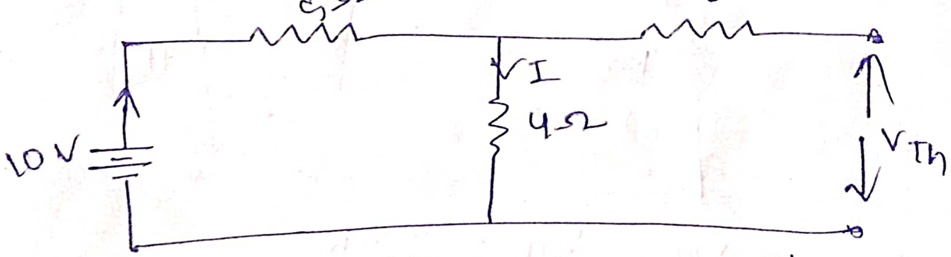


Find the current through 2Ω Resistance using Thevenin Theorem.

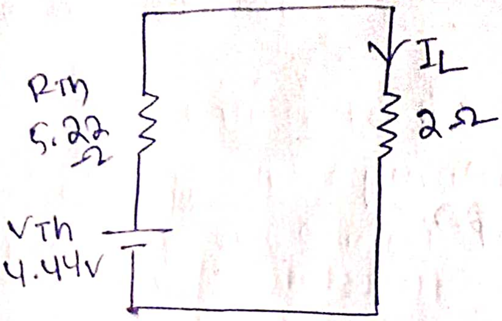
A.



$$R_{Th} = \frac{5\times4}{5+4} + 3$$
$$= \frac{20}{9} + 3$$
$$= 5.22\,\Omega$$



$I = \frac{10}{9}$ $V_{4\Omega} = I \times 4$
$= \frac{10}{9}\times4 = \frac{40}{9}\,V = 4.44V$

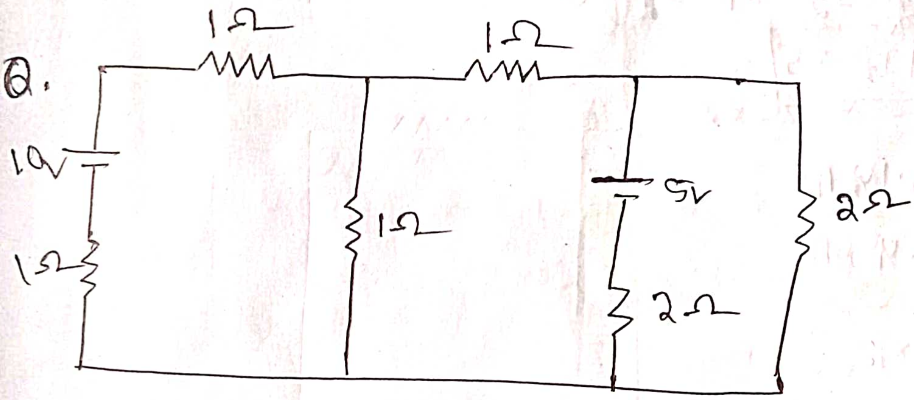


$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{4.44}{5.22+2}$$

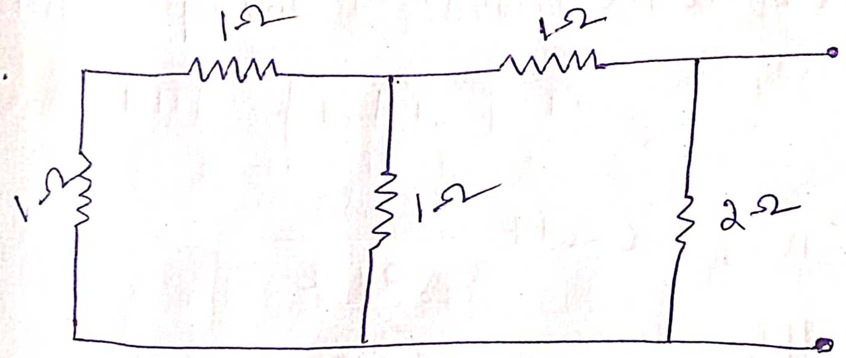
$$= 0.615\ A.$$

Q.



Draw the Thevenin. Equivalent circuit and find the ~~circuit~~ current in 2Ω resistance

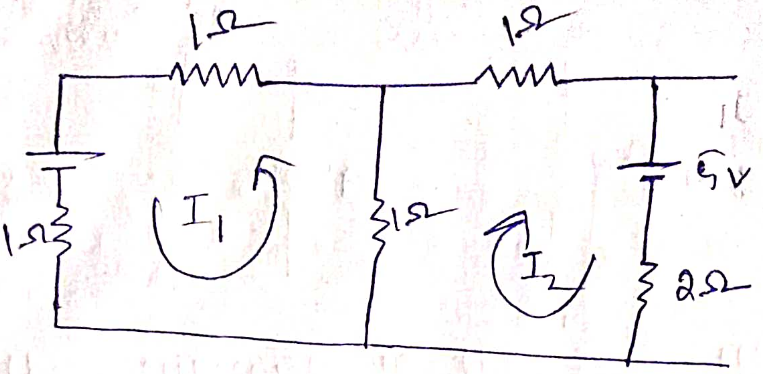
A.



$$R_{Th} = \frac{2 \times 1}{2+1} = \frac{2}{3} + 1$$

$$= \frac{\frac{5}{3} \times 2}{\frac{5}{3} + 2} = 0.91$$

$$R_{Th} = 0.91\ \Omega$$



$$-10 - I_1 \times 1 - (I_1 + I_2) \times 1 - 1 \times I_1 = 0$$

$$\Rightarrow -10 - I_1 - I_1 - I_2 - I_1 = 0$$

$$\Rightarrow \boxed{-3I_1 - I_2 = 10}$$

$$-5 - 2I_2 - (I_2 + I_1) \times 1 - I_2 \times 1 = 0$$

$$\Rightarrow -5 - 2I_2 - I_2 - I_1 - I_2 = 0$$

$$\Rightarrow \boxed{-I_1 - 4I_2 = 5}$$

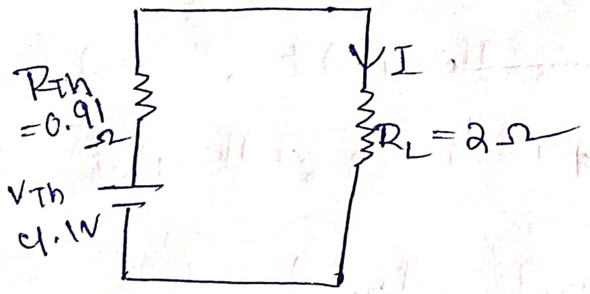
$$I_1 = -3.18\,A$$

$$I_2 = -0.45\,A.$$

$$V_{2\Omega} = I_2 \times 2$$
$$= 0.45 \times 2$$
$$= 0.90\,V$$

$$V_{BC} = 5 - 0.9$$
$$= 4.1\,V$$

$$V_{Th} = 4.1\,V$$

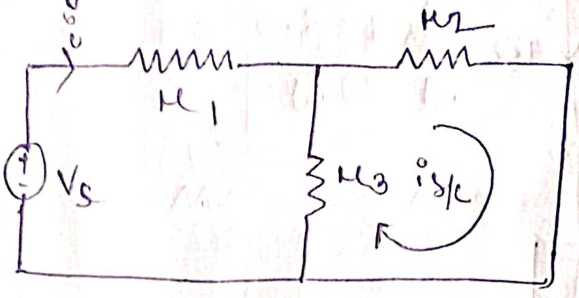


$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{4.1}{0.91 + 2} = 1.41\,A$$

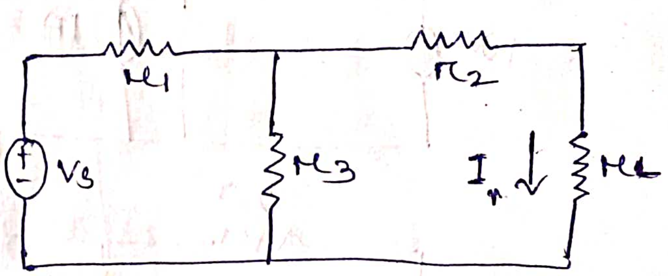
Norton's Theorem :-

Statement :-

A linear active network consisting of independent & or dependant voltage and current sources and linear bilateral network elements can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance, the current source being the short circuited current across the load terminal and the resistance being the internal resistance of the source network looking through the open circuited load terminals.



S/C

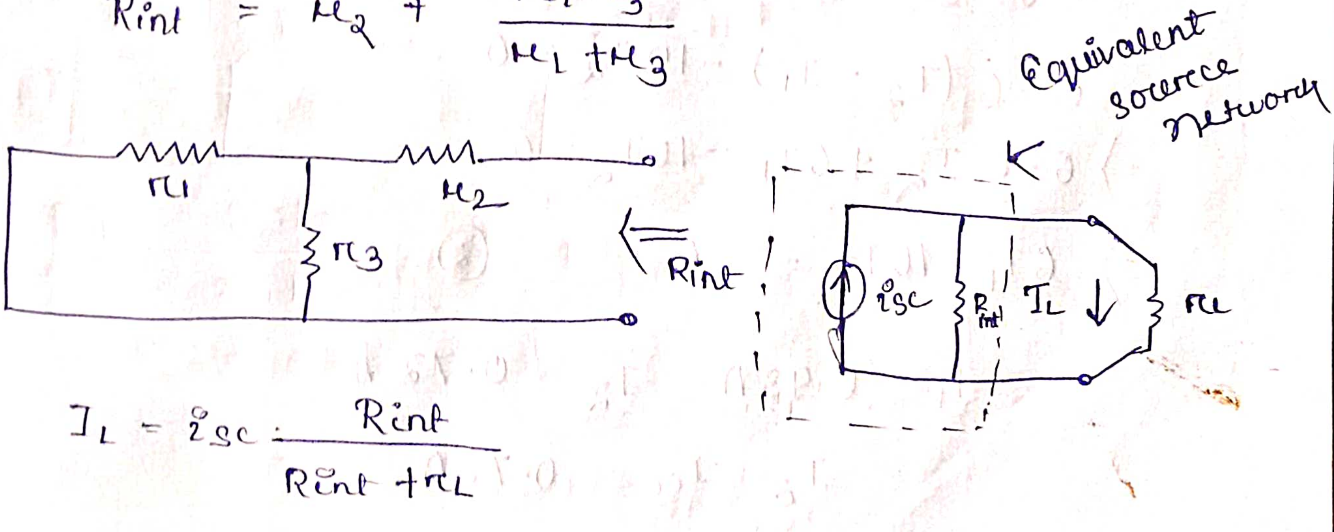


In order to find the current through RL, the load resistance by Norton's theorem, let us replace RL by short circuit.

$$i = \frac{V_s}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$i_{sc} = i \frac{R_3}{R_3 + R_2}$$

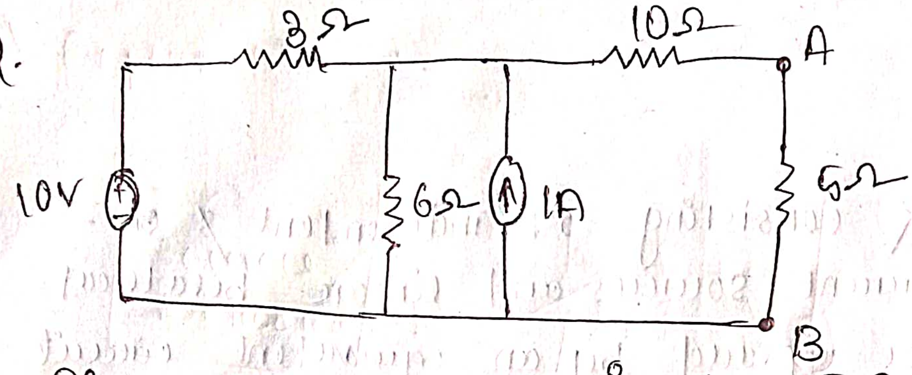
$$R_{int} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$



$$I_L = i_{sc} \cdot \frac{R_{int}}{R_{int} + R_L}$$

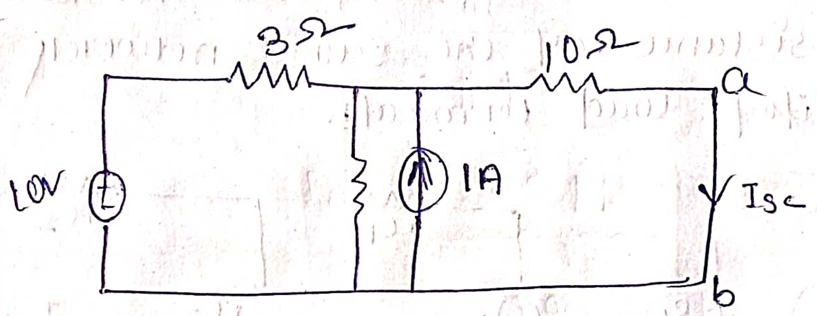
$$\boxed{I_L = i_{sc} \frac{R_{int}}{R_{int} + R_L}}$$

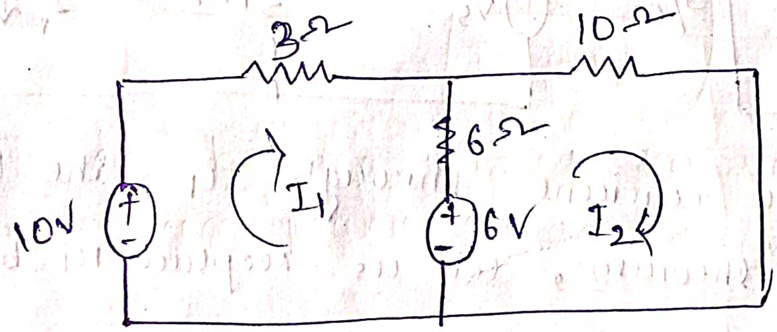
Q.



