GOVERNMENT POLYTECHNIC,DHENKANAL

LECTURE NOTES

ON

CONTROL SYSTEM ENGINEERING

6th SEMESTER

PREPARED BY

TUKURAJ SOREN

CH-01

[Fundamental Of Control System]

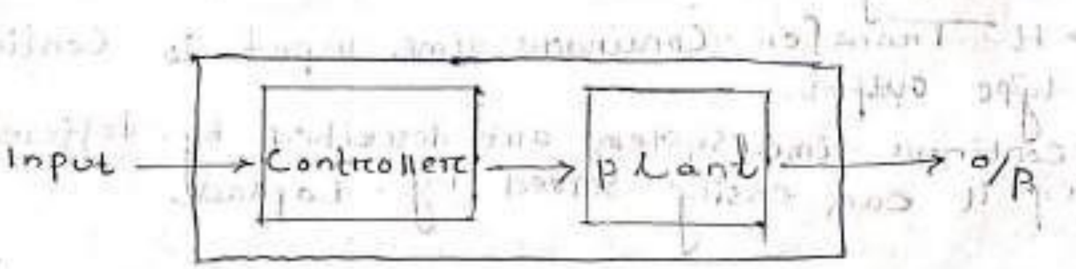
System :→
Arrangement or Combination of different physical Component that are connect together to form a entire unit to achieve a certain objective is called System.

Control :→
The meaning of Control to regulate, direct or command a system, so that desired objective
e.x - Speed Control of a dc motor can control by controlling the Input dc voltage.

Plant :→
It is defined as the part of the system which is to be controlled or regulate it is also called process.

Controller :→
The element of the system itself may be external to the system. It controls the plant or process.

Control system :→
The system which physical element linked in such a way that so as regulate, direct or command itself to obtain a certain objective. It must have input, output, controller and plant.



Control system

Classification of control system :→

It is three types.

1. Natural Control System
2. Man-made Control System
3. Combinational Control System

Natural Control System :→

The system inside a human being or biological system are known as Natural Control System.

Ex- Solar system, Planetary atmosphere Circulation system.

Man-made Control System :→

Some Control system which are designed or developed by men are called man-made Control system

Ex- Automobile system.

Combinational Control System :→

Combinational of natural Control system and manmade Control system is called Combinational Control system.

Ex- Driver driving a car

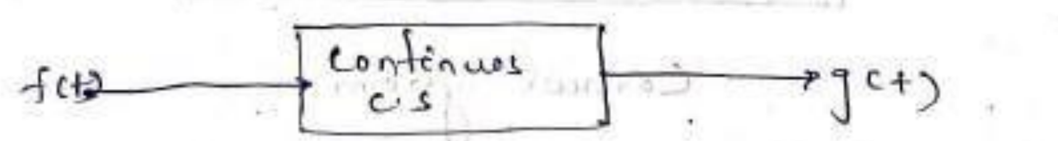
Continuous and discriptive type Control system :→

Continuous type Control System →

If all the system variable of Control system are function of time it is called Continuous control system.

→ It Transfer Continuous time input to Continuous type output.

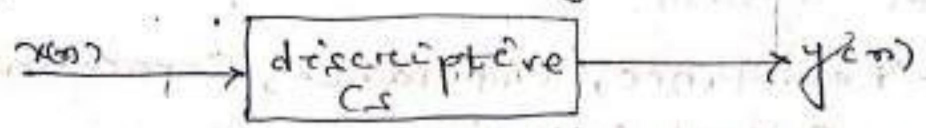
→ contenous time system are described by differential eqn it can easily solved by Laplace



(sinusodial signal)

Discriptive Control system:-

If one or more system variable of a Control System are known at certain it is called Discriptive time control system.



e.x - Microprocessor or computer are example of discriptive time Control system: It converts discriptive time Input to discriptive time output

→ Discriptive time system are described by differential eqn it can easily solved by Z-transform.

SISO

Single Input Single output

- If a Control System has one Input and one output it called SISO Control system.

MIMO

Multi Input multi output

- If a control system has multi enput and multi output it called as MIMO Control System.

Time varying Control system:→

If a parrameter of Control System vary with time the Control system is termed as time varying Control system.

e.x - Space Vehicle leaving (satelite)

Time invarying Control System :-

If parameter of Control system at not vary with time is called time-invarying Control System.

e.x- Resistance, inductance, Capacitance.

Linear Control System :-

A control system is known as linear if it Satisfied the additive property as well as homogenous property

→ It hold the principle of superposition.

Additive

If $x, y \rightarrow$ domain of function 'f'

$$f(x+y) = f(x) + f(y)$$

Homogenous

for a variable 'x' → domain of function, and any Scalar Constant (β)

$$f(\beta x) = \beta f(x)$$

Open Loop Control System :-

→ Open Loop Control system is known as without feedback control system

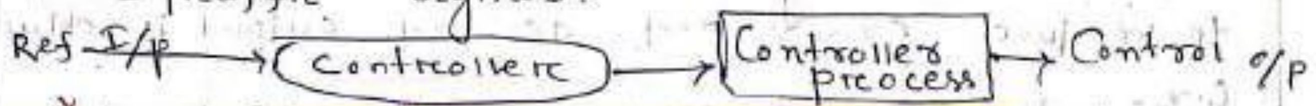
* The open loop Control System the control action is independent of desired output.

* In this System output is not Compare with referece input.

→ The Component of open loop system are Controller and process.

→ The Controller may be amplifier, filter. depends upon the system.

Ex - Automatic Washing Machine.
- Electric Water Heater
- Traffic Signal.



Advantages of open loop Control system :→

* This system is simple in construction and design.
* This System are economic.
* It required less maintance and not difficult.
* This System are not much trouble in Problem in Stability.
* This System is coinvienent to use output is difficult to measure.

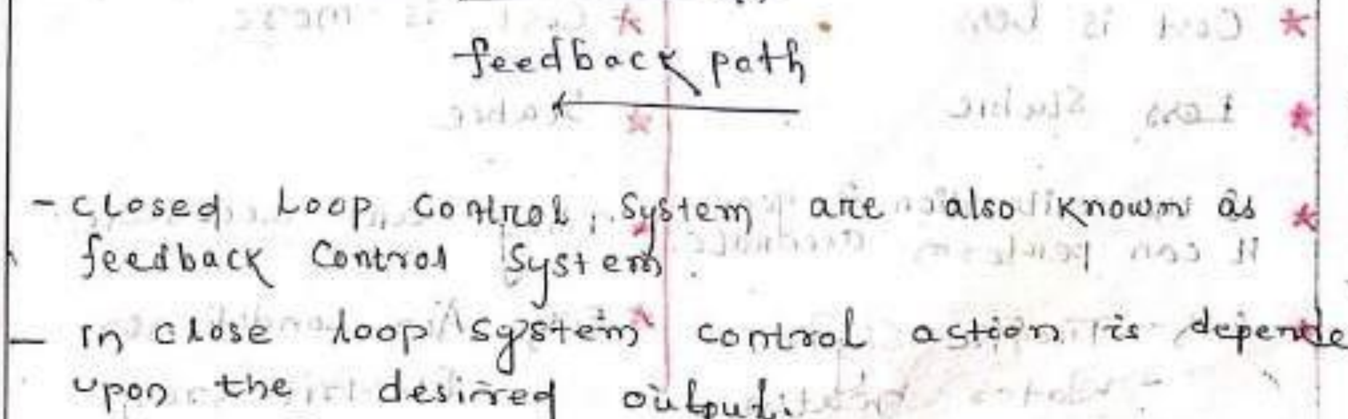
Disadvantages of open loop control system :→

→ This system are not accurate & reliable, because there accuracy is depend on the accurately of calibration.

→ These are Slow

→ Optimatisation is not possible.

Close loop control system :→



- Closed Loop Control System are also known as feedback Control System.
- In close loop system control action is dependent upon the desired output.

- In a closed loop system compare with reference input.
- The error signal is fed to the controller to reduce error and desired output is obtained
e.x.- 1. Air Conditioner
2. Electric Iron

Advantages of closed loop control system :→

- This system are more reliable
* Closed loop system are faster.
* In number of variable can be handle simultaneously
* optimisation is possible
* Accuracy is very high due to correction of any error analysing.

Disadvantages of ~~contr~~ closed Loop Control System

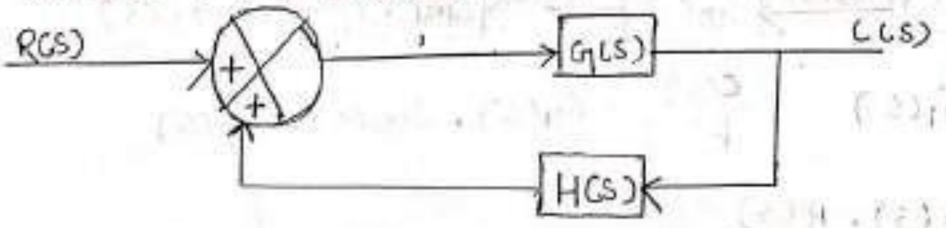
- This system are expensive
- Maintaince are difficult.
- Complicated installisation.

Comparision between open loop & closed loop C S

Open loop system	Closed loop system
* These are not reliable	* These are reliable
* easier to built	* difficult to built
* optimisation not possible.	* Optimisation is possible.
* Cost is less	* Cost is more.
* Less Stable	* Stable
* If the calibration is good it can perform accurate.	* They are accurate -
* e.x - Traffic Signal - Water heater	* E.x - Air Conditioner - Electric Iron

Effect of
Feedback (closed loop)

positive feedback



$= R(S) + H(S)\cdot$

$= \frac{C(s)}{G(s)} = R(S) + H(S)\cdot C(S)$

$\Rightarrow \frac{C(s)}{G(S)} - H(S)\cdot C(S) = R(S)$

$\Rightarrow C(S)\left[\frac{1}{G(S)} - H(S)\right] = R(S)$

$\Rightarrow C(S)\left[\frac{1 - G(S)\cdot H(S)}{G(S)}\right] = R(S)$

gain
$G(S) \Rightarrow \boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)\cdot H(S)}}$

Negative feedback

$= R(S) - H(S)$

$\Rightarrow \frac{C(S)}{G(S)} = R(S) - H(S)\cdot C(S)$

$\Rightarrow \frac{C(S)}{G(S)} + H(S)\cdot C(S) = R(S)$

$\Rightarrow C(S)\left[\frac{1}{G(S)} + H(S)\right] = R(S)$

$\Rightarrow C(S)\left[\frac{1 + G(S)\cdot H(S)}{G(S)}\right] = R(S) \Rightarrow$ gain $G(S)$ $\boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)\cdot H(S)}}$

CH-02

CH-02 [Mathematical Model of a system]

Transfer function :→

→ It is defined as the ratio of the laplace transformation of output response to the laplace transformation of Input response. assuming all the Initial Condition to be Zero

$$R(S) \longrightarrow \boxed{G(S)} \longrightarrow C(S)$$

$$T.F = G(S) = \frac{C(S)}{R(S)} \quad \{ \text{all initial Cond}^n \text{ are Zero} \}$$

Impulse Response :→

→ It has been proved that the Laplace Transformation of an impulse function

→ The transfer function between an input Variable and output variable of a system is defined as the laplace transform of the impulse response.

Properties of Transfer Function:→

1. The transfer function is defined only for a linear time-invariant system. It is not defined for non-linear system.

2. The Transfer function between an input variable and an output variable of a system is defined as the Laplace transform of the impulse response.