Find the current in the 5 Ω resistor for the circuit shown below.

A.





$$10 - 3I_1 - 6(I_1 - I_2) - 6 = 0$$

$$\Rightarrow 10 - 3I_1 - 6I_1 + 6I_2 - 6 = 0$$

$$\Rightarrow -9I_1 + 6I_2 = -4 \quad \text{——— (1)}$$

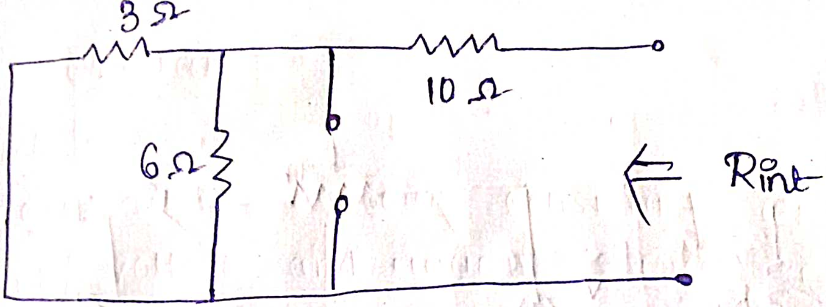
$$6 - 6(I_2 - I_1) - 10I_2 = 0$$

$$\Rightarrow 6 - 6I_2 + 6I_1 - 10I_2 = 0$$

$$\Rightarrow 6I_1 - 16I_2 = -6 \quad \text{——— (2)}$$

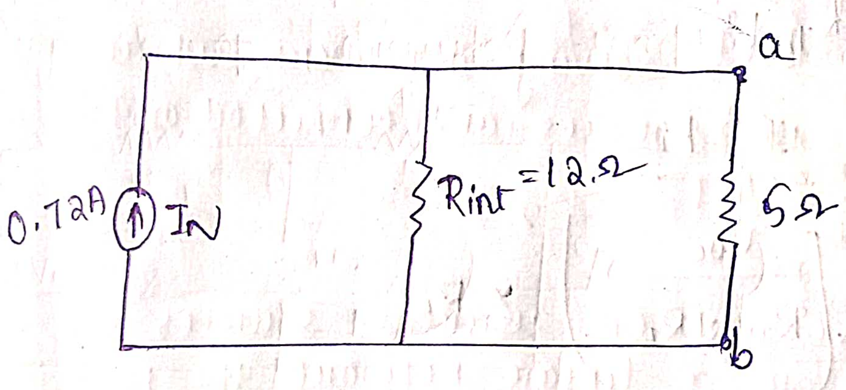
$I_1 = 0.93\,A \qquad I_2 = 0.72\,A$

$I_2 = I_{sc} = 0.72A$



$$R_{int} = [3 \| 6] + 10$$

$$= \frac{3 \times 6}{3+6} + 10 = 12\,\Omega$$



$$I_{5\Omega} = I_N \times \frac{R_{int}}{R_{int}+5}$$

$$= 0.72 \times \frac{12}{12+5} = 0.51\ A$$

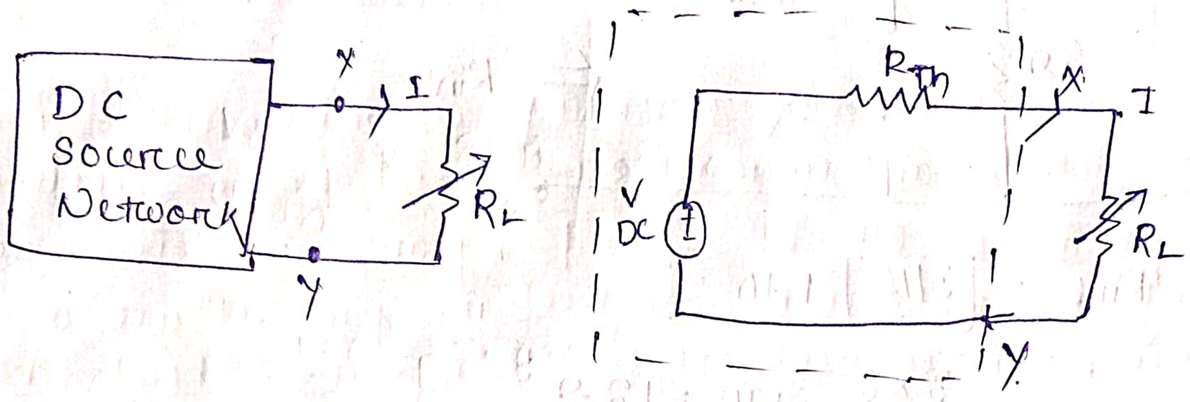
Maximum Power Transfer Theorem :-

-* This theorem is used to find the value of load resistance for which there would be maximum amount of power transfer form source to load.

Statement :-

A resistive load being connected to a DC network receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent Theorem) of the source network as seen from the load terminals.

→ Explanation :-



$$I = \frac{V_{oc}}{R_{Th} + R_L}$$

$$P_L = I^2 R_L$$

$$= \left[\frac{V_{oc}}{R_{Th} + R_L}\right]^2 \times R_L$$

P_L can be maximized by vary R and Hence maximum power can be deliver.

$$\frac{dP_L}{dR_L} = 0$$

$$\Rightarrow \frac{d}{dR_L}\left(\frac{V_0^2 R_L}{(R_{Th} + R_L)^2}\right)$$

$$\Rightarrow \frac{(R_{Th} + R_L)^2 \cdot \frac{d}{dR_L}(V_0^2 R_L) - V_0^2 R_L \frac{d}{dR_L}\left[(R_{Th} + R_L)^2\right]}{\left[(R_{Th} + R_L)^2\right]^2}$$

$$\Rightarrow \frac{V_0^2 (R_{Th} + R_L)^2 - V_0^2 R_L (R_{Th} + R_2)}{[R_{Th} + R_L]^4}$$

$$= \frac{V_0^2 (R_{Th} + R_L) \left[(R_{Th} + R_L) - 2R_L \right]}{(R_{Th} + R_L)^4}$$

$$= \frac{V_0^2 \left[R_{Th} - R_L \right]}{(R_{Th} + R_L)^3}$$

$$\frac{V_0^2 \left[R_{Th} - R_L \right]}{\left[R_{Th} + R_L \right]^3} = 0$$

$$\Rightarrow V_0^2 \left[R_{Th} - R_L \right] = 0$$

$$\Rightarrow R_{Th} - R_L = 0$$

$$\Rightarrow \boxed{R_{Th} = R_L}$$

$$P_{max} = \frac{V_0^2 R_{Th}}{(R_{Th} + R_{Th})^2}$$

$$= \frac{V_0^2 R_{Th}}{(2R_{Th})^2} = \frac{V_0 \cancel{R_{Th}}}{4R_{\cancel{Th}}^2}$$

$$= \frac{V_0^2}{4R_{Th}}$$

$$\boxed{P_{max} = \frac{V_0^2}{4R_{Th}}}$$

The total Power supply

$$P = \frac{\cancel{2} \times V_0^2}{\cancel{4} R_{Th}}$$

$$= \frac{V_0^2}{2R_{Th}}$$

$$\eta = \frac{P_{max}}{P} \times 100$$

$$= \frac{\cancel{V_o^2}}{\cancel{4}_{2}\cancel{R_{th}}} \times \frac{\cancel{2}\cancel{R_{th}}}{\cancel{V_o^2}} \times 100$$

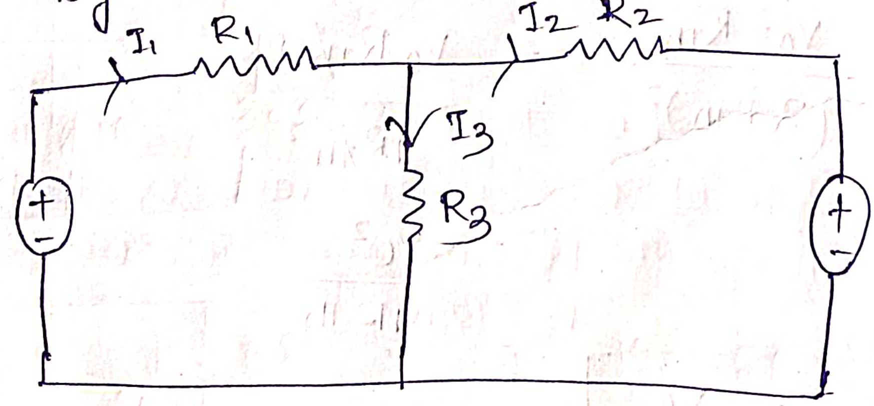
$$= \frac{100}{2} = 50\%.$$

Superposition Theorem :-

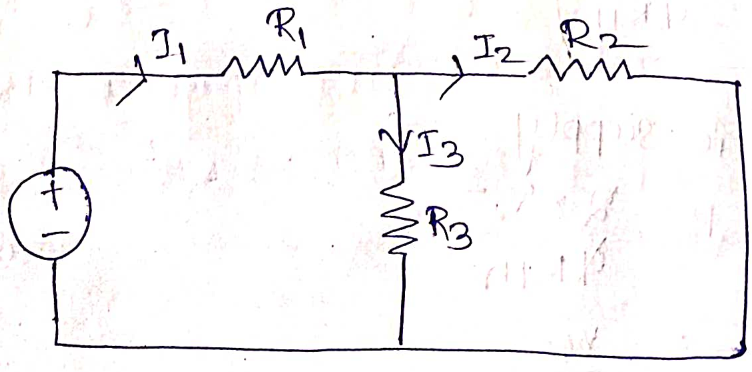
This Theorem is used in solving a network where there are two or more sources and connected not in series or in parallel.

Statement :-

If a number of voltage or current sources are acting simultaneously in a linear network the resultant current in any branch is the algebric sum of the currents that would be produce in it when each sources acts alone replacing all other independent sources by their external resistances.



Step 1



$$I_1' = \frac{V_1}{R_{eq}}$$

$$R_{eq} = [R_2 \| R_3] + R_1$$

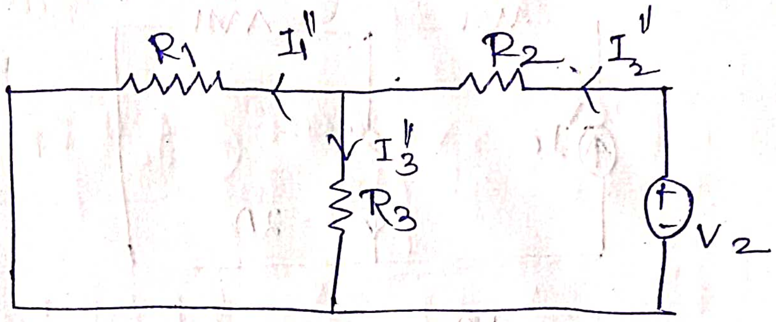
$$= \frac{R_2 R_3}{R_2 + R_3} + R_1$$

$$I_1' = \frac{V_1}{\frac{R_2 R_3}{R_2 + R_3} + R_1}$$

$$I_2' = I_1' \times \frac{R_3}{R_2 + R_3}$$

$$I_3' = I_1' \times \frac{R_2}{R_2 + R_3}$$

Step - 2



$$I''_2 = \frac{V_2}{R_{eq}}$$

$$R_{eq} = [R_1 \| R_3] + R_2$$

$$= \frac{R_1 R_3}{R_1 + R_3} + R_2$$

$$I''_2 = \frac{V_2}{\frac{R_1 R_3}{R_1 + R_3} + R_2}$$

$$I_1' = I_2'' \times \frac{R_3}{R_1 + R_3}$$

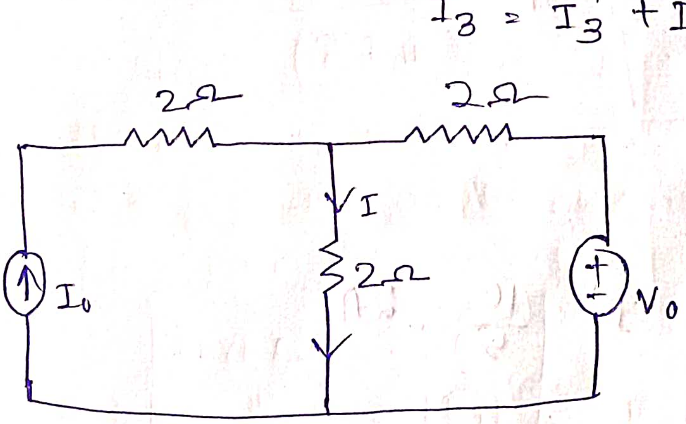
$$I_3'' = I_2'' \times \frac{R_1}{R_1 + R_3}$$

$$I_1 = I_1' - I_1''$$

$$I_2 = I_2' - I_2''$$

$$I_3 = I_3' + I_3''$$

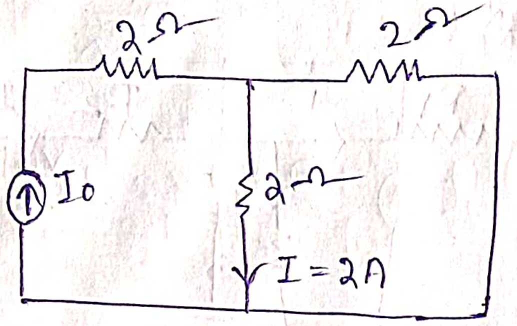
Q.

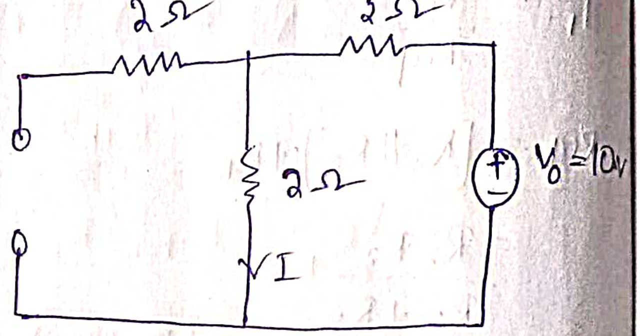


from the above circuit, $V_0 = I_0$, $I = 2A$
find I when $V_0 = 10V$

soln.

A. when $V_0 = 0$ $I = 2A$

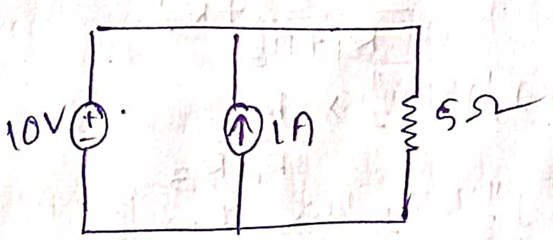




$$I = \frac{10}{2+2} = \frac{10}{4} = 2.5A$$

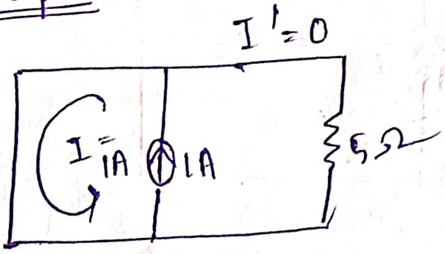
∴ Net current through the required resistor
i.e $I = 2 + 2.5 = 4.5A$

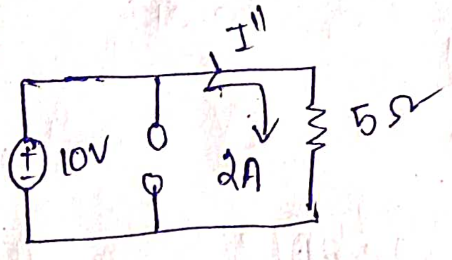
Q. In the circuit shown find the current through the 5Ω resistor using the principle of superposition.



A.

<u>Step 1</u>

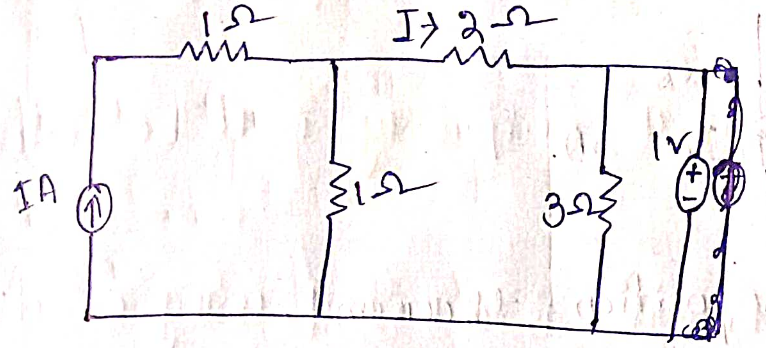




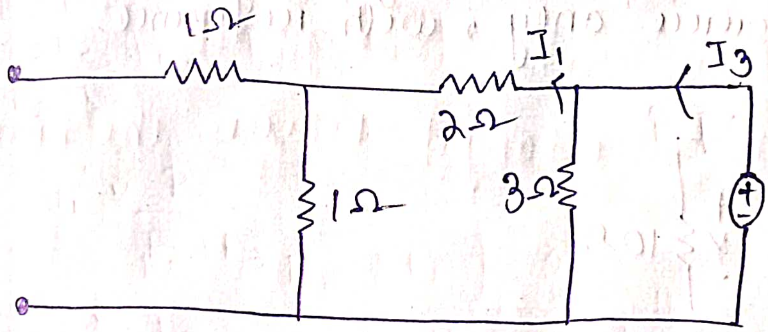
$$I'' = \frac{10}{5} = 2A$$

current through 5Ω Resistor
$= I' + I'' = 0 + 2 = 2A$

Q. Find the value of current I in circuit shown.

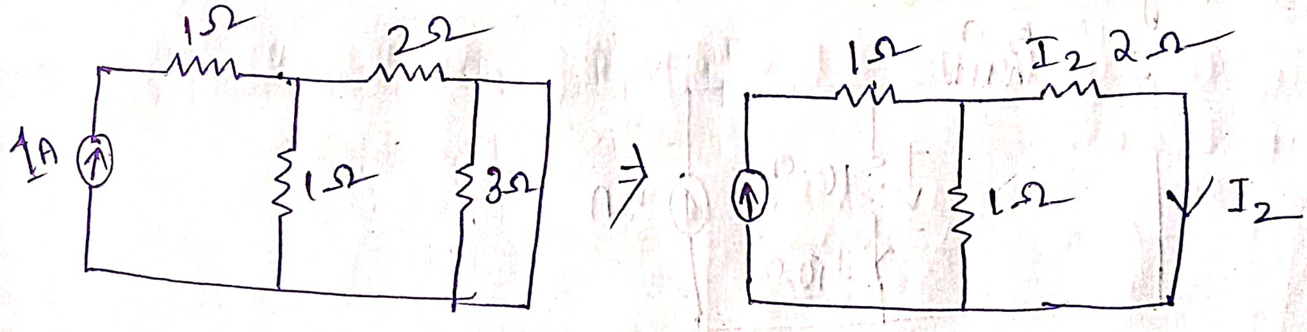


A.



$$I_s = \frac{1}{[(1+2)//3]} = \frac{1}{3//3} = \frac{1}{\frac{3\times3}{3+3}} = \frac{6}{9} = 0.67A$$

$$I_1 = I_s \times \frac{3}{3+2+1} = 0.67 \times \frac{3}{6} = 0.335\,A$$

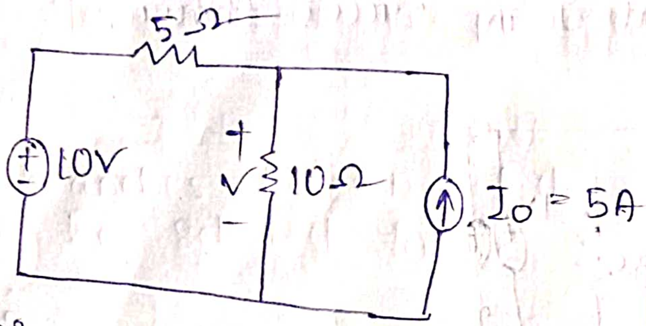


$$I_2 = 1 \times \frac{1}{1+2} = \frac{1}{3} = 0.33A$$

So the current through 2Ω resistor

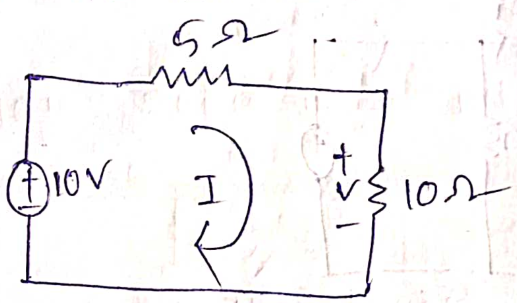
$$I = I_3 - I_2 = 0.335 - 0.33$$
$$= 0.005A$$

Q.



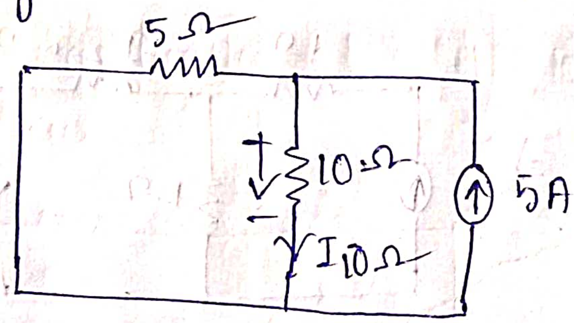
Find V by Superposition theorem.

A. Taking the 10V source only, with reference



$$V_1 = 10 \times I$$
$$= 10 \times \frac{10}{5+10} = 6.67V$$

Taking the 5A source only



$$I_{10\Omega} = 5 \times \frac{5}{10+5} = 1.67A$$

$$V_2 = 1.67 \times 10 = 16.70V$$

By superposition theorem

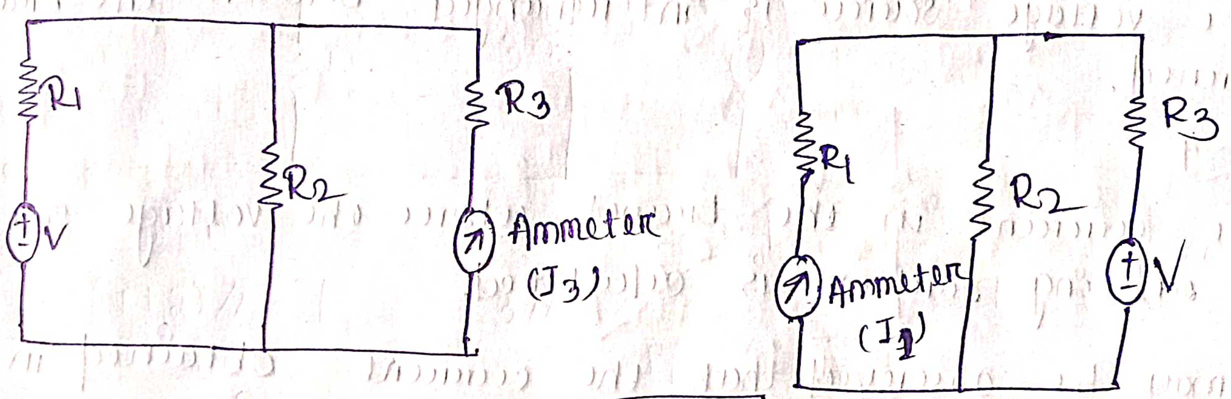
$$V = V_1 + V_2 = 6.67 + 16.70 = 23.37 V.$$

RECIPROCITY THEOREM :-

Statement of Reciprocity Theorem :-

In any branch of a network, the current (I) due to a single source of voltage (V) else where in the network is equal to the current through the branch in which the source was originally placed when the source is placed in the branch in which the current (I) was originally obtained.

Explanation :-



$$\frac{V}{I_3} = \frac{V}{I_1} \quad \text{i.e.} \quad \boxed{I_3 = I_1}$$

In simple sense, the location of the voltage source and the through current may be interchanged without a change in the current.

However the polarity of the voltage source should have a the identicality with the direction of branch current in each position.

The limitation of this theorem is that it is applicable only to single source networks and not in multi source network.

Moreover, the network where reciprocity theorem is applied should be a linear one and containing resistor, inductors, capacitors & coupled circuits.

The network should not have any time v. element.

Steps for Solving a Network utilising Reciprocity Theorem

Step - 1

The branches between which reciprocity is to be established are to be selected first.

Step - 2

The current in the branch is obtained using conventional network analysis.

Step - 3

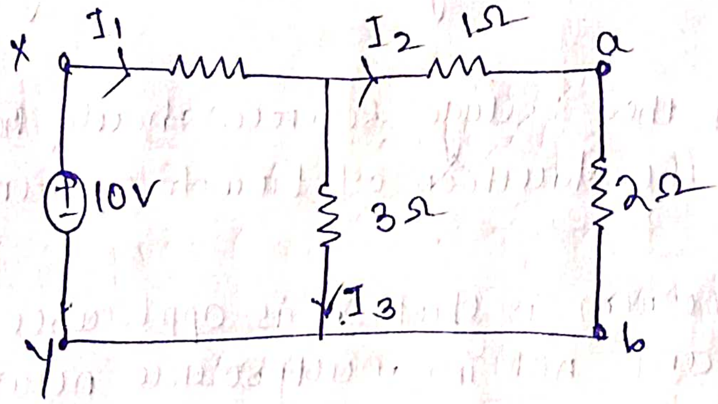
The voltage source is interchanged between the branches concerned.

Step - 4

The current in the branch where the voltage source was existing earlier is calculated.

It may be observed that the current obtained in step-2 and step-4 are identical to each other.