3. All initial conditions of the system are set to zero.

4. The transfer function is independent of the input of the system.

5. Stability can be found form characteristics open ratio.

Advantages of Transfer Function:-

↳ A Transfer function is a mathematical model and it gives the gain of the system.

↳ Transfer function helps in the study of stability another of the system.

↳ The Response of the system to any input can be determined very easily.

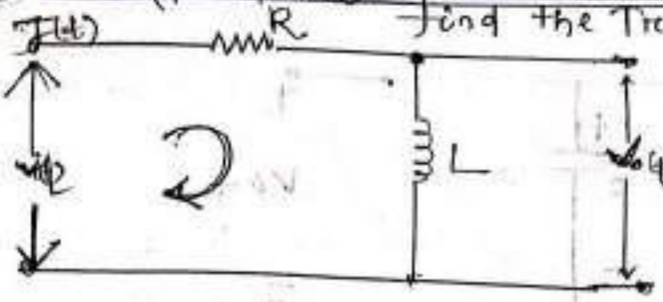
Disadvantages:-

↳ Transfer function does not take into account the initial conditions.

↳ The transfer function can be defined for linear systems only.

↳ No inferences can be drawn about the physical structure of the system.

Find the Transfer function of given network



$\rightarrow V_i(t) - I(t)R - L\frac{di(t)}{dt} = 0$

$\Rightarrow V_i(t) = I(t)R + L\frac{di(t)}{dt}$ ——— (1)

$\Rightarrow V_o(t) = L\frac{di(t)}{dt}$ ——— (2)

Taking Laplace of equation — (1)

$L[V_i(t)] = L[I(t)R] + L\left[L\frac{di(t)}{d(t)}\right]$

$\Rightarrow V_i(s) = R \cdot I(s) + L \cdot s \cdot I(s)$

Taking Laplace of eqn - (2)

$\Rightarrow V_o(t) = L\frac{di(t)}{dt}$

$\Rightarrow L\{V_o(t)\} = L\left[L\frac{di(t)}{dt}\right]$

$V_o(s) = Ls \cdot Is.$

Transfer Function

$= \frac{V_o(s)}{V_i(s)} = \frac{Ls \cdot Is}{R \cdot I(s) + Ls \cdot I(s)}$

$= \frac{Ls \cdot I(s)}{I(s)[R+Ls]}$

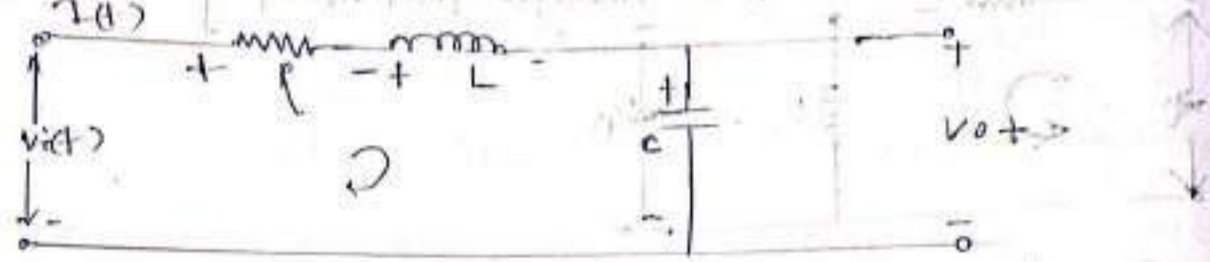
$= \frac{Ls}{R+Ls}$

$L\left[\frac{dx(t)}{d(t)}\right] = s \cdot (s)$

$L\left[\int x(t)\,dt\right] = \frac{1}{s} \cdot x(s)$

$L\left[\frac{d^2x(t)}{d(t)^2}\right] = s^2 X s$

Find the Transfer function of Given network.



$\Rightarrow Vi(t) - I(t)R - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt = 0$

$\Rightarrow Vi(t) = I(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int I(t)\cdot dt = 0$

Taking Laplace

$= L\left[vi(t)\right] = L\left[I(t)\cdot R\right] + L\left[L\cdot\frac{di(t)}{dt}\right] + L\left[\frac{1}{C}\int I(t)\cdot dt\right]$

$V = L\frac{di}{dt}$

$C = \frac{1}{V}\int v(t)dt$

$I = C\frac{dv}{dt}$

$V = \frac{1}{C}\int i(t)\,dt$

$vi(s) = I(s)R + L(s)\cdot I(s) + \frac{1}{Cs}\times I(s)$

$vo(t) = \frac{1}{C}\int I(t)\cdot dt$

Taking Laplace $\Rightarrow Vo(s) = \frac{1}{Cs}\times I(s)$

$$\frac{vo(s)}{vi(s)} = \frac{1/cs \times I(s)}{I(s)R + L(s)\cdot I(s) + \frac{1}{Cs}\times I(s)}$$

$$= \frac{\frac{1}{Cs}\times I(s)}{I(s)\left[R + L(s) + \frac{1}{Cs}\right]}$$

$$= \frac{1/Cs}{\frac{RCs + Ls^2C + 1}{Cs}}$$

$$= \frac{1}{Ls^2C + RCs + 1}$$

Poles & Zeroes of Transfer Function :-

- Poles & Zeros of transfer function are the frequencies for which value of the denominator and numerator of transfer function becomes zero respectively.
- Transfer function of a control system can also represented as.

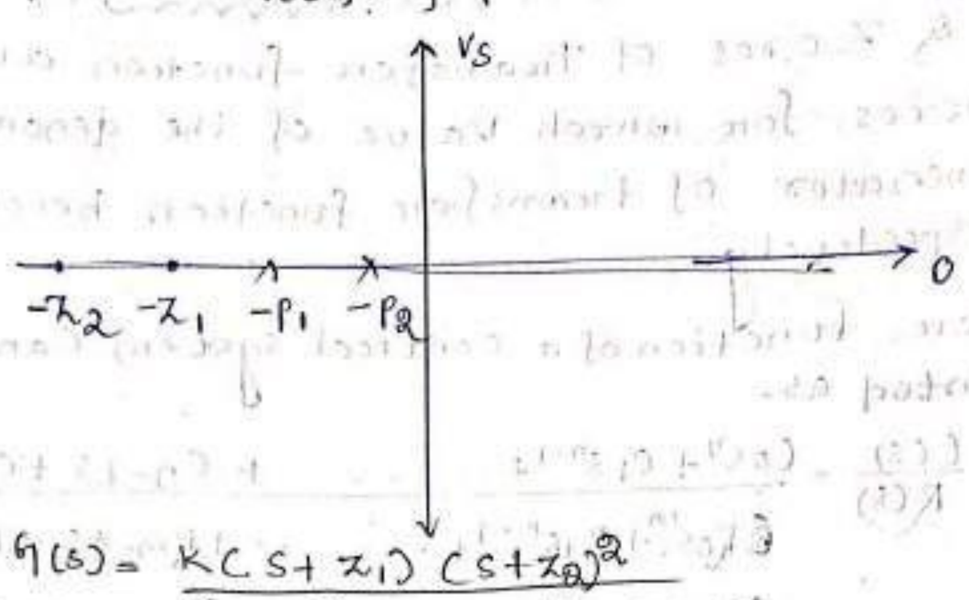
$$G(s) = \frac{C(s)}{R(s)} = \frac{C_0 s^n + C_1 s^{n-1} + \cdots + C_{n-1} s + C_n}{R_0 s^m + R_1 s^{m-1} + \cdots + R_{m-1} s + R_m}$$

$$= K \frac{(s-z_1)(s-z_2)(s-z_3)\cdots(s-z_n)}{(s-p_1)(s-p_2)(s-p_3)\cdots(s-p_m)}$$

Where, K = gain factor of the transfer function.

- $z_1, z_2, z_3 \ldots z_n$ are roots of the numerator polynomial.
- As for these polynomial, the transfer function becomes zero, these roots are called zeros of the transfer function.
- $p_1, p_2, p_3 \ldots p_m$ → The value of T.f becomes infinite Thus the roots of denominator are called the poles of the function.
- Zero of a transfer function are needed are termed circuit zero.
- Pole of a transfer function are defined as the value of magnitude of the transfer function becomes infinity.

Representation of pole and zero :→



$$G(s) = \frac{K(s+z_1)(s+z_2)^2}{(s+p_1)(s+p_2)^2}$$

Mathmatical Modeling of Electrical system :-

↳ Most of the electrical systems can be modelled by three basic elements.

1. Resistor.
2. Capacitor
3. Inductor.

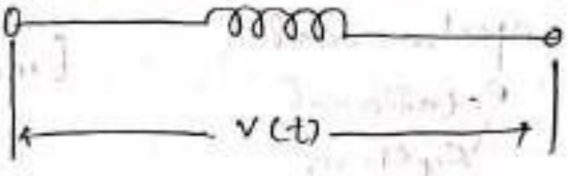
Resistor :→

↳ The mathmatical model is given by the ohm's law relationship.

$$V(t) = i(t)R$$

$$i(t) = \frac{V(t)}{R}$$

Inductor (L):→

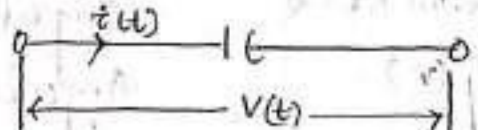


↳ The input, output relations ar given by Faraday's Law.

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int v\,dt$$

Capacitor:→



$$v(t) = \frac{1}{C}\int i\,dt$$

$$i(t) = C\frac{dv}{dt}$$

Analogous System:→

↳ Comparing equations for the mechanical translational system or for the mechanical rotational system and for the series electrical system.

↳ Such systems whose differential equations are of identical form are called analogous system.

Analogous quantities in force - voltage analogy-

Mechanical translational system	Mechanical rotational system	Electrical system.
Force (F)	Torque (T)	Voltage.
Mass (M)	Moment of inertia J	Inductance
Viscous friction coefficient (f)	viscous friction of coefficient (f)	Resistance
Spring stiffness (K)	Torsional spring stiffness K	Capacitance
Displacement (x)	Angular displacement (θ)	Charge
velocity (v)	Angular velocity (ω)	Current

Analogous quantities in force current analogy:

Mechanical translational system	Mechanical Rotational system	Electrical system
Force (F)	Torque T	Current i
Mass (M)	Moment of Inertia J	Capacitance C
Viscous friction Coefficient (f)	Viscous friction Coefficient (f)	Resistance 1/R
Spring stiffness (K)	Torsional Spring (K)	Inductance 1/L
Displacement (x)	Angular displacement θ	→ magnetic flux Linkage
Velocity (v)	Angular Velocity (ω)	Voltage (v)