Q. Show the application of reciprocity theorem in the network shown below.

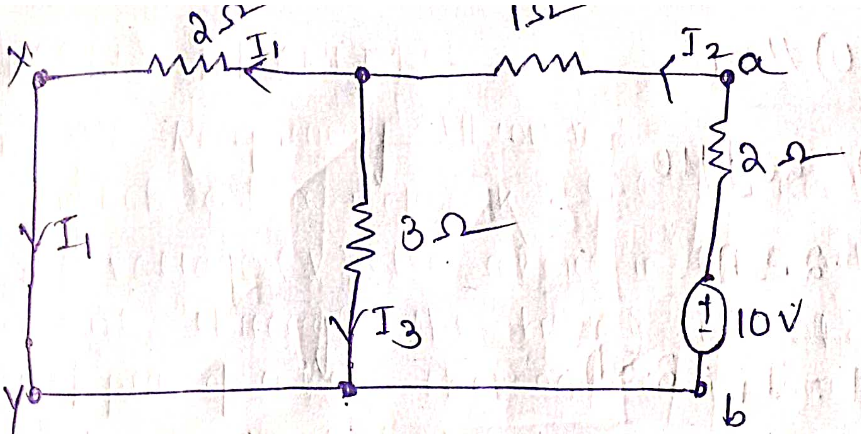


A. Equivalent Resistance across xy

$$R_{eq} = [(2+1) \parallel 3] + 2 = 3.5\,\Omega$$

$$I_1 = \frac{10}{3.5} = 2.86\,A$$

$$I_2 = 2.86 \times \frac{3}{3+2+1} = 1.43\,A.$$



From the above figure

$R_{eq} = [(2 // 3) + 1 + 2] = \frac{6}{5} + 3 = \frac{21}{5} = 4.2\,\Omega$

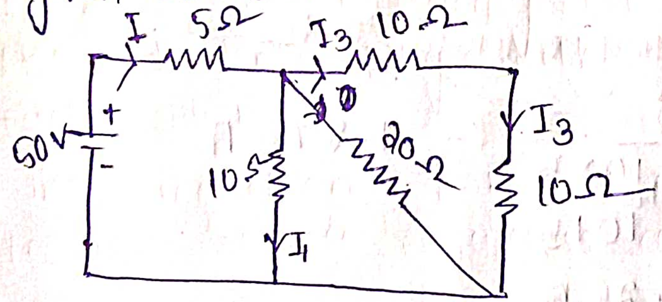
$I_2 = \frac{10}{4.2} = 2.381\,A$

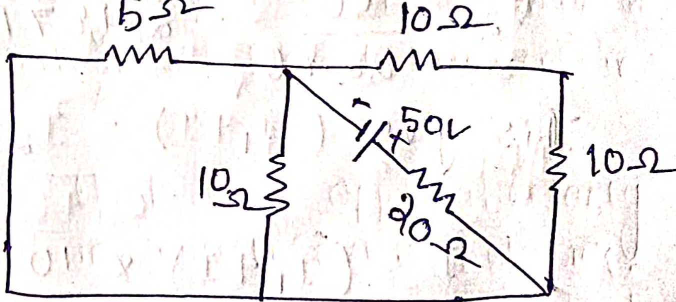
$I_1 = I_2 \times \frac{3}{3+2} = 2.381 \times \frac{3}{5} = 1.43A$

Hence we observe that when the source was in branch x-y the a-b branch current is 1.43A ; again when the source is in branch a-b the x-y branch current becomes 1.43A.

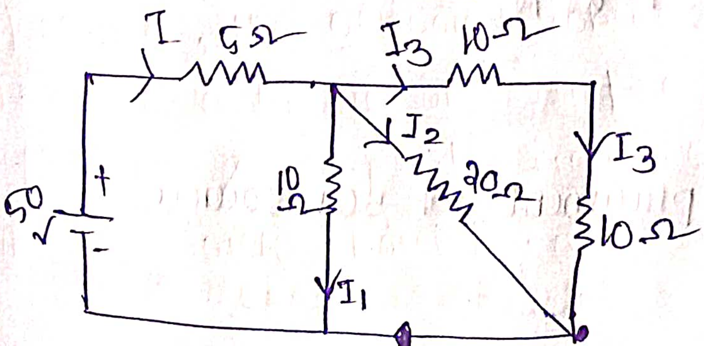
This proves the ~~Rec~~ Reciprocity theorem.

Q. Shown the validity of reciprocity theorem in the figure given below.





A.



$R_{eq} = [((10+10) // 20) // 10] + 5$

$= 10\,\Omega$

$I = \frac{50}{10} = 5A$

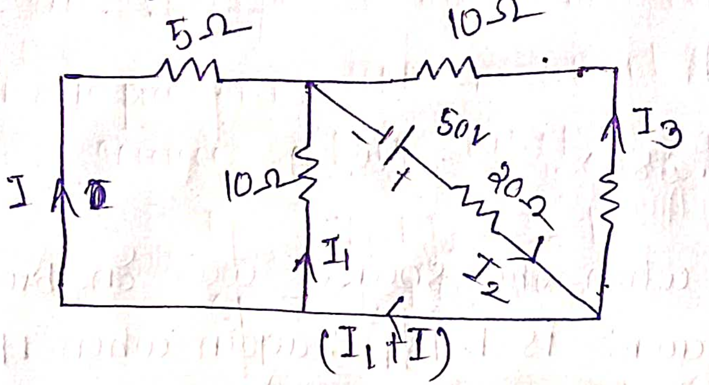
$$I_1 = I \times \frac{(10+10)//20}{[(10+10)//20]+10}$$

$$= 5 \times \frac{10}{20} = 2.5A.$$

$\therefore (I_2 + I_3) = I - I_1 = 2.5A$

while $I_2 = (I_2 + I_3) \times \frac{10+10}{10+10+20} = 2.5 \times \frac{20}{40} = 1.25A$

$I_3 = 1.25A$



Equivalent resistance across the 50V source

$$R_{eq} = \left[\left[\frac{10 \times 5}{10+5}\right] // (10+10)\right] + 20 = (3.33//20) + 20$$

$$= 2.855 + 20$$

$$= 22.86\ \Omega$$

$$I_2 = \frac{50}{22.86} = 2.187A$$

This gives $(I_1 + I) = I_2 \times \frac{20}{20 + \frac{10 \times 5}{10+5}} = 1.875A$

$$I = (I_1 + I) \times \frac{10}{10+5} = 1.25A$$

$I_2 = 1.25A$

On the other hand with placement of 50v source in I_2 branch now $I = 1.25A$

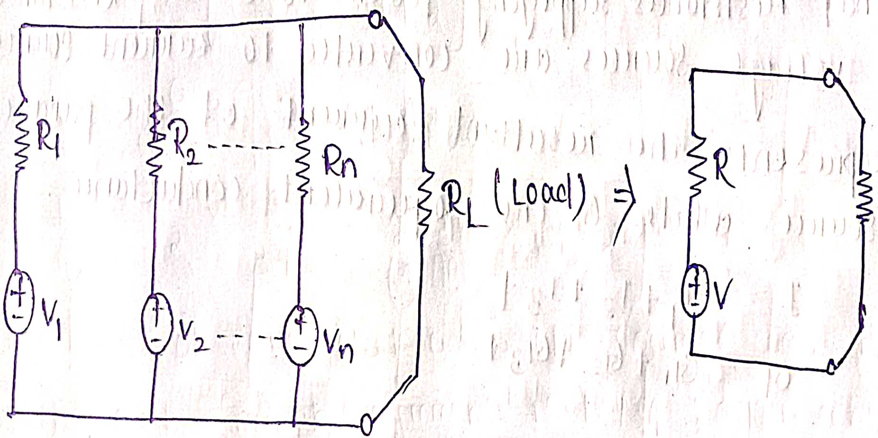
Thus the Reciprocity theorem is proved.

Millman's Theorem :-

The utility of this theorem is that, any number of parallel voltage sources can be reduced to one equivalent one.

Statement of Millman's Theorem :-

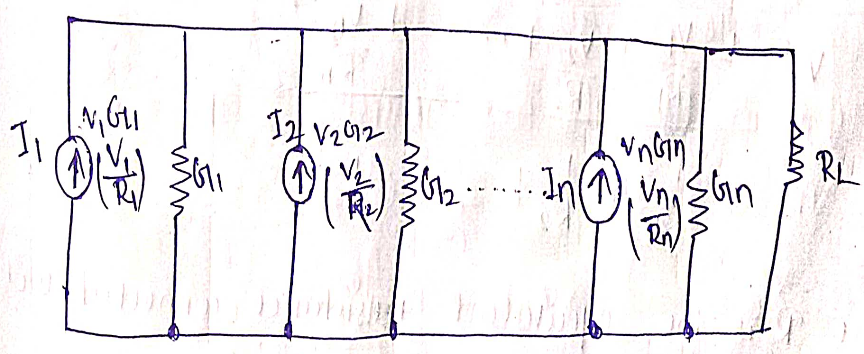
When a number of voltage sources ($V_1, V_2 \cdots V_n$) are in parallel having internal resistances ($R_1, R_2 \cdots R_n$) respectively, the arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R.

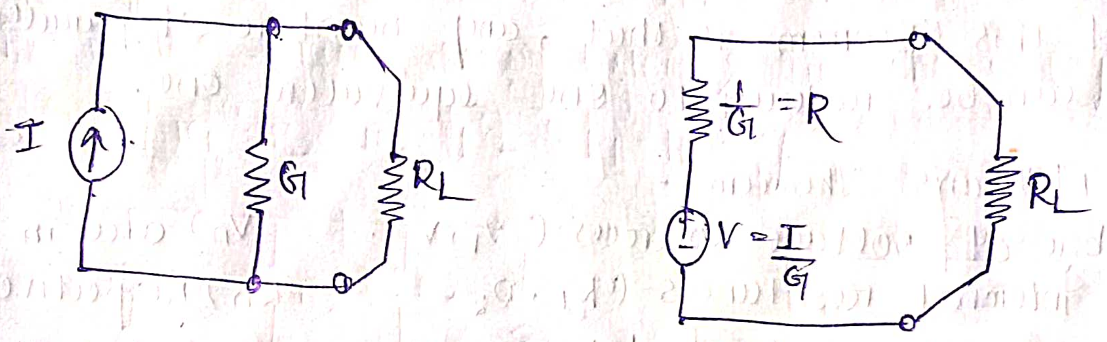


As per Millman's Theorem

$$\boxed{V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \pm V_n G_n}{G_1 + G_2 + \cdots + G_n}}$$

$$\boxed{R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots + G_n}}$$





Assuming a dc network of numerous parallel voltage sources with internal resistances supplying power to a load Resistance R_L, all voltage sources are converted to current sources. Let I represents the resultant current of the parallel current sources while G the equivalent conductance.

$$I = I_1 + I_2 + I_3 + \cdots$$
$$G = G_1 + G_2 + G_3 + \cdots$$

Next the resulting current source is converted to an equivalent voltage source.

Thus $$V = \frac{I}{G} = \frac{\pm I_1 \pm I_2 \pm \cdots \pm I_n}{G_1 + G_2 + \cdots + G_n}$$

[(+) & (−) sign appeared to include the cases where the sources may not be supplying current in the same direction]

Also $$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots + G_n}$$

&

$$V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \cdots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

where R is the equivalent resistance connected with the equivalent voltage source in series.

Thus finally $V = \dfrac{\pm V_1G_1 \pm V_2G_2 \pm \cdots \pm V_nG_n}{G_1 + G_2 + \cdots + G_n}$

$$V = \frac{\sum_{k=1}^{n} V_k G_k}{\sum_{k=1}^{n} G_k} \quad \& \quad G_k = \frac{1}{R_k}$$

Steps for solving problems Relating to Millman's Theorem :-

Following steps can be executed to get a direct solution of the problems relating Millman's Theorem.

Step 1 :-

Obtain the conductance (G_1, G_2 ......) of each voltage source (V_1, V_2, ...) and Find G, the equivalent conductance removing the load.

Step-2

Apply Millman's theorem to find V, the equivalent voltage source given by.

$$V = \frac{V_1G_1 + V_2G_2 + \cdots + V_nG_n}{G_1 + G_2 + \cdots + G_n}$$

Step-3

Determine (R), the equivalent series resistance with the equivalent voltage source (V)

$$R = \frac{1}{G}$$

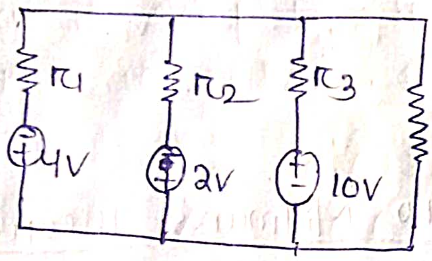
Step-4

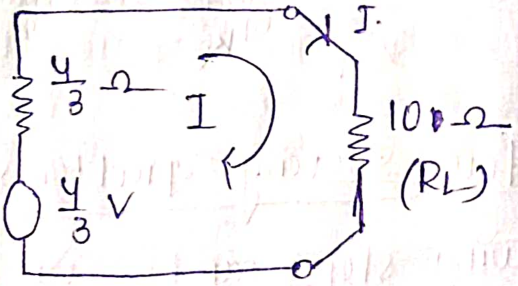
The current through the load is then given by

$I_L = \dfrac{V}{R + R_L}$; R_L being the load resistance.

Q. Using Millman's theorem, find the current through R_L in the circuit shown below & find the voltage drop

$[r_1 = r_2 = r_3 = 4\,\Omega,\; R_L = 10\,\Omega]$





A. Here $V = \dfrac{-V_1G_1 - V_2G_2 + V_3G_3}{G_1 + G_2 + G_3}$

& $R = \dfrac{1}{G} = \dfrac{1}{G_1+G_2+G_3} = \dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$

$V = \dfrac{-4\times\frac{1}{4} - 2\times\frac{1}{4} + 10\times\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \dfrac{-1-\frac{1}{2}+\frac{5}{2}}{\frac{3}{4}} = \frac{4}{3}\,V$

& $R = \dfrac{1}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \frac{4}{3}\,\Omega$

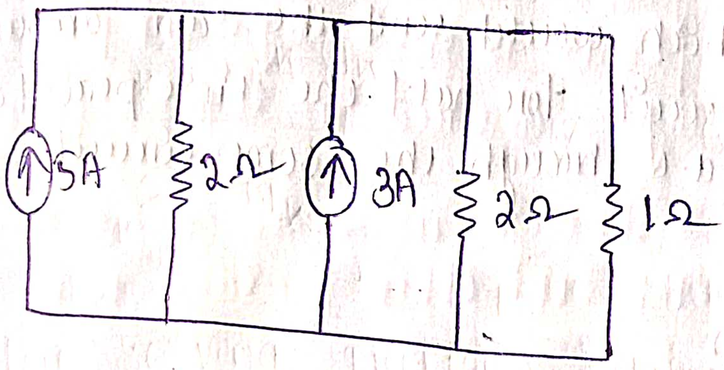
From the equivalent circuit diagram

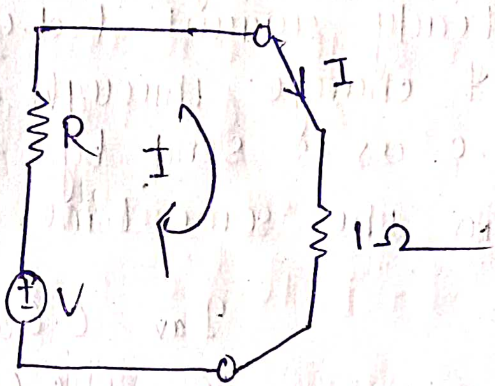
$I = \dfrac{4/3}{\frac{4}{3}+10} = \frac{4}{3}\times\frac{3}{34} = 0.12\,A$

The current through $10\,\Omega$ resistor is thus $0.12\,A$ while the drop across $10\,\Omega$ resistor

$V_{10\Omega} = 10 \times 0.12 = 1.2\,V$

Q. Find the current through the 1Ω resistor using Millman's Theorem.





A.

$I = I_1 + I_2 = 5 + 3 = 8A$

$G = G_1 + G_2 = \frac{1}{2} + \frac{1}{2} = 1$

converting current source to an equivalent voltage source

$V = \frac{I}{G} = \frac{8}{1} = 8V$

$R = \frac{1}{G} = \frac{1}{1} = 1\Omega$

current through 1Ω resistor

$I_{1\Omega} = \frac{V}{R+1} = \frac{8}{1+1} = \frac{8}{2} = 4A$

Average value of sinusoidal current (I_{avg})

The half-cycle average value of a.c is that value of steady current (d.c) which would send the same amount of charge through a circuit for half the time period of a.c as is send by the a.c through the same circuit in the same time.

$$I_{av} = 0.637\, I_m$$

$$V_{av} = 0.637\, V_m$$

RMS or Effective value :-

The effective or r.m.s value of a an alternating current is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2}{\sqrt{2}}}$$

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \cdots + V_n^2}{\sqrt{2}}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}} \quad \text{or} \quad I_{rms} = 0.707\, I_m$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \text{or} \quad V_{rms} = 0.707\, V_m$$

Form Factor :-

The ratio of r.m.s value to the average value of an alternating quantity is known as form factor.

$$\text{Form Factor} = \frac{\text{R.M.S value}}{\text{Average value}}$$

$$\Rightarrow \text{Form factor} = \frac{0.707 \times \text{Max value}}{0.637 \times \text{Max value}} = 1.11$$

The form factor gives a measure of the peakiness of the wave form. The peakier the wave, the greater is its form factor & vice-versa.

Triangular wo here form factor = 1.15

Peak factor :-

The ratio of maximum value to the rms value of an alternating quantity is known as peak factor.

i.e $\text{Peak factor} = \dfrac{\text{Max. value}}{\text{RMS Value}}$

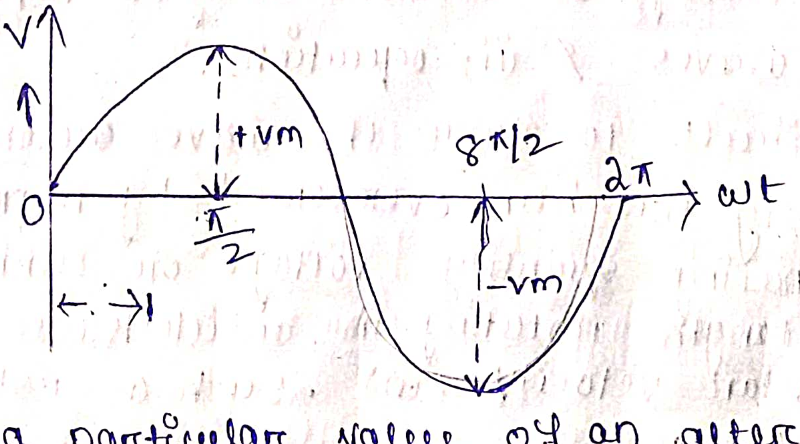
The value of peak factor also depends upon the wave form of the alternating quantity. For an alternating voltage or current varying sinusoidally, ~~value~~ its value is 1.414.

For a sinusoidal voltage or current

$$\text{Peak factor} = \frac{\text{Max. value}}{0.707 \times \text{Max value}} = 1.414$$

The peak factor is also called crest factor or amplitude factor.

Phase :-

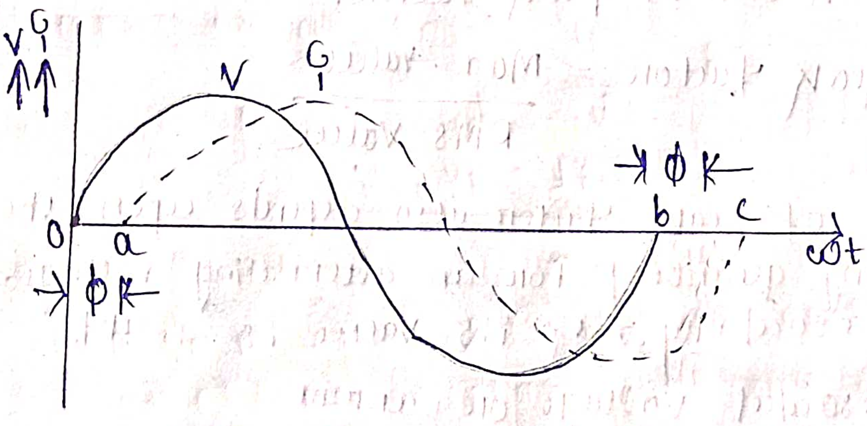


Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase difference :-

When two alternating quantity of the same frequency have different zero points, they are said to have a phase difference. The angle between zero points is the angle of phase - difference ϕ. It is generally measured in degrees or radians.

The quantity which passes through its zero point earlier is said to be leading while the other is said to be lagging.



$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t - \phi)$$

The above eqn's indicates that current is logging behind the voltage by phase angle ϕ.

Representation of Alternating voltages and currents :-

An alternating voltage or current may be represented in the form of (i) waves & (ii) equations.

But it is difficult to draw the wave accurately. The above difficulty has been overcomed by representing sinusoidal alternating ~~quantity~~ voltage or current by a line of definite length rotating in anticlockwise direction at a constant angular velocity (ω). Such a rotating line is called a phasor.

The length of the phasor is taken equal to the maximum value of the alternating quantity and angular velocity equal to the angular velocity of the alternating quantity.

Instantoneous power :-

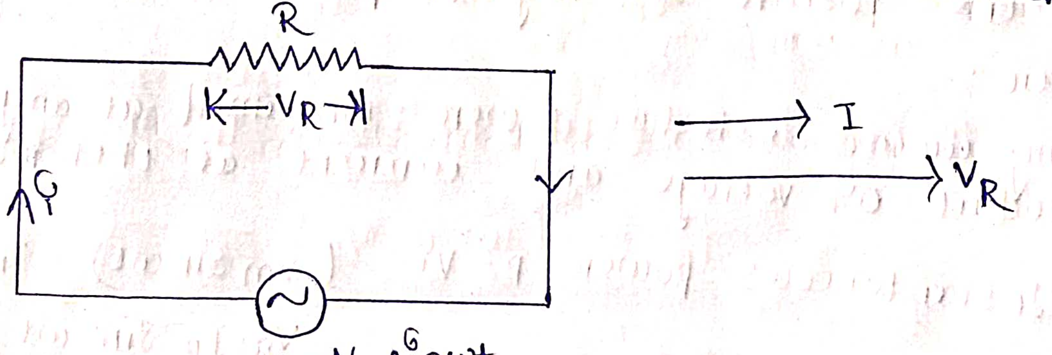
The instantoneous power supplied to a circuit is simply the product of the instantoneous voltage and instantoneous current. The instantoneous ~~voltage~~ power is always expressed in watts, irrespective of the type of circuit used.

The instantoneous power may be positive or negative.

A positive value means the power flows from the source to the load. consequently, a negative value means that power flows from the load to the source

Ac circuit containing Resistance only :-

when an alternating voltage is applied across pure resistance, then free electrons flow (i.e. current) in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle, thus constituting alternating ~~circuit~~ current in the circuit.



$V = V_m \sin\omega t$

consider a circuit containing a pure resistance of $R\,\Omega$ connected across an alternating voltage source.

Let the alternating voltage be given by the equation

$$v = V_m \sin\omega t \quad \text{——— (1)}$$

As a result of this voltage an alternating current i will flow in the circuit. The applied voltage has to overcome the drop in the resistance only i.e

$$v = iR$$

$$\Rightarrow i = \frac{v}{R}$$

Substituting the value of v.

$$i = \frac{V_m}{R} \sin \omega t \quad \text{--- (ii)}$$

The value of i will be maximum ~~value~~ (i.e I_m) when $\sin \omega t = 1$

$$\therefore I_m = \frac{V_m}{R}$$

Eqn-(2) becomes $i = I_m \sin \omega t$ ——— (iii)

In terms of r.m.s values $\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \times R$

$$\Rightarrow \boxed{V = V_R = IR}$$

(i) Phase angle :-

It is ~~clear~~ clear from eqn-(1) & (3) that the applied voltage and the circuit current are in phase with each other i.e they pass through their zero values at the same instant and attain their positive and negative peaks at the same instant.

(ii) Power :-

In any circuit, electric power consumed at any instant is the product of voltage and current at that instant i.e.

Instantaneous power $p = vi = (V_m \sin \omega t)(I_m \sin \omega t)$

$$= V_m I_m \sin^2 \omega t$$

$$= V_m I_m \left[\frac{1 - \cos 2\omega t}{2}\right]$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Thus power consists of two parts, a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$.

Since power is a scalar quantity average power over a complete cycle is to be considered.

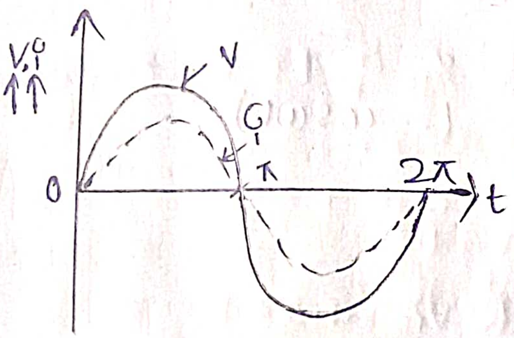
$$\therefore \text{Power consumed } P = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \, d(\omega t) + \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \cos 2\omega t \, d(\omega t)$$

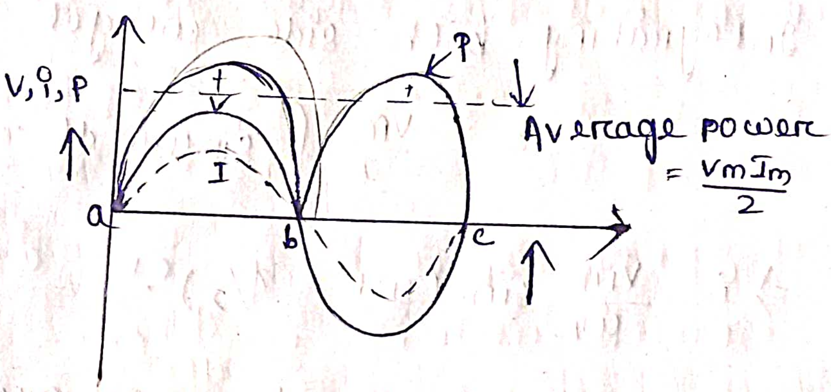
$$\Rightarrow P = \frac{V_m I_m}{2} + 0 = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\Rightarrow P = V_R I = VI$$

where $V = V_R$ = rms value of the applied voltage.
I = rms value of the circuit current

$$\boxed{P = VI}$$





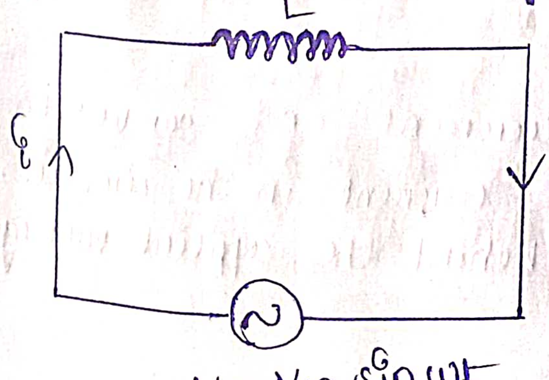
Average power $P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$
$= VI$

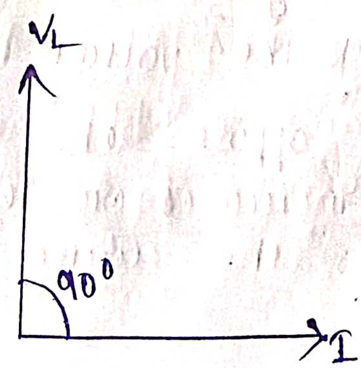
A.C circuit containing pure Inductance Only

When an alternating current flows through a pure ~~inductance~~ inductive coil a back emf ($= L\frac{di}{dt}$) is induced due to the inductance of the coil. This back emf at every instant opposes the change in current through the coil.