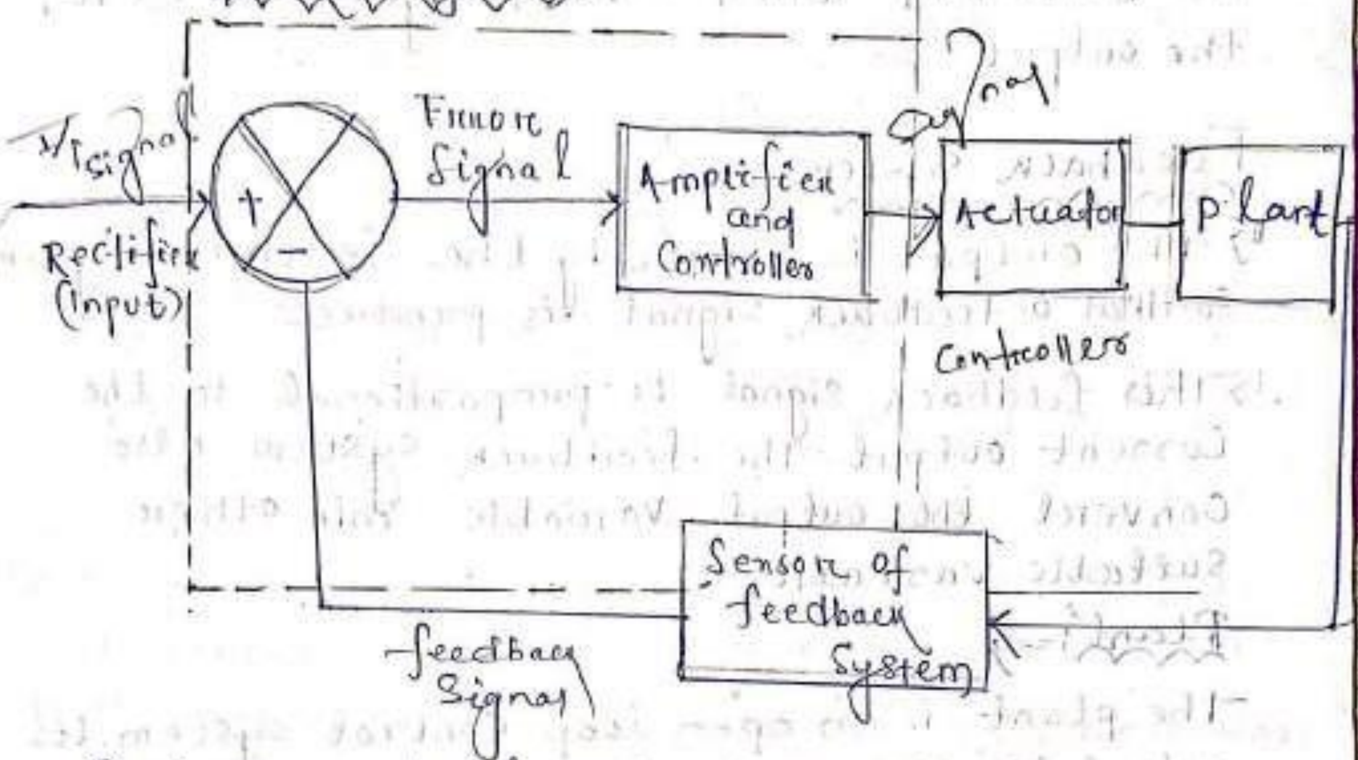
→ The Concept of analogous system is a useful technique for the study of various systems like electrical, mechanical, thermal, hydraulic etc.

Chapter-03 Control system Components :→

↳ Components of Control system :→

↳ Error detector amplifier and Controller actuator plant and sensor of feedback system are the basic Components of automatic Control system.

Block Diagram



Reference Input :→

↳ The reference Input becomes on input Signal Proportional to the desired output automatic Control system.

Error Detector :→

↳ The error detector is a become the reference I/p and feed signal. an error is produced by if it there is a difference

↳ example - synchronous, LVDT.

Actuator:→

↳ The function of the fan is formaly the Controller output and Convert the required from of energy which is accept for the plant output is on the input of the plant.

↳ If there is difference between reference input and feedback signal, the process will be continued when error signal is zero, the output s.

Feedback System:→

↳ The output is simple by the feedback system So that a feedback signal is producess.

↳ This feedback signal is proportional to the Current output. The feedback system also Convert the output Variable into other Suitable Variable.

Plant:→

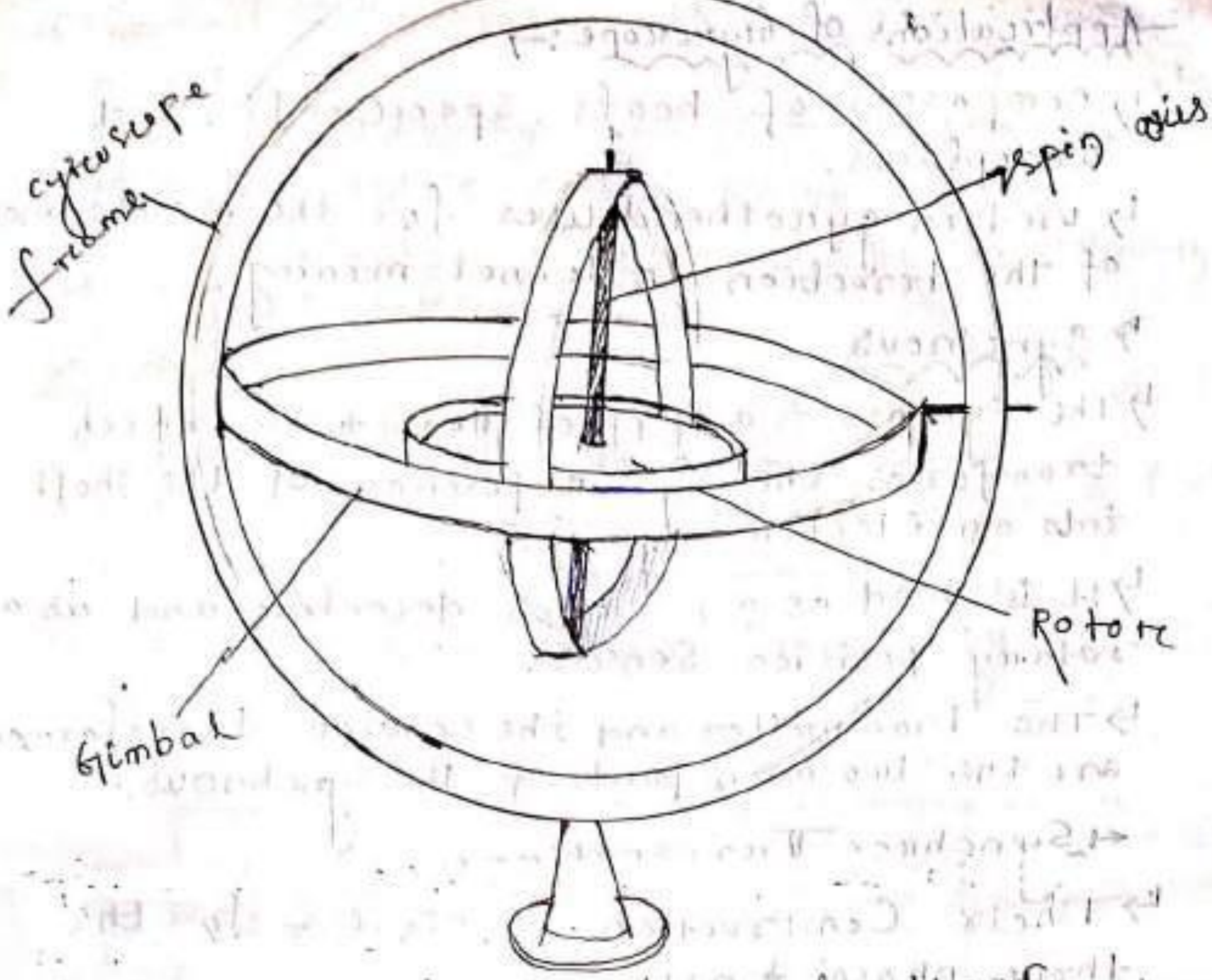
The plant is an open loop Control system, its output is controlled closed loop system.

Gyroscopes:→

↳ A Gyroscope is a device used for measuring or maintaining orientation and angular Velocity

↳ It is a Spinning wheel or disc in which the axis of rotation is free to assume any orientation by itself.

↳ When rotating, the orientation of this axis is unaffected by tilting or rotation of the mounting according to the Conservation of angular momentum.



↳ A gyroscope in operation, Note the freedom of rotation in all three axes.

↳ The rotor will maintain its spin axis direction regardless of the orientation of the outer frame.

★ Parts of Gyroscope :-

- Spin axis
- Gimbal
- Rotor
- Gyroscope frame.

★ Working Principle :→

↳ Gyroscope is based on gravity and is explained as the product of angular momentum which is experienced by the torque on a disc to produce a gyroscopic precession in the spinning wheel.

Applications of Gyroscope:→

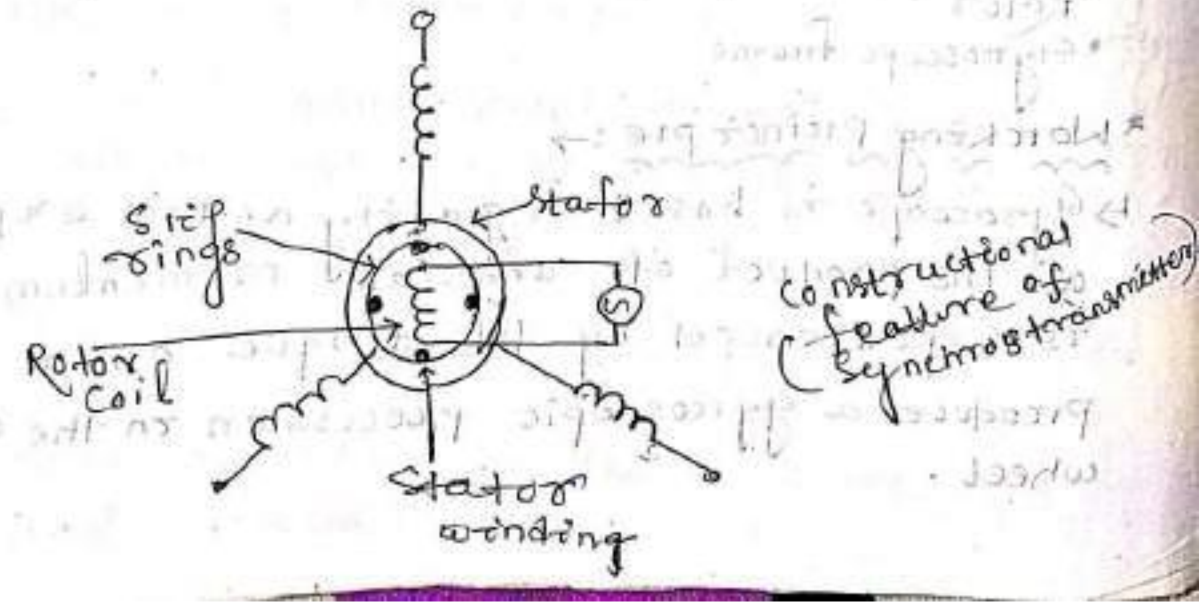
- Compasses of boats, spacecraft, and aeroplanes.
- used in gyrotheodolites for the maintenance of the direction in tunnel mining.

Synchrous

- The synchro is a type of transducer which transforms the angular position of the shaft into an electric signal.
- It is used as an error detector and as a rotary position sensor.
- The Transmitter and the control transformer are the two main parts of the synchrous.