Since there is no ohmic drop, the applied voltage has to overcome the back emf only.

$\therefore$ Applied alternating voltage = Back emf





consider an alternating voltage applied to a pure inductance of L henory. Let the equation of the applied alternating voltage be.

$$V = V_m \sin\omega t \quad \text{——— (1)}$$

clearly $V_m \sin\omega t = L\frac{di}{dt}$

$$\Rightarrow di = \frac{V_m}{L} \sin\omega t \, dt.$$

Integrating both sides we get

$$i = \frac{V_m}{L}\int \sin\omega t \, dt = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$\Rightarrow i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2) \quad \text{——— (2)}$$

The value of i will be maximum (i.e Im) when $\sin(\omega t - \pi/2)$ is unity.

$$\therefore I_m = \frac{V_m}{\omega L}$$

Substituting the value of $\frac{V_m}{\omega L} = I_m$ in eq^n ——— (2)

$$i = I_m \sin(\omega t - \pi/2) \quad \text{——— (3)}$$

$$\boxed{X_L = \omega L}$$

i) Phase angle :-

It is clear from eq^n — (1) & (3) that current lags behind the voltage by $\frac{\pi}{2}$ radians or 90°. Hence in a pure inductance current lags the voltage by 90°.

Inductance opposes the change in current and serves to delay the increases or decreases of current in the circuit. This causes the current to lag behind the applied voltage.

(ii) Inductive reactance :-

Inductance not only causes the current to lag behind the voltage but it also limits the magnitude of current in the circuit.

$$I_m = \frac{V_m}{\omega L}$$

$$\Rightarrow \frac{V_m}{I_m} = \omega L$$

ωL is the opposition offered by inductance to current flow. The quantity ωL is called the inductive reactance X_L of the coil.

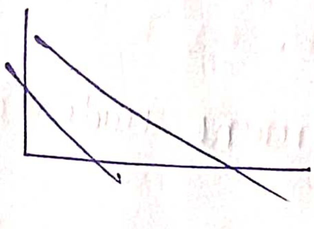
$$I_m = \frac{V_m}{X_L}$$

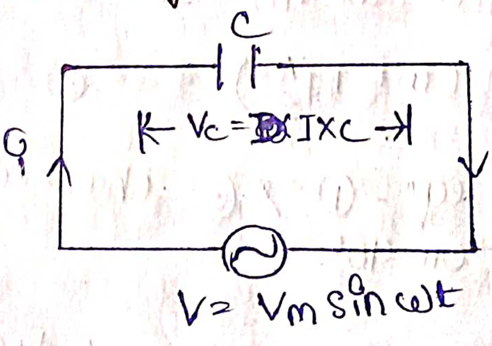
$$\cancel{\Rightarrow \frac{V_m}{I_m} = \omega L}$$

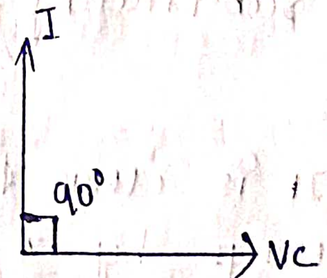
$$\Rightarrow \frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{X_L} \Rightarrow I = \frac{V_L}{X_L} \quad (V = V_L)$$

$$\Rightarrow \boxed{X_L = \omega L = 2\pi f L}$$

AC circuit containing capacitance only







When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electrons move to and fro around the circuit, ~~connecting~~ connecting the plates, thus constituting alternating current.

consider an alternating voltage applied to a capacitor of capacitance c farad.

Let the equation of the applied ~~to a capacitor of capa~~ voltage be

$$v = V_m \sin \omega t \quad \text{———} \quad (1)$$

As a result of this alternating voltage, alternating current will flow through the circuit.

Let at any instant i be the current & q be the charge on the plates.

charge on capacitor $q = cv = cV_m \sin \omega t$

$\therefore$ circuit current $i = \frac{d}{dt}(q) = \frac{d}{dt}(cV_m \sin \omega t)$

$= \omega C V_m \cos \omega t$.

$$\Rightarrow i = \omega C V_m \sin(\omega t + \frac{\pi}{2}) \quad \text{———} \quad (2)$$

The value of i will be maximum (i.e I_m) when $\sin(\omega t + \frac{\pi}{2})$ is unity.

$$I_m = \omega C V_m$$

Substituting the value $\omega C V_m = I_m$ in eqn - (2)

$$i = I_m \sin(\omega t + \frac{\pi}{2}) \quad \text{———} \quad (3)$$

It is clear ~~for~~ from eqn - (1) & (3) that current leads the voltage by $\frac{\pi}{2}$ radian or 90°.

Hence in a pure capacitance, current leads the voltage by 90°. capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across the capacitor. This causes the voltage to lag behind the current.

Capacitive reactance :-

capacitance not only causes the voltage to lag behind current but it also limits the magnitude of current in the circuit.

$$I_m = \omega C V_m$$

$$\Rightarrow \frac{V_m}{I_m} = \frac{1}{\omega C}$$

If V_c & I are the r.m.s values then

$$\frac{V_m}{I_m} = \frac{V_c}{I} = \frac{1}{\omega C}$$

Clearly the opposition offered by capacitance to current flow is $\frac{1}{\omega C}$. This quantity $1/\omega C$ is called the capacitive reactance X_C of the capacitor.

$$I = \frac{V_C}{X_C}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Power :-

Instantaneous power

$$P = Vi = V_m \sin\omega t \times I_m \sin(\omega t + \tfrac{\pi}{2})$$

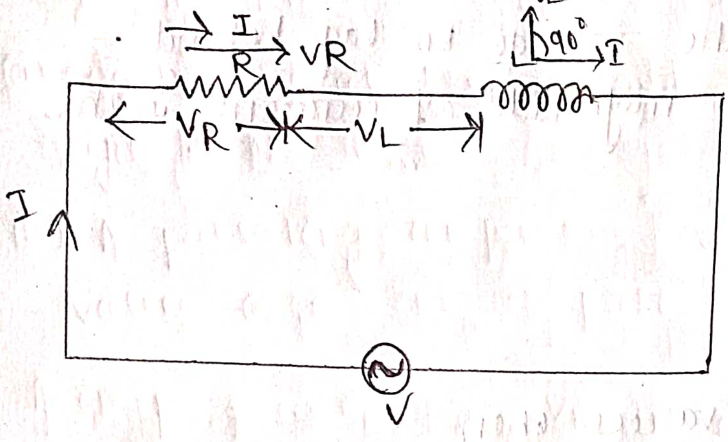
$$= V_m I_m \sin\omega t \cos\omega t$$

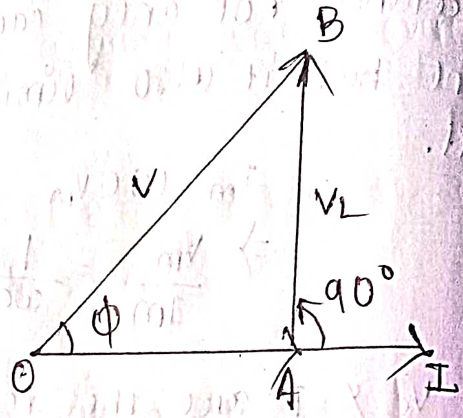
$$\Rightarrow P = \frac{V_m I_m}{2} \sin 2\omega t.$$

Average power, P = Average of P over one cycle = 0

Hence power absorbed in a pure capacitance is zero.

R-L series A.C circuit :-





Let V = rms value of the applied voltage.
I = rms value of the circuit current
V_R = IR where V_R is in phase with I.
$V_L = I X_L$ where V_L leads I by 90°.

Taking current as the reference phasor, the phasor-diagram of the circuit can be drawn.

The voltage drop $V_R = IR$ is in phase with current and is represented in magnitude and direction by the phasor OA.

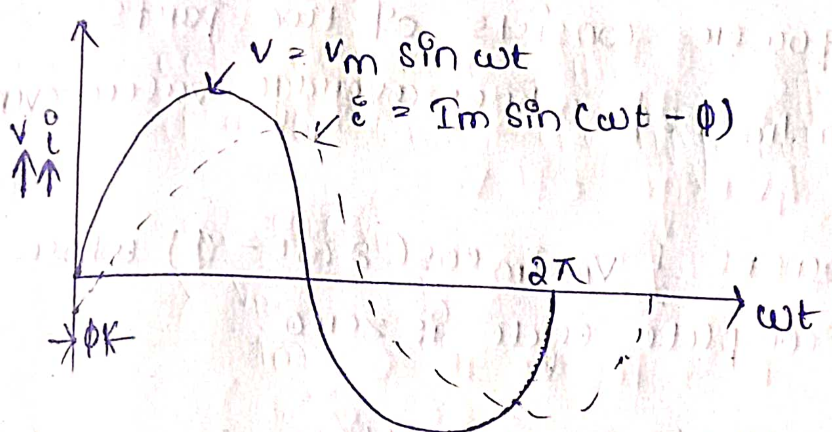
The voltage drop $V_L = I X_L$ leads the current by 90° and is represented in magnitude and direction by the phasor AB.

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (I X_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called impedance of the circuit. It is represented by Z and is measured in ohms (Ω)

$$I = \frac{V}{Z} \quad \text{when } Z = \sqrt{R^2 + X_L^2}$$



It is clear from the phasor diagram that circuit current I lags behind the applied voltage V by 90°.

$$\tan\phi = \frac{V_L}{V_R} = \frac{I X_L}{I R} = \frac{X_L}{R}$$

$V = V_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$ where $I_m = \frac{V_m}{Z}$

The angle of lag (i.e ϕ) is greater than 0° but less than 90°. ~~The~~

It is determined by the ratio of ~~inductance~~ inductive reactance to resistance.

The greater the value of this ratio, the greater will be the phase angle ϕ & vice-versa.

Impedance :-

The total opposition offered to the flow of alternating current by a circuit is called impedance Z of the circuit.

$$Z = \sqrt{R^2 + X_L^2} \text{ where } X_L = 2\pi f L.$$

Power :-

Instantaneous power $P = vi = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$$P = \frac{1}{2} V_m I_m [2 \sin\omega t \sin(\omega t - \phi)]$$

$$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$$

$$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

The instantaneous power consists of two parts..

(a) constant part $\frac{1}{2} V_m I_m \cos\phi$ whose average value over a cycle is the same

(b) A pulsating component $-\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ whose average value over one complete cycle is zero.

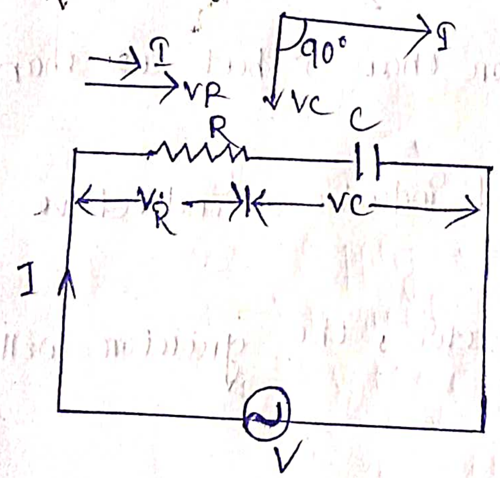
Average power $P = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$

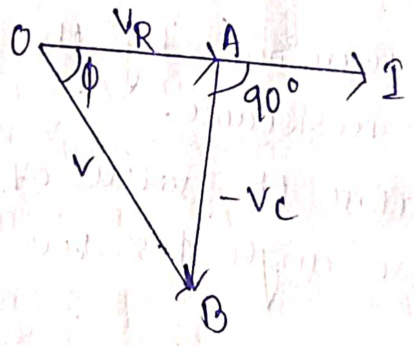
$\Rightarrow P = VI\cos\phi$

$\cos\phi = \frac{IR}{IZ} = \frac{R}{Z}$

$P = VI\cos\phi = (IZ)\, I\, (R/Z) = I^2 R$

R-C series A.C circuit :-





The above figure shows a resistance of R ohms connected in series with a capacitor of C farad

Let $V \rightarrow$ rms value of applied voltage

$I \rightarrow$ rms value of circuit current

$V_R \rightarrow IR \rightarrow$ where V_R is in phase with I

$V_C \rightarrow I X_C \rightarrow$ where V_C lags I by 90°.

Taking current as the reference phasor, the phasor diagram of the circuit can be drawn.

The voltage drop $[V_R = IR]$ is in phase with current and is represented in magnitude and direction by the phasor OA.

The voltage drop $V_C = IX_C$ lags behind the current by $90°$ and is represented in magnitude and direction by the phasor OA.

~~The voltage drop~~

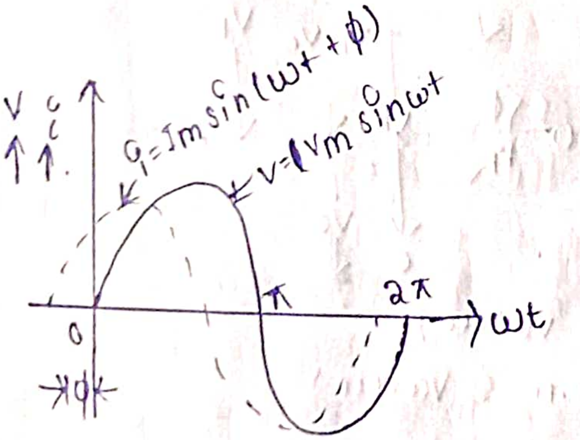
The applied voltage V is the phasor sum of these two drops.

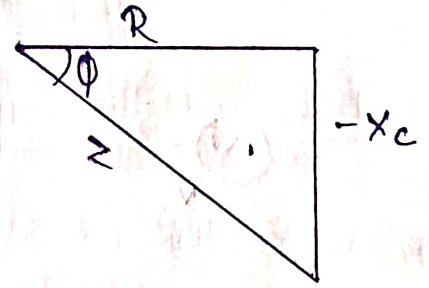
$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + X_C^2}}$$

The quantity $\sqrt{R^2 + X_C^2}$ offers opposition to current flow and is called impedance of the circuit.

$$I = \frac{V}{Z} \quad \text{where } Z = \sqrt{R^2 + X_C^2}$$





Impedance triangle

Phase angle :-

It is clear from the phasor diagram that circuit current I leads the applied voltage V by $\phi°$.

$$\tan\phi = \frac{-V_C}{V_R} = \frac{-IX_C}{IR} = \frac{-X_C}{R}$$

Since current is taken as the reference phasor, negative phase angle implies that voltage lags behind the current.

If the applied voltage is $v = V_m \sin \omega t$

then circuit current $i = I_m \sin(\omega t + \phi)$

where $I_m = \frac{V_m}{Z}$

Power :-

$V = V_m \sin \omega t \qquad i = I_m \sin(\omega t + \phi)$

Average power P = Average of vi

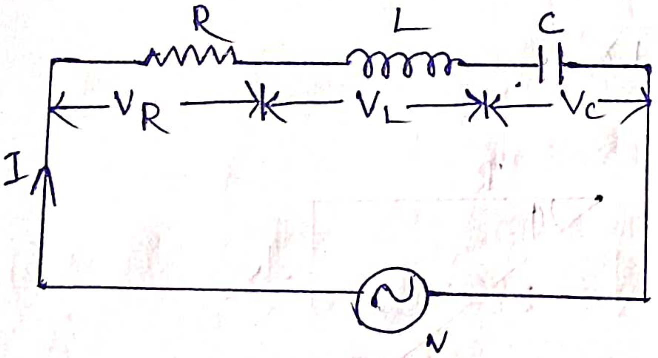
$$\boxed{P = VI\cos\phi}$$

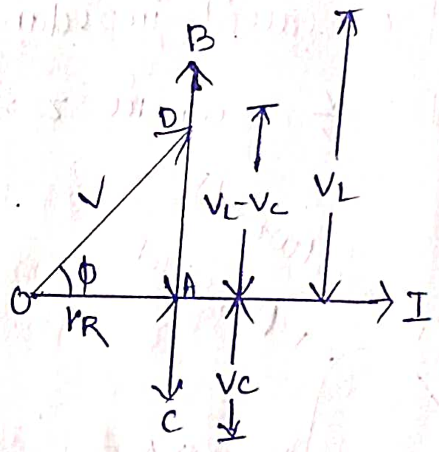
P = Power in R + Power in C

$= I^2R + 0 = IR \times I = IR \times \frac{V}{Z}$

$= VI \times \frac{R}{Z} = VI\cos\phi$

R-L-C Series A.C circuit :-





$V \rightarrow$ supply voltage (r.m.s value)

$I \rightarrow$ resulting current (r.m.s value)

$V_R \rightarrow$ voltage drop across resistor

$V_R = IR$ (V_R is in phase with I)

$V_L \rightarrow$ voltage drop across inductor

$V_L = I X_L$ (V_L leads I by 90°)

$V_C \rightarrow$ voltage drop across capacitor

$V_C = I X_C$ (V_C lags I by 90°)

current is taken as the reference phasor. OA represents V_R, AB represents V_L & AC represents V_C. It may be seen that V_L is in phase opposition to V_C. It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant.

For the case considered $V_L > V_C$ so that net voltage drop across L-C combination is $V_L - V_C$ and is represented by AD.

Therefore the applied voltage V is the phasor sum of V_R & $V_L - V_C$ and is represented by OD.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\therefore I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The quantity $\sqrt{R^2 + (X_L - X_C)^2}$ offers opposition to current flow and is called impedance of the circuit.

circuit power factor $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

Power consumed $= \boxed{P = VI\cos\phi}$

or $P = I^2 R$

We have seen that the impedance of a R-L-C series circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(i) When $X_L - X_C$ is positive (i.e $X_L > X_C$), phase angle ϕ is positive and the circuit will be inductive. In such case the circuit current I will lag behind the applied voltage V by ϕ.

(ii) When $X_L - X_C$ is negative (i.e $X_C > X_L$), phase angle ϕ is nagative and the circuit ~~will be~~ is capacitive. The circuit current I ~~and app~~ leads the applied voltage V by ϕ.

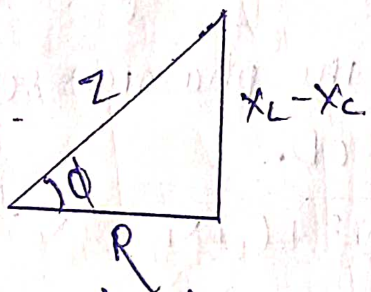
(iii) When $X_L - X_C$ is zero (i.e $X_C = X_L$) the circuit is purely resistive.

The circuit current I and applied voltage V will be in phase i.e. $\phi = 0°$.

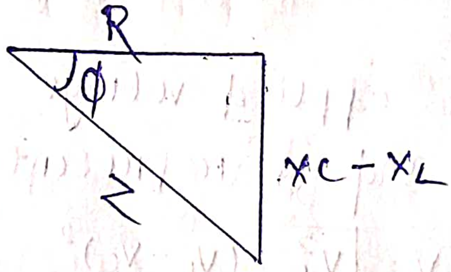
The circuit will then have unity power factor.

If $V = V_m \sin \omega t$

$i = I_m \sin(\omega t \pm \phi)$ where $I_m = V_m/Z$



$(X_L > X_C)$



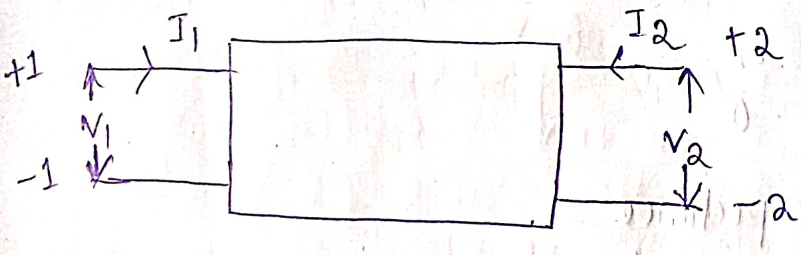
$(X_C > X_L)$

Two Port Network Analysis   Date - 16.12.2022

two port Network

1) Z - parameters (impedance/ open circuit)
2) Y - Parameters (Admitance/ Short circuit)
3) H - Parameters (Hybrid)

1) Z - Parameters or impedance parameters or open circuit parameters :-

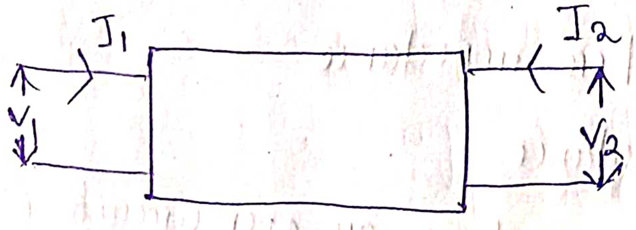


$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (2)$$

$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Step-1



Open circuiting the output port

$$I_2 = 0$$

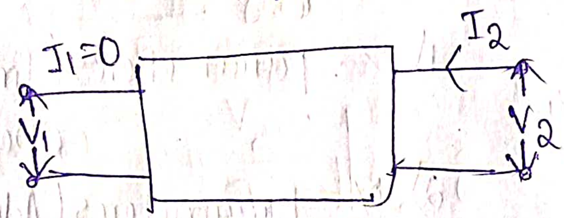
$$V_1 = Z_{11} I_1$$
$$V_2 = Z_{21} I_1$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2 = 0} \ \Omega$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2 = 0} \ \Omega$$

Step - 2

Open circuiting the port (1 1')



$\boxed{I_1 = 0}$

$$V_1 = Z_{12} I_2$$

$$V_2 = Z_{22} I_2$$

$$Z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1 = 0} \ \Omega$$

$$Z_{22} = \frac{V_2}{I_2}\Bigg|_{I_1 = 0} \ \Omega$$

Z_{11} = Open circuit input impedance

Z_{21} = Open circuit forward transfer impedence.

Z_{12} = Open circuit reverse transfer impedance.

Z_{22} = Open circuit output impedance.

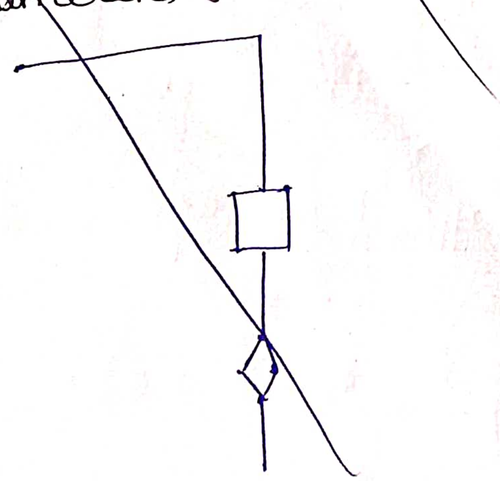
or

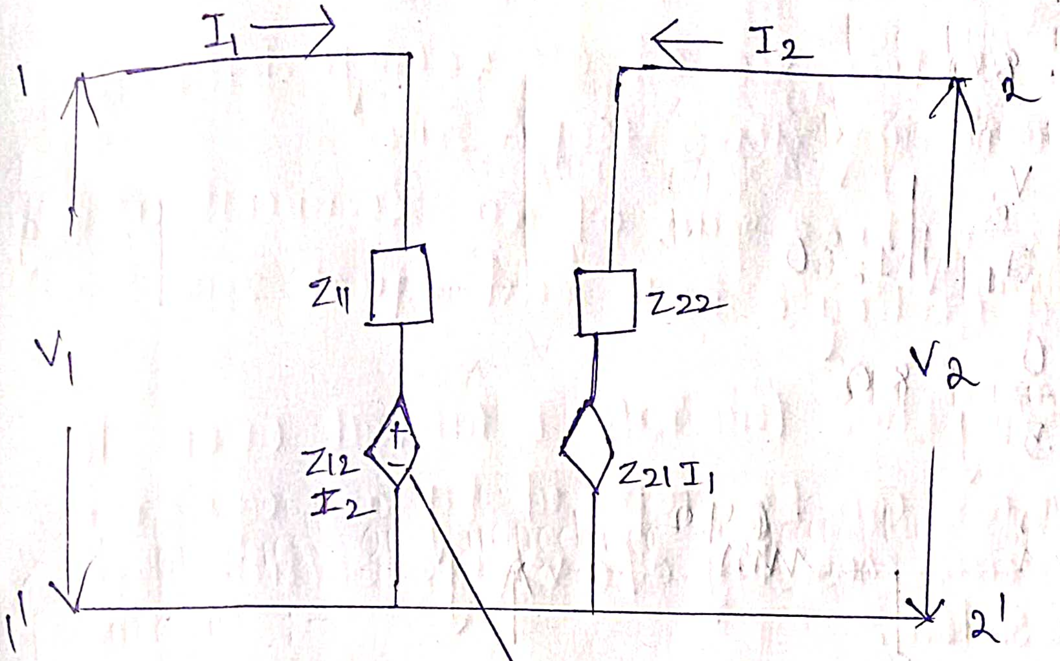
Open circuit driving point impedance.

common name - transfer impedance

driving point impedance

Equivalent circuit for z parameters or open circuit parameters :-

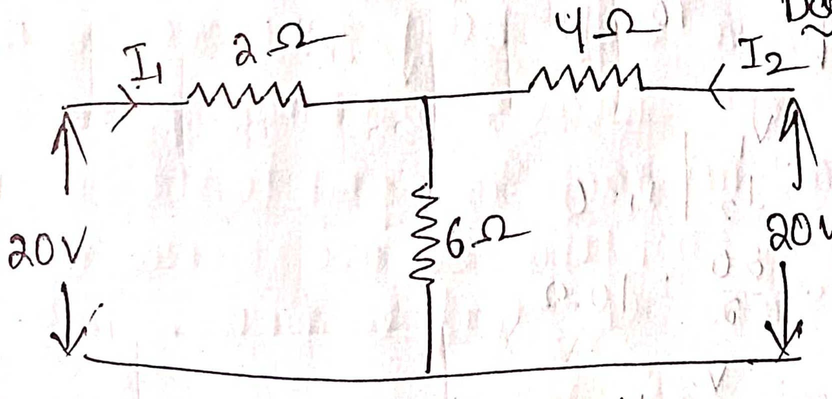




dependent source
current dependent
(voltage source)

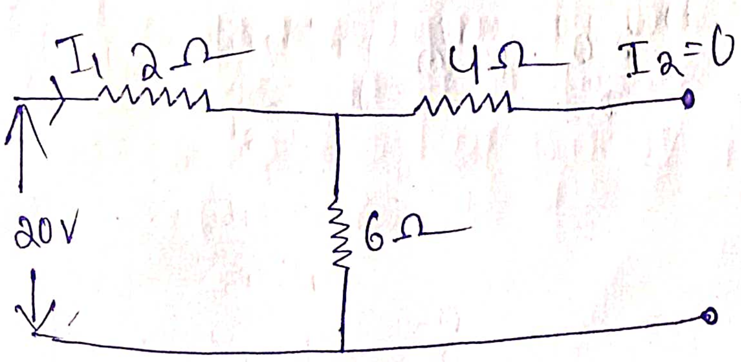
Date - 17.12.2022

Q.



find the Z parameters.