Synchro Transmitters:→

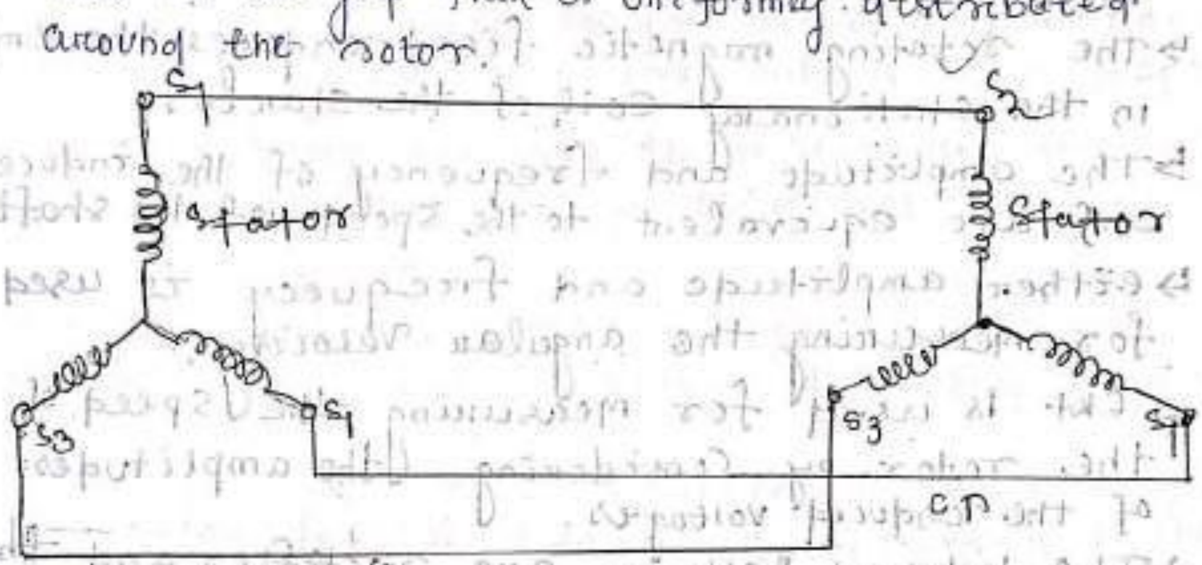
- Their Construction is similar to the three phase alternator.
- The stator of the synchros is made of steel for reducing the iron losses.
- The stator is slotted for housing the three phase windings.
- The axis of the stator winding is kept 120° apart from each other.



↳ The coil of the stator windings are connected in star.
↳ The root of the synchros is a dumbbell in shape and a concentric coil is wound on it.
↳ The AC voltage is applied to the rotor with the help of slip rings.

Synchro Control Transformer:-

A synchro control transformer is used in conjuction with a synchro transmitter to act as error sensor of mechanical components.
↳ except that the rotor is cylindrically shaped so that the air gap flux is uniformly distributed around the rotor.



↳ The essential to a control transformer since its rotor terminals are usually connected to an amplifie
↳ The cylindrical shape of the rotor of the synchro control transformer helps to keep the change of impedance in the rotor coil ckt with the change of angular position.

Application

- For the application of detection of the error
- Used as rotory position sensors by using synchro TX and CT.

Tachometer :→

→ It is an electromechanism device production o/p voltage proportional to its shaft speed tachometers can be used as an analog speed indicator velocity feedback device or signal integrator.

→ It can be AC or DC tachometer.

AC Tachometer :→

→ The AC Tachometer has stationary armature & rotating magnetic field.

→ The commutator & brushes are absent in AC Tachometer.

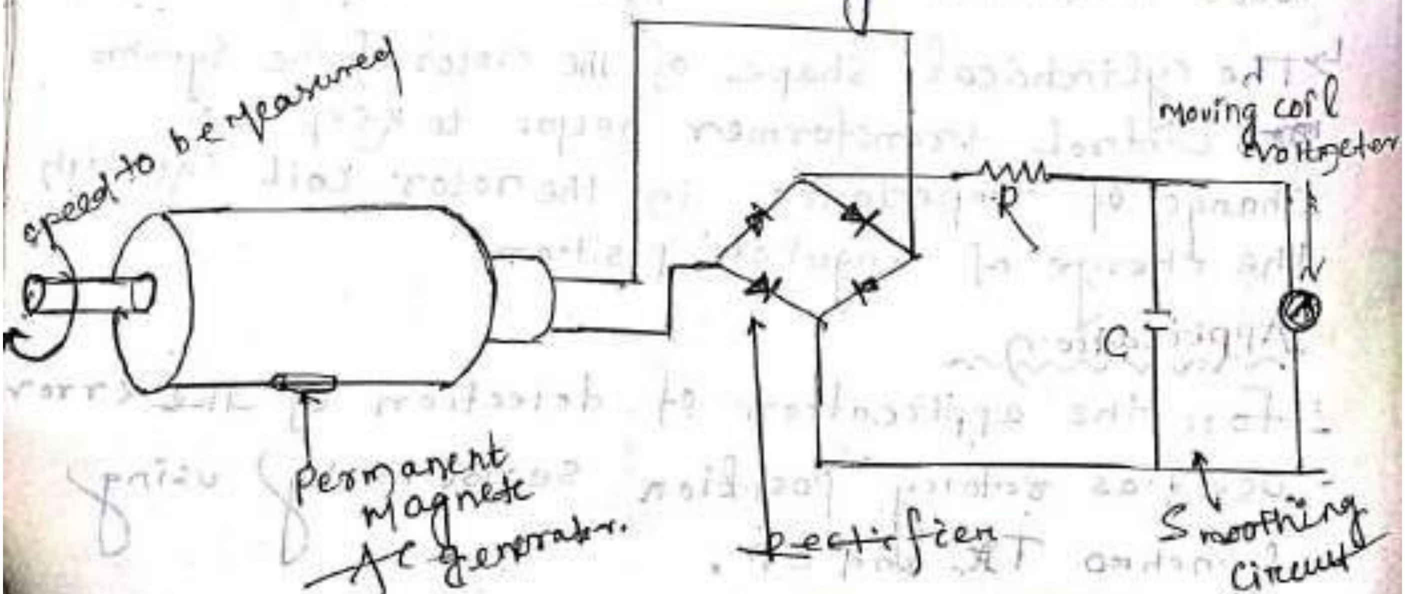
→ The rotating magnetic field induces the emf in the stationary coil of the stator.

→ The amplitude and frequency of the induced emf are equivalent to the speed of the shaft.

→ Either amplitude and frequency is used for measuring the angular velocity.

→ Ckt is used for measuring the speed of the rotor by considering the amplitudes of the induced voltages.

→ The induced voltage are rectifier and then passes to capacitor filter for smoothening the ripples of rectified voltage.



DC Tachometer Generator :→

→ Main parts of the DC Tachometer - permanent magnet, armature, Commutator, brushes, Variable resistor and moving coil voltmeter.

↳ The machine whose speed is to be measured is coupled with the shaft of the DC tachometer.

Working Principle :→

↳ When the closed conductor moves in the magnetic field, emf induced in the conductor.

↳ The magnitude of the induced emf depends on the flux link with the conductor and speed of the shaft.

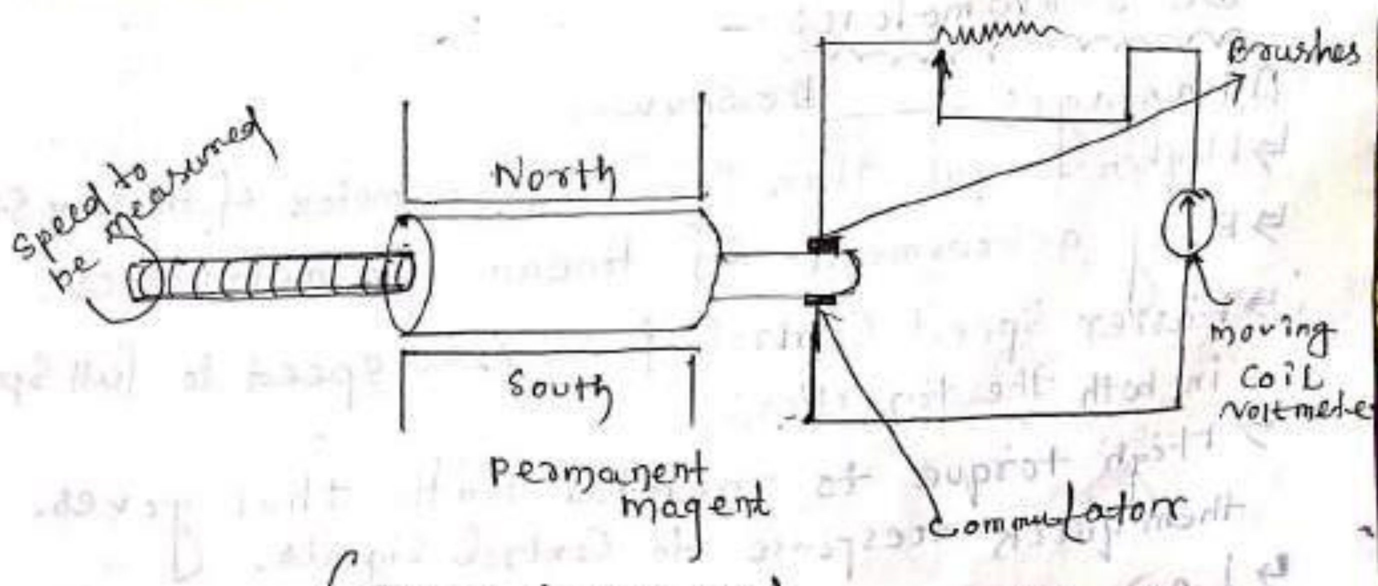
↳ The rotation induces the emf in the coil the magnitude of the induced emf is proportional to the shaft speed.

↳ The commutator converts the alternating current of the armature coil to the direct current with the help of the brushes.

↳ The moving coil voltmeter measures the induced emf.

→ The polarity of induced voltage determines the direction of motion of the shaft.

→ The resistance is connected in series with the voltmeter for controlling the heavy current of the armature.



(DC Tachometer)

Servo Motors:-

- The Control system which are used to Control the position or time derivatives of position, i.e velocity and acceleration are called Servomechanism.
- The motors which are used in automatic Control Systems are called Servomotors.
- The servomotors are used to Convert an electrical signal applied to them into an angular displacement of the shaft.
- In general, a servomotor should have the following features:-
  1. Linear Relationship between speed and of electric Control Signal.
  2. Steady State stability.
  3. Wide range of speed Control.
  4. Linearity of mechanical Characteristics througout the entire speed range.
  5. Low Mechanical and Electrical inertia.
  6. Fast response.

DC Servomotors:-

Advantages:-

- Higher output than from an ac motor of the same size.
- Easy achievment of linear Characteristics.
- Easier speed Control from zero speed to full speed in both the directions.
- High torque to inertia ratio that gives them quick response to Control signals.
- Low electrical & Mechanical time Constants

Dc Servomotors

- Sliding Contact motors with Commutator and brushes
  - Permanent Magnet motors
    - Cylindrical armature motors.
    - Disc armature Motors.
    - Moving Coil motors.
  - Electromagnetic field motors.
    - Armature Controlled motors.
    - Field Controlled motors.
    - Series Motors.
    - Split field motors.
- brushless or Contact less motors with SCR/ transistor Commutator.

Permanate Magnetic Dc motors :-

↳ Permanate Magnetic used in these motor to replace the field winding to produce the required magnetic field.

↳ Permanent magnet motors are Economical for power ratings upto a few Kilowatts

Advantages of Permanent Magnet Motors :-

↳ A Simpler and more reliable motor because the field power supply is not required

↳ Higher Operating efficiency as the motor has no field losses.

↳ Field flux is less affected by temperature rise.

↳ Higher torque / inertia ratio

→ A more Linear Torque / speed Curve.

Disadvantages permanent magnet motor :→

→ The magnets deteriorate with time and demagnetized with large current transients.

→ These drawbacks are eliminated by high-grade magnetic material such as Ceramic magnets & rare earth magnets.

→ The cost of these materials is very high.

Electromagnetic field Motors :→

→ Electromagnetic motors are economical for higher power ratings generally above 1KW.

→ This type of servomotor is similar to a conventional Dc motor constructially :-

- The number of slots and commutator ~~Strength~~ segments is large to improve commutation.
- Compoles and compensating windings are provided to eliminate sparking.
- The diameter to length ratio is kept low to reduce inertia.
- Oversize shafts are employed to withstand the high torque stress.

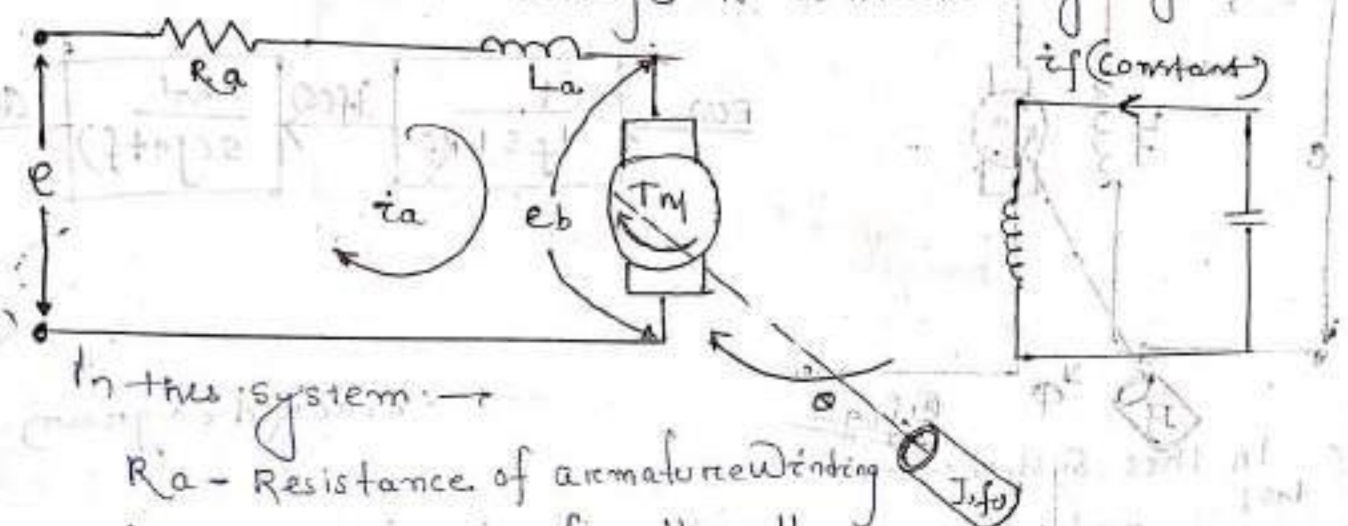
→ In this type of motor, the torque and speed may be controlled by varying the armature current and field current.

→ In armature controlled mode of operation, the field current is held constant and the armature current is varied to control the torque

→ It except for minor differences in constructional features, a dc servomotor is essentially an ordinary dc motor.

Armature-Controlled DC Servomotor: →

- An armature-Controlled DC servomotor is a dc Shunt motor designed to satisfy the requirement of a servomotor. If the field current constant.
  - Speed ∝ armature voltage
  - Torque ∝ armature current.
  - Torque & Speed Can be Controlled by armature Voltage.
  - The armature voltage is Controlled by a Variable resistance.
  - But in Large motors in order to reduce power loss, armature Voltage is Controlled by thyristor.



In this system: →

Ra - Resistance of armature winding

La - Inductance of " "

Ia = Armature Current

if = field Current

e = applied voltage

eb = back emf

TM = Torque developed by motor

θ = Angular displacement

J = equivalent moment of inertia of motor and load referred to motor Shaft.

fo = equivalent Viscous friction Coefficient of motor and load referred to motor Shaft

Field-Controlled DC Servomotor

↳ A field Controlled DC Servomotor is a dc Shunt motor designed to Satisfy the requirement of a Servomotor.

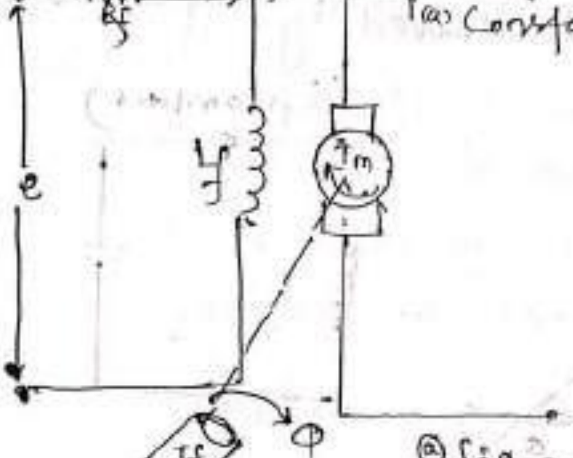
↳ The armature is ~~required~~ Supplied with a Constant Current or voltage.

↳ Armature voltage Constant

↳ Torque ∝ field flux

↳ field Current is proportional to flux

↳ The torque of the motor is Controlled by Controlling the field Current



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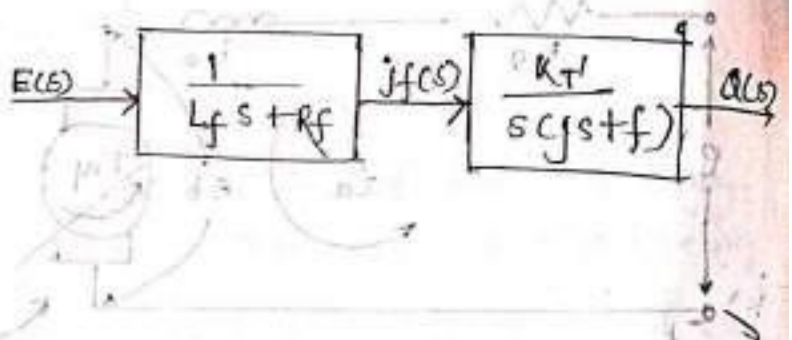


Block diagram

In this system:—

Rf = field winding resistance

Lf = field winding Inductance

e = field Control voltage

If = field Current

TM = torque developed by motor

J = equivalent moment of inertia of motor and load referred to motor Shaft.

θ = Angular displacement

f = equivalent viscous friction Coefficient of motor & load referred to motor Shaft.

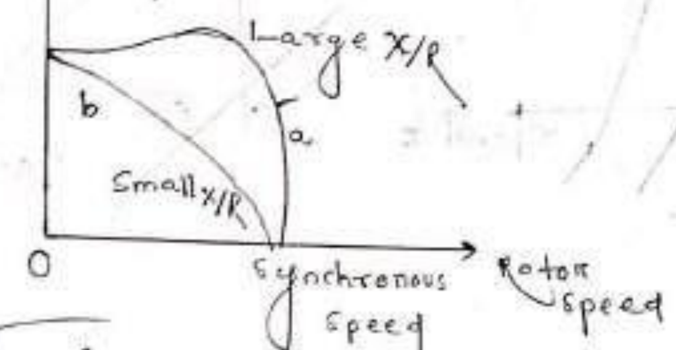
AC Servomotors :→

↳ An ac Servomotor is basically a two phase Induction motor except for Certain Special design features.

↳ A two phase induction motor Consists of two ways from a normal Induction motor.

↳ The rotor of the Servomotor is built with high resistance so that its X/R ratio is small which results in Linear, Speed-torque Characteristics.

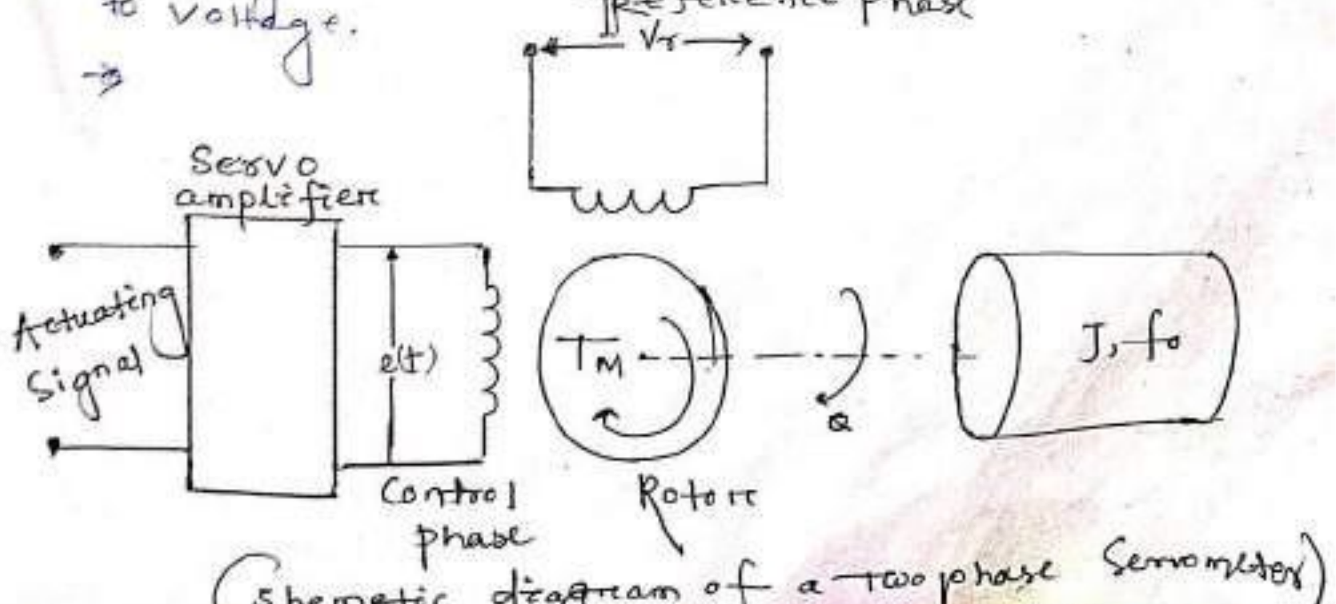
↳ The excitation voltage applied to two stator windings should have a phase difference of 90°



Working of an ac Servomotor :→

↳ One of the phase known as the reference phase is excited by a Constant Voltage, and the other Phases known as the Control phase, is energized by a voltage which is 90° out of phase with respect to voltage.

→

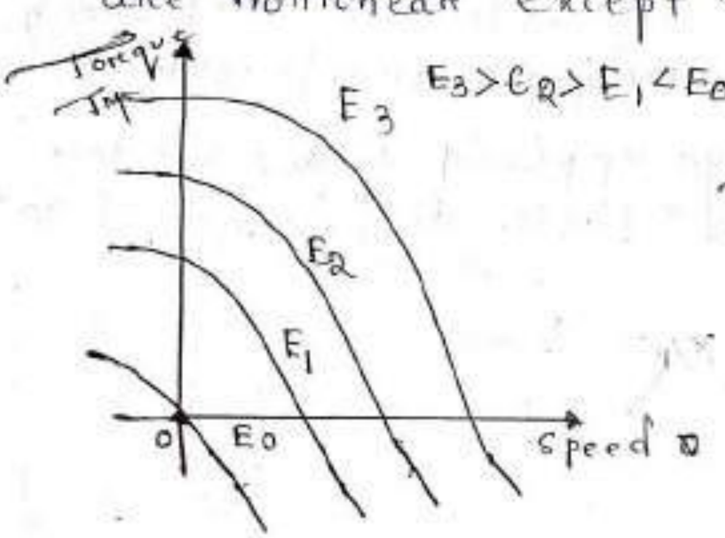


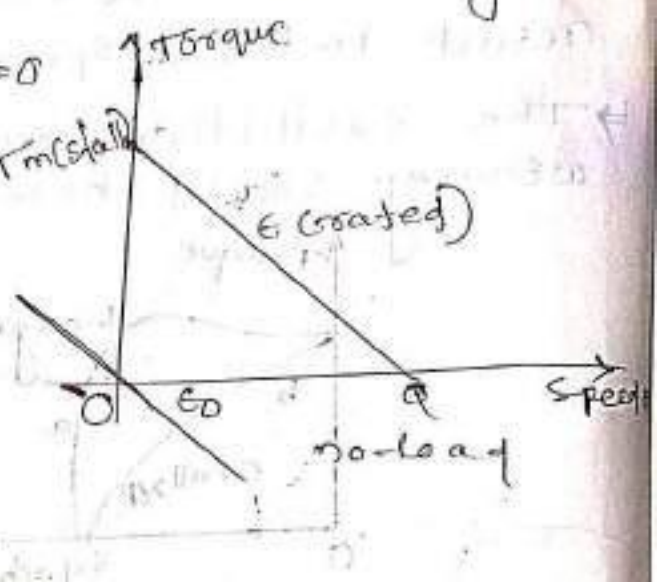
(Shematic diagram of a two phase Servomotor)

↳ The Control signals in Control System are usually of low-frequency, in the range of 0 to 20 Hz.

↳ for production of rotating magnetic field, the Control phase voltage must be of the same frequency.

↳ The torque-speed curves of ac servomotors are nonlinear except in the low speed region.



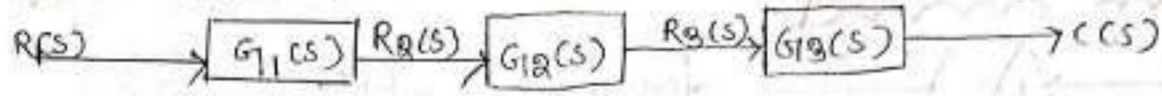


Ch.09

Block Diagram Algebra & Signal Flow graphs

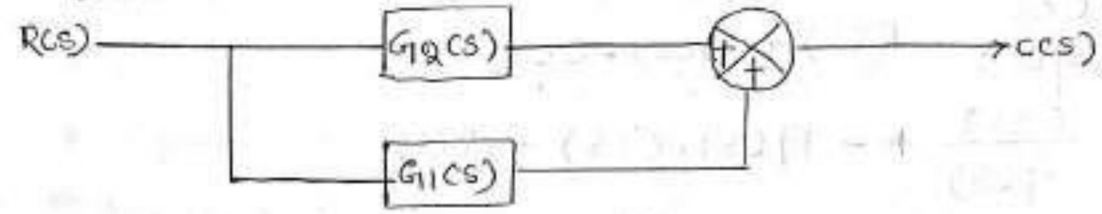
Rules for block diagram Reduction :→

1. For blocks in cascade/series :→



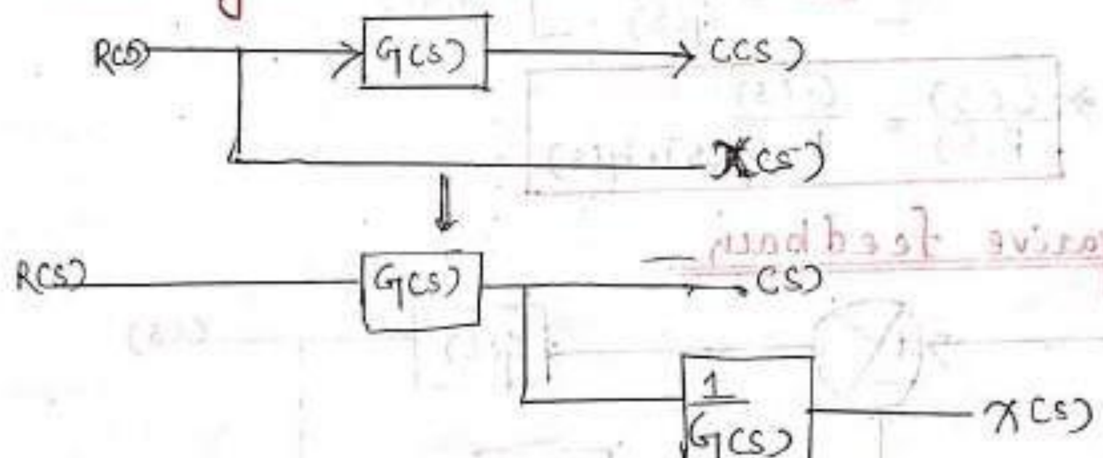
$$\frac{C(S)}{R(S)} = G_1(S) \cdot G_2(S) \cdot G_3(S)$$

2. For Block in parallel :→



$$\frac{C(S)}{R(S)} = G_1(S) + G_2(S)$$

3. Shifting a Take-off point before the block :-

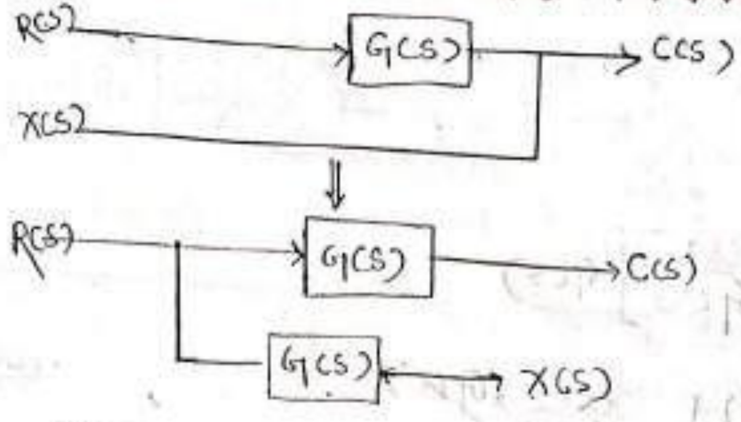


$$C(S) = R(S) \cdot G(S)$$

$$X(S) = \frac{1}{G(S)} \times R(S) \cdot G(S)$$

$$= R(S)$$

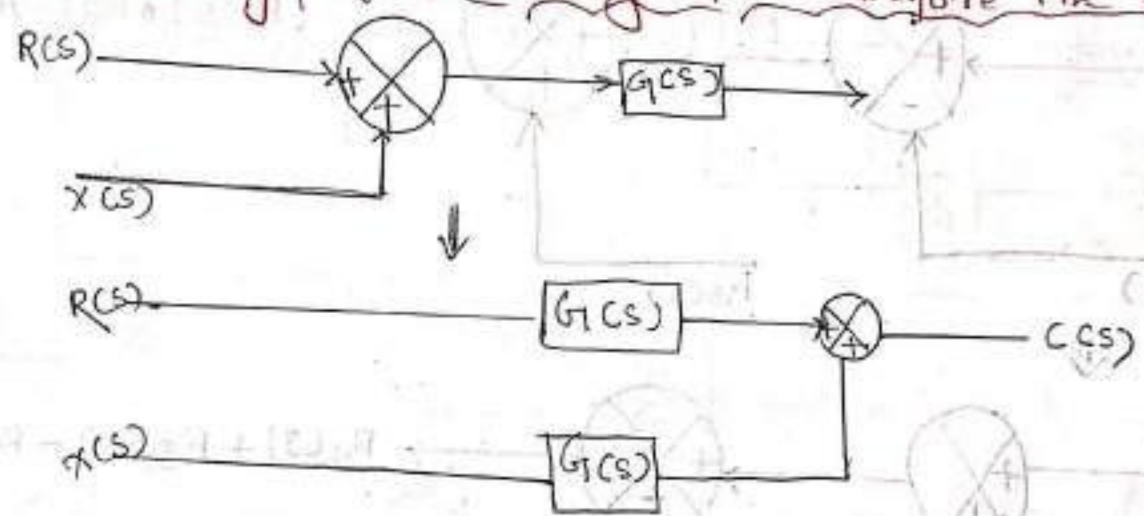
4. Shifting of take off point after the block :→



$C(S) = R(S) \cdot G(S)$

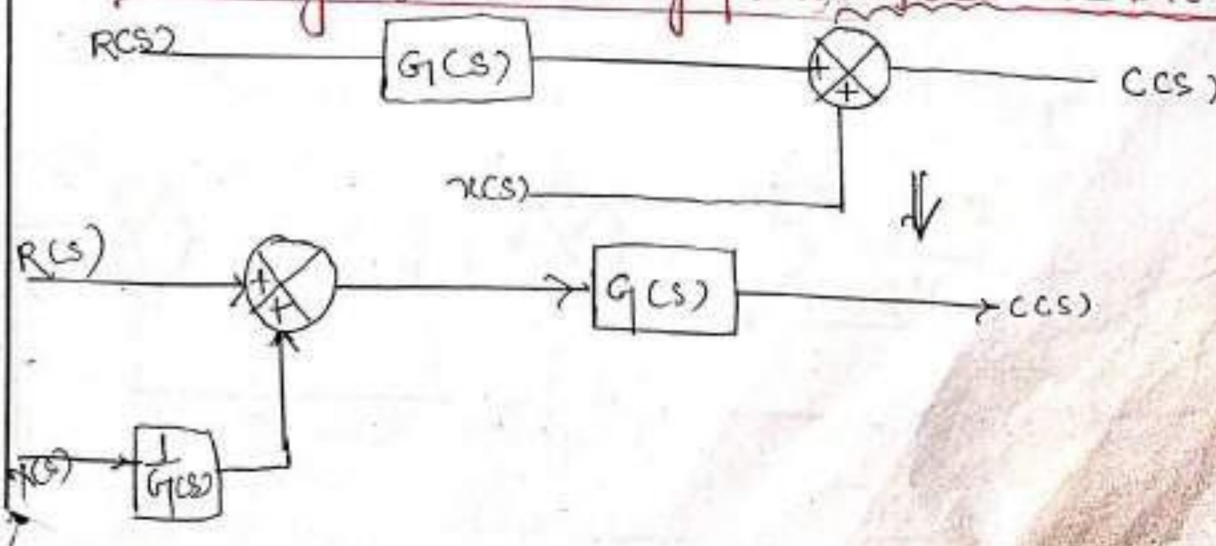
$X(S) = G(S) \times R(S)$

5. Shifting of a summing point before the block :-



(5) $C(S) = G(S)\left[R(S) + X(S)\right]$

(6) Shifting of a summing point after the block :-



Step-1

$$\frac{x(s)}{G(s)}$$

Step-2

$$R(s) + \frac{x(s)}{G(s)}$$

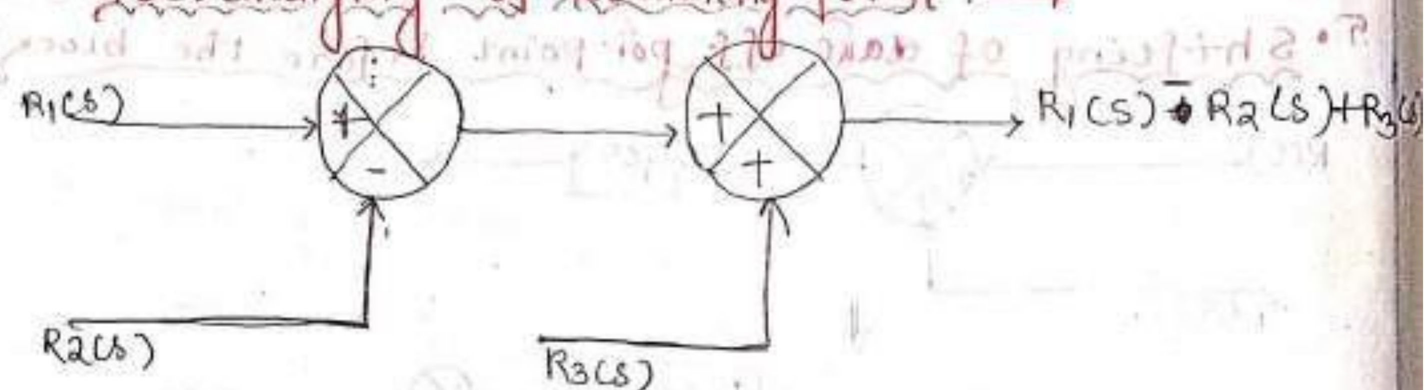
Step-3

$$C(s) = \left[R(s) + \frac{x(s)}{G(s)}\right] G(s)$$

$$= R(s) \cdot G(s) + \frac{x(s)}{G(s)} \times G(s)$$

$$= R(s) \cdot G(s) + x(s)$$

07. Interchanging of summing point :→



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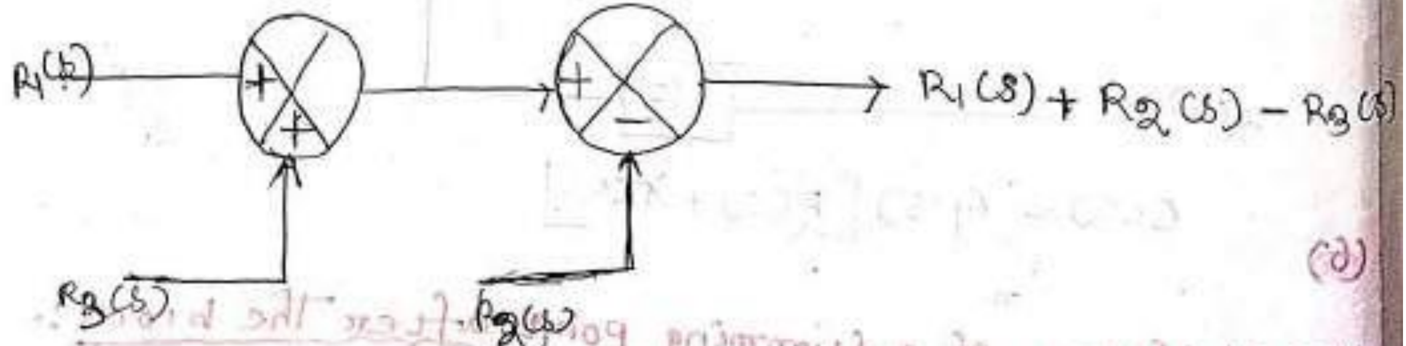
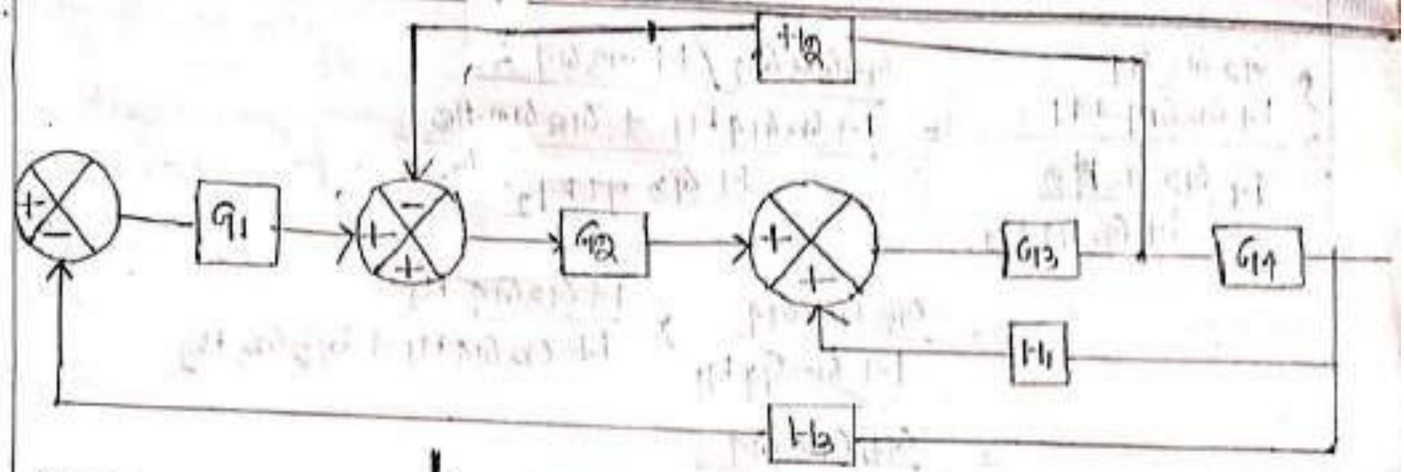
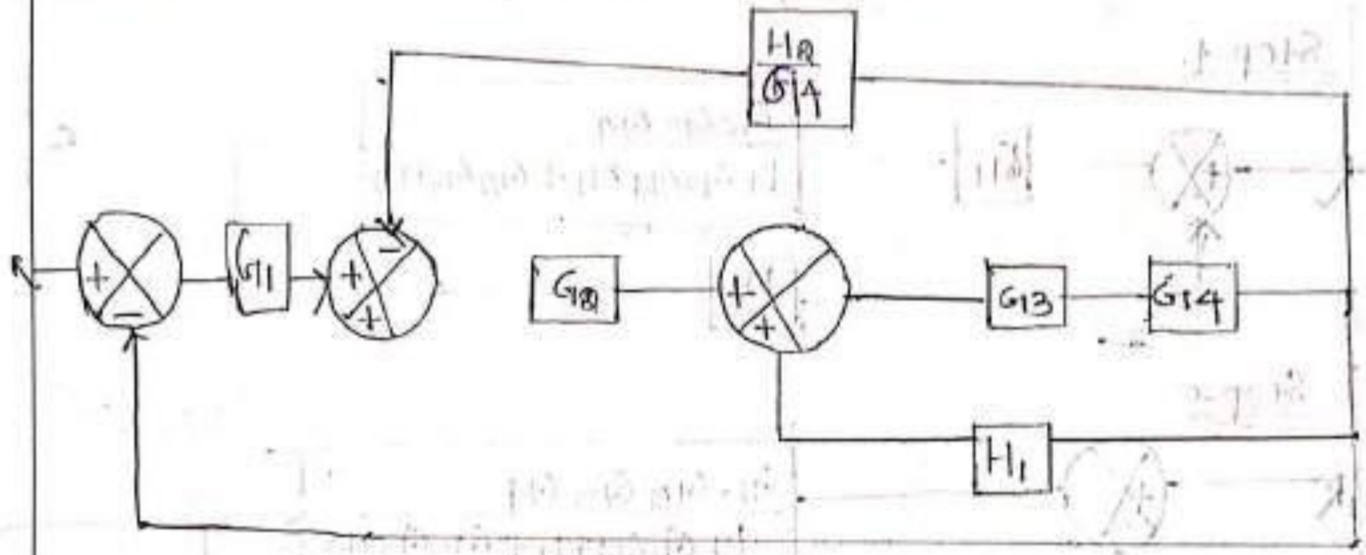


Problem,

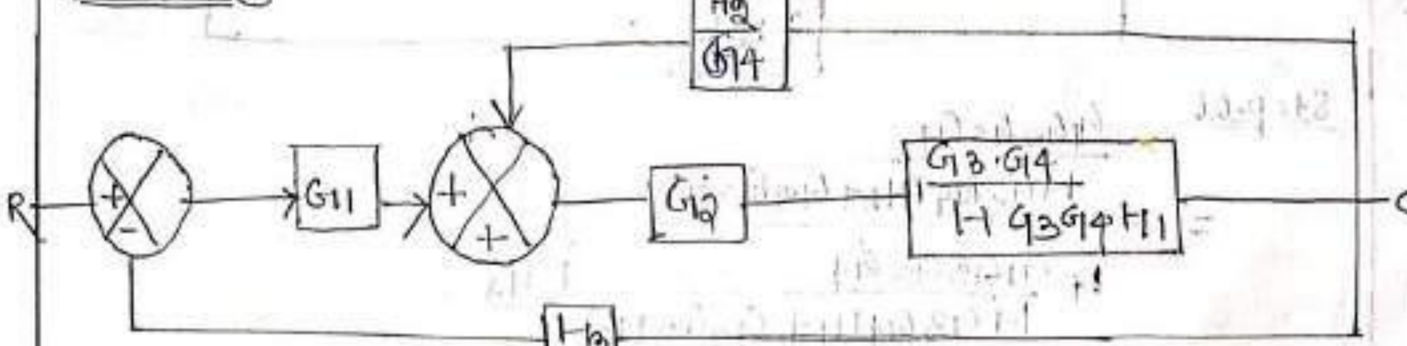
1.



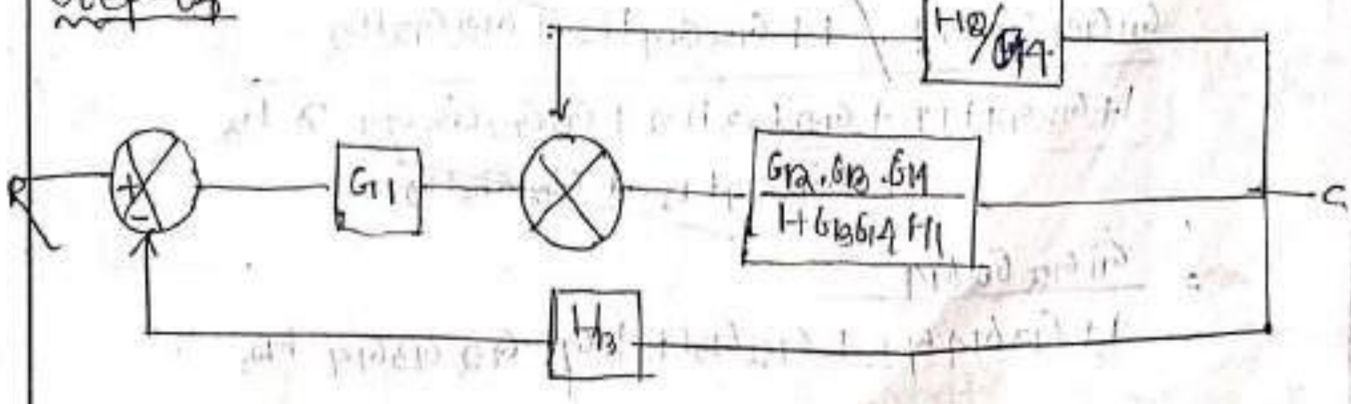
Step-1



Step-02



Step-03

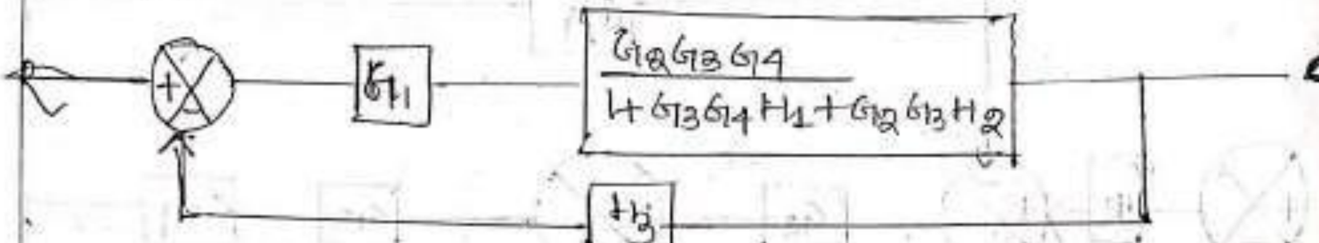


$$\frac{\frac{G_2 G_3 G_4}{1+G_3 G_4 H_1}}{1+\frac{G_2 G_3 H_2}{1+G_3 G_4 H_1}} = \frac{G_2 G_3 G_4 / 1+G_3 G_4 H_1}{\frac{1+G_3 G_4 H_1 + G_2 G_3 H_2}{1+G_3 G_4 H_1}}$$

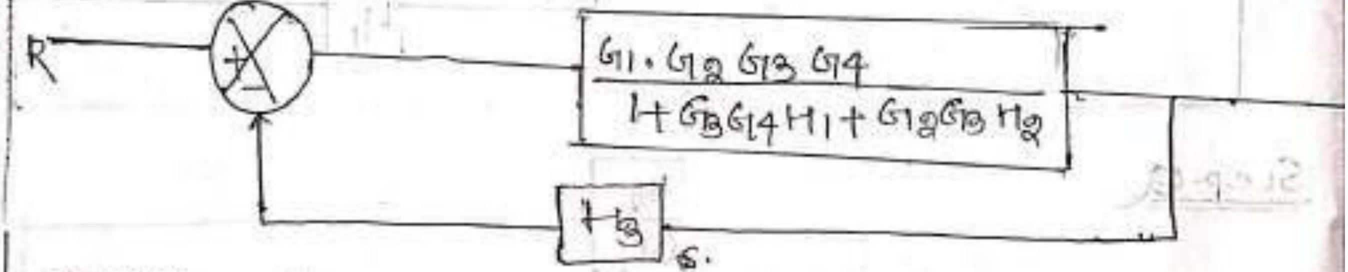
$$= \frac{G_2 G_3 G_4}{1+G_3 G_4 H_1} \times \frac{1+G_3 G_4 H_1}{1+G_3 G_4 H_1 + G_2 G_3 H_2}$$

$$= \frac{G_2 G_3 G_4}{1+G_3 G_4 H_1 + G_2 G_3 H_2}$$

Step-4



Step-5



Step-06

$$= \frac{\frac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_1 + G_2 G_3 H_2}}{1+\frac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_1 + G_2 G_3 H_2} H_3}$$

$$= \frac{G_1 G_2 G_3 G_4 / 1+G_3 G_4 H_1 + G_2 G_3 H_2}{\frac{1+G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 \times H_3}{1+G_3 G_4 H_1 + G_2 G_3 H_2}}$$

$$= \frac{G_1 G_2 G_3 G_4}{1+G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$

Procedure for Reduction of block diagram :-

→ follow these rules for simplifying the block diagram, which is having many blocks, summing points & take-off points.

Rule-1 Check for the blocks connected in series and simplify.

Rule-2 check for the blocks connected in parallel and simplify.

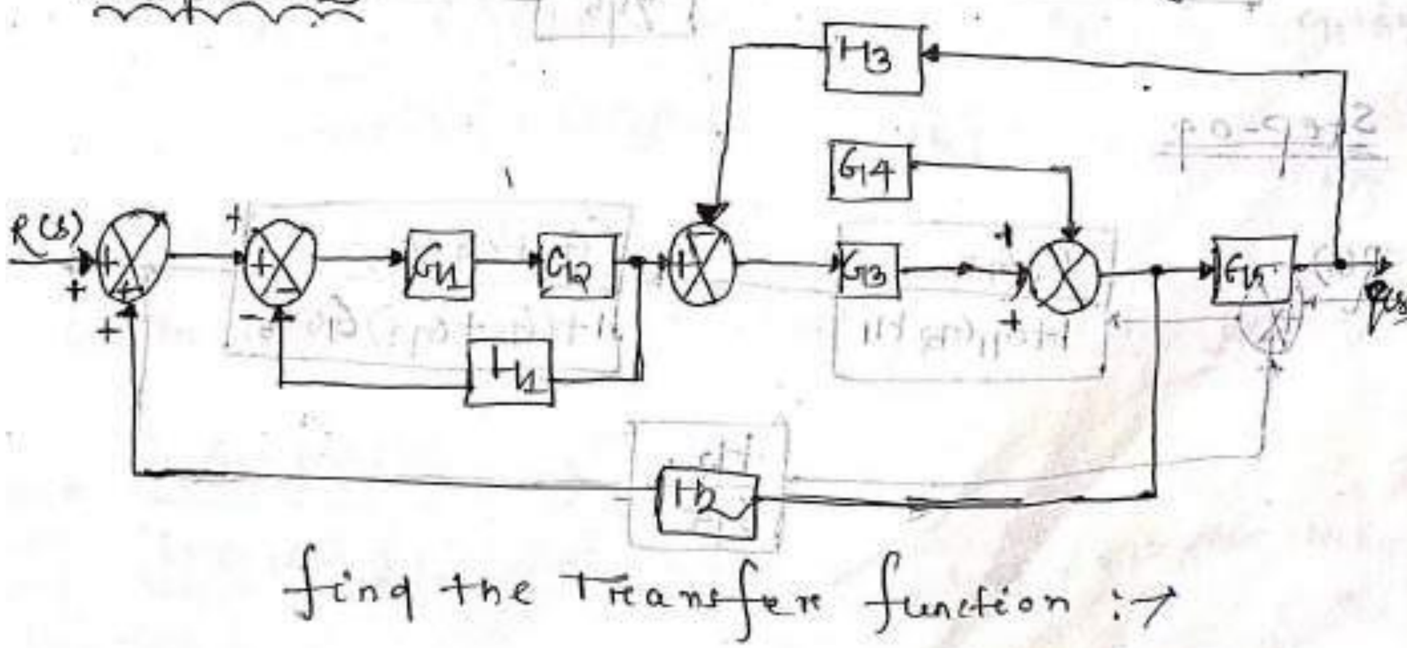
Rule-3 Check for the blocks connected in feedback loop & simplify.

Rule-04 If there is difficulty with take-off point while simplifying, shift it towards right.

Rule-05 If there is difficulty with summing point while simplifying, shift it towards left.

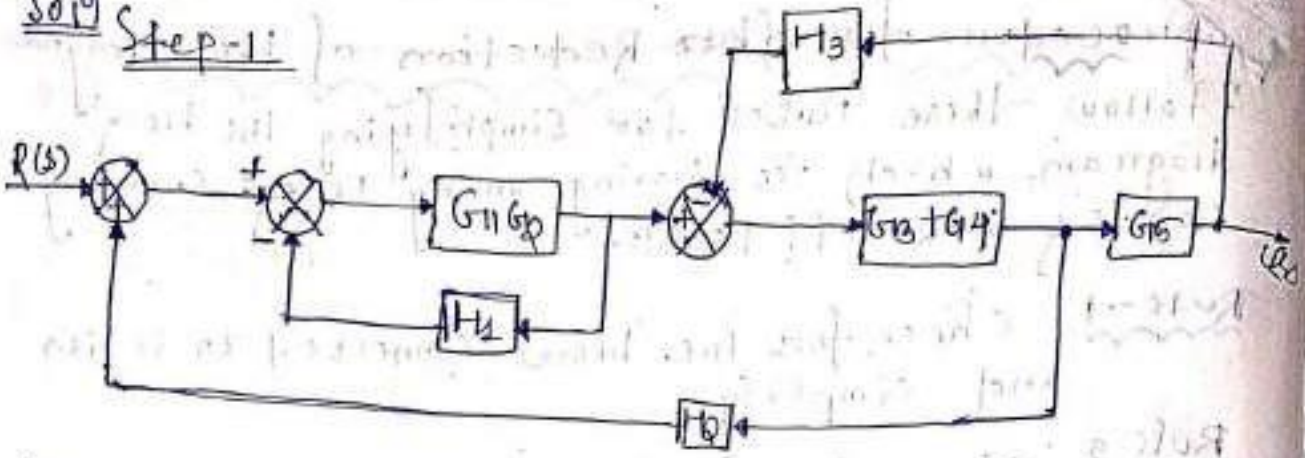
Rule-06 Repeat the above steps till you get the simplified form i.e single block.

Example-02

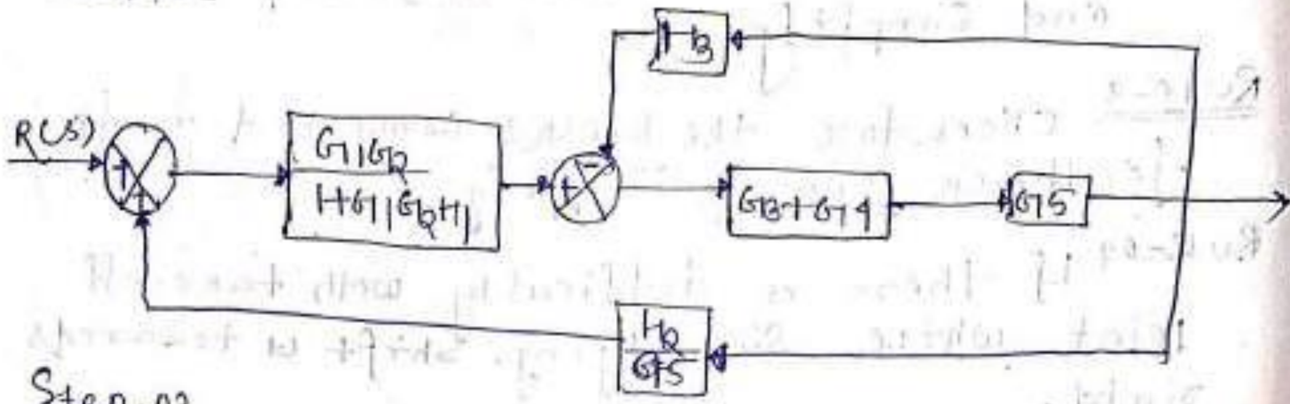


find the Transfer function :-

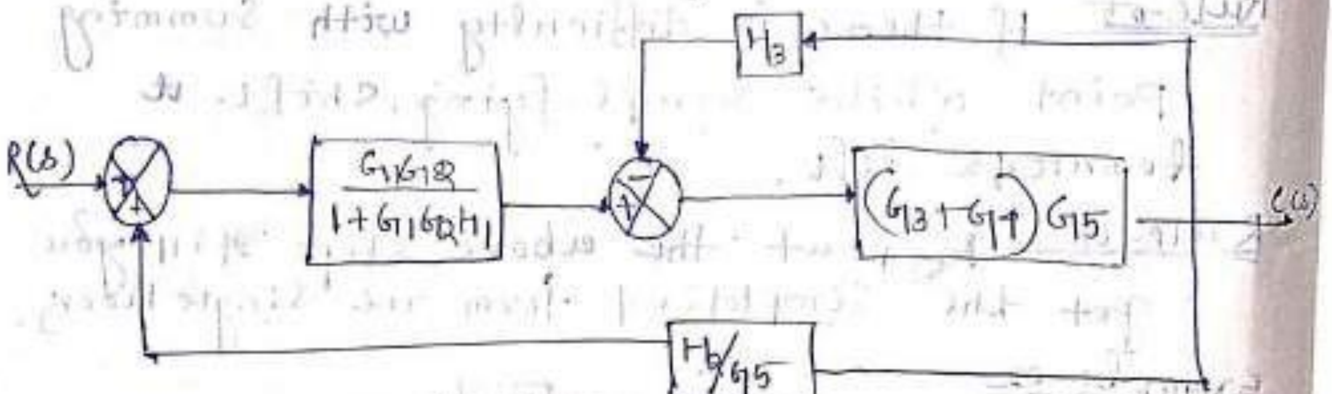
Sol^n **Step-1:**