A.



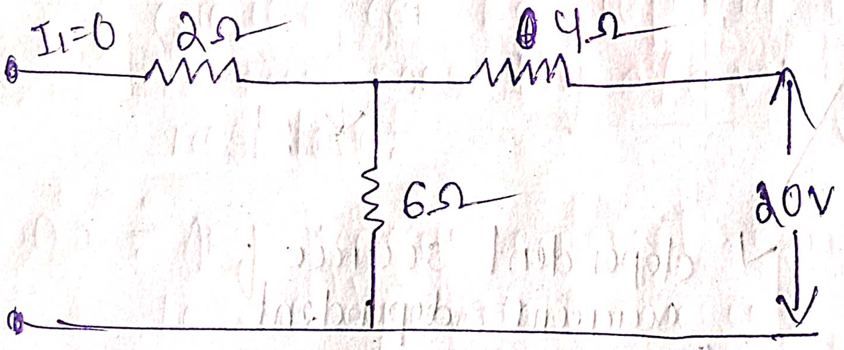
$$I_1 = \frac{V}{8} = \frac{20}{8} = 2.5A$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$= \frac{20}{2.5} = 8\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$= \frac{20}{2.5} = 8\Omega$$



$R_{eq} = 10\Omega$

$V = 20V$

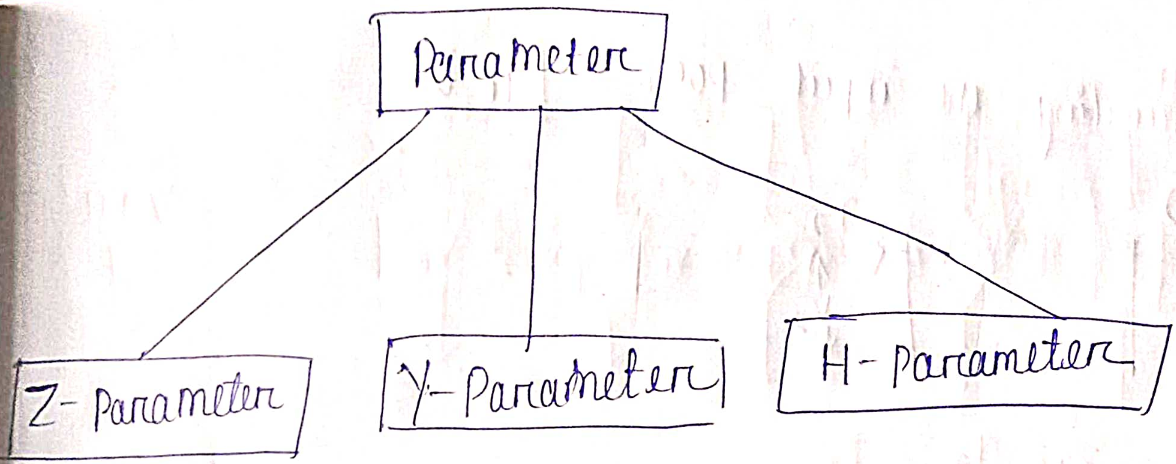
$$I_2 = \frac{V}{R_{eq}} = \frac{20}{10} = 2A$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

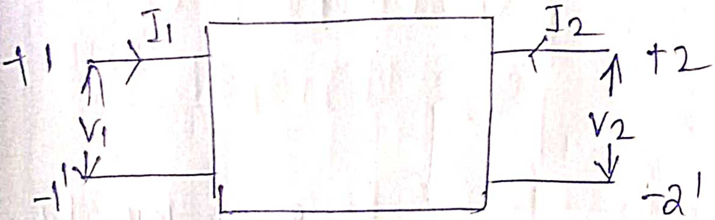
$$= \frac{20}{2} = 10\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$= \frac{20}{2} = 10\Omega$$



① Z-Parameters/ Impedance parameters/open circuit parameters :-



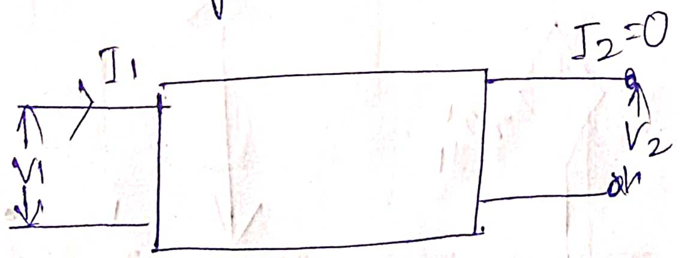
$$\boxed{\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}}$$

$$[V] = [Z]\,[I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Step 1:

Open circuiting the output port $\boxed{I_2 = 0}$



$$V_1 = Z_{11} I_1$$

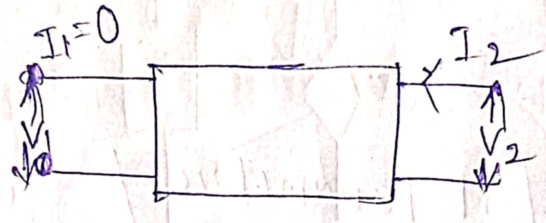
$$V_2 = Z_{21} I_1$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

Step 2 :

Open circuiting the input port $\boxed{I_1=0}$

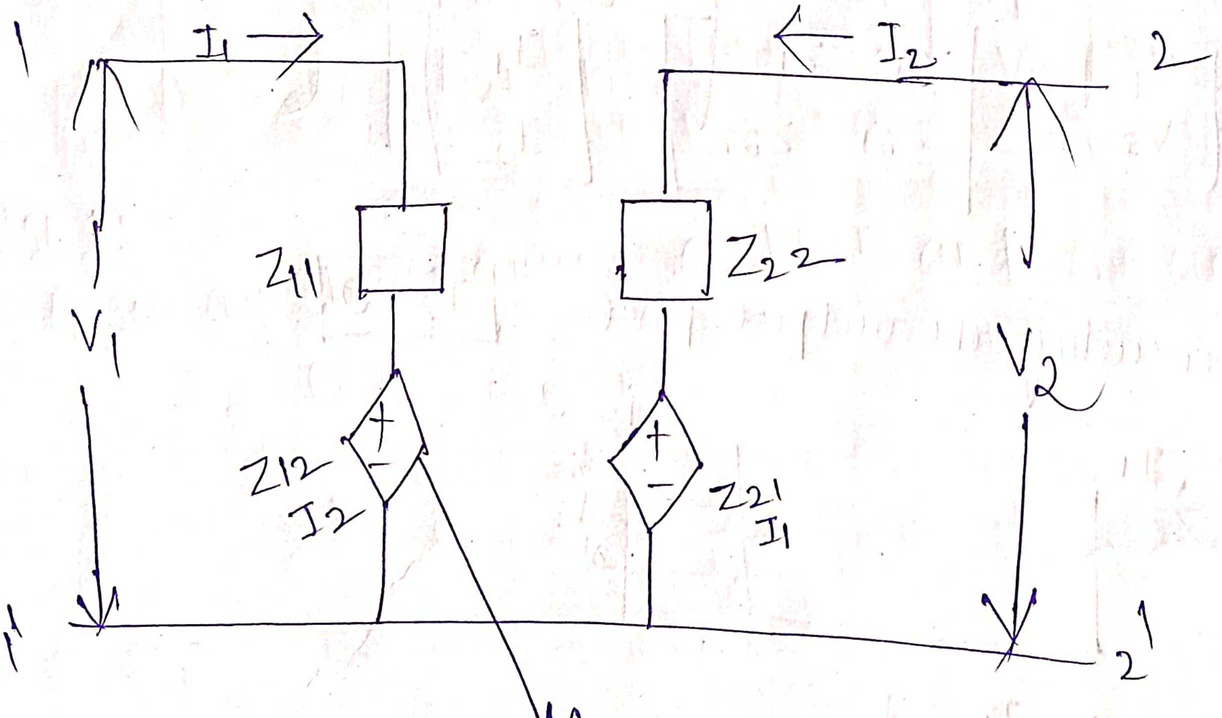


$$V_1 = Z_{12} I_2$$
$$V_2 = Z_{22} I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Equivalent circuit for z parameters or open circuit parameters:-



(dependent source)

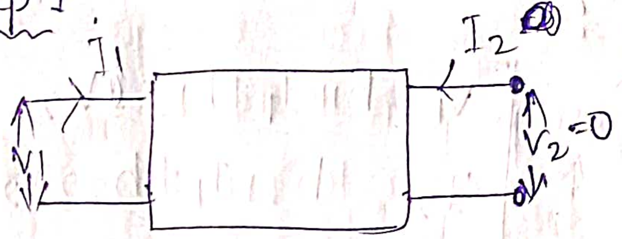
② Y- Parameters / Short circuit parameters / Admittance parameters :-

$$\boxed{\begin{aligned} I_1 &= Y_{11} V_1 + Y_{12} V_2 \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}}$$

Admitance unit
Ω = ohm
$\mho$ = mho

$$[I] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Step 1 :



$$I_1 = Y_{11} V_1$$

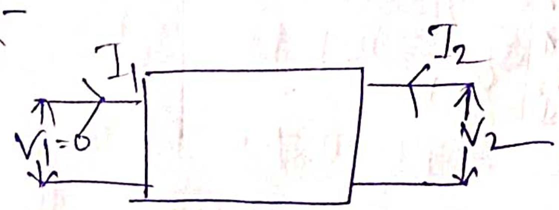
$$I_2 = Y_{21} V_1$$

$$\cancel{Y_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}}$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

Step 2:-

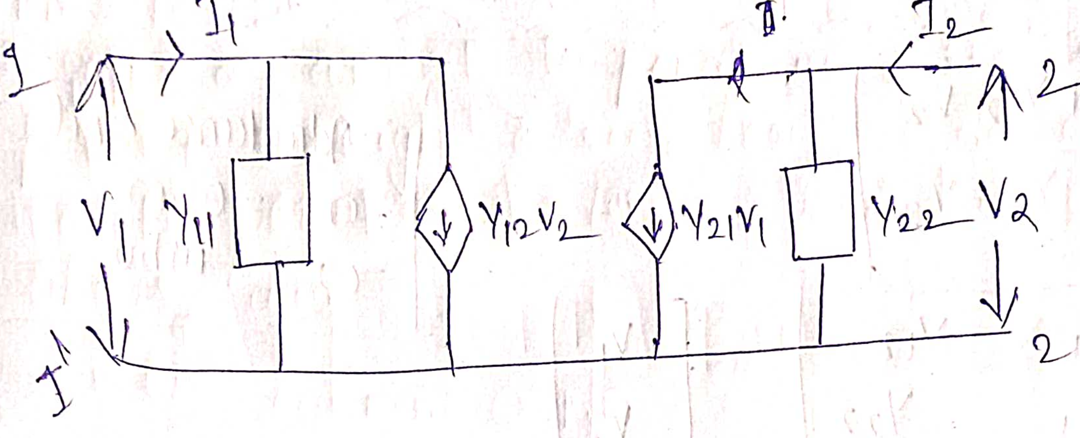


$$I_1 = Y_{12} V_2$$

$$I_2 = Y_{22} V_2$$

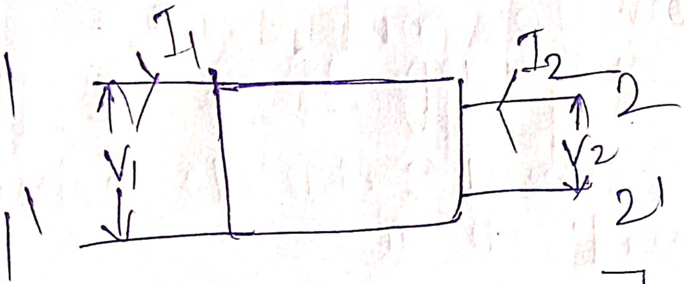
$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$



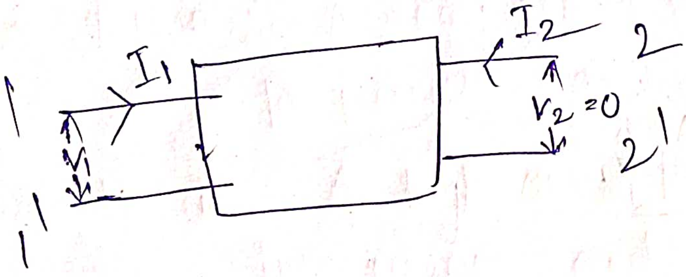
③ H-Parameters / Hybrid parameters :-

$$\boxed{\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}}$$



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Step 1:
Assuming short circuit condition and the output
$V_2 = 0$



$$V_1 = h_{11} I_1$$

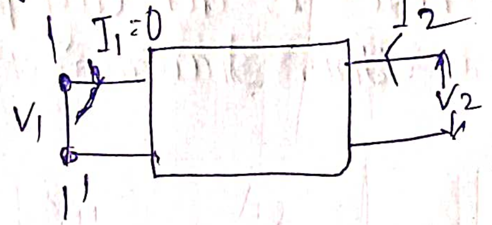
$$I_2 = h_{21} I_1$$

$$h_{11} = \frac{V_1}{I_1} \bigg|_{V_2 = 0}$$

$$h_{21} = \frac{I_2}{I_1} \bigg|_{V_2 = 0}$$

Step 2:

Assuming open circuited condition for input port.



$V_1 = h_{12} V_2$

$I_2 = h_{22} V_2$

$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$

Step 3: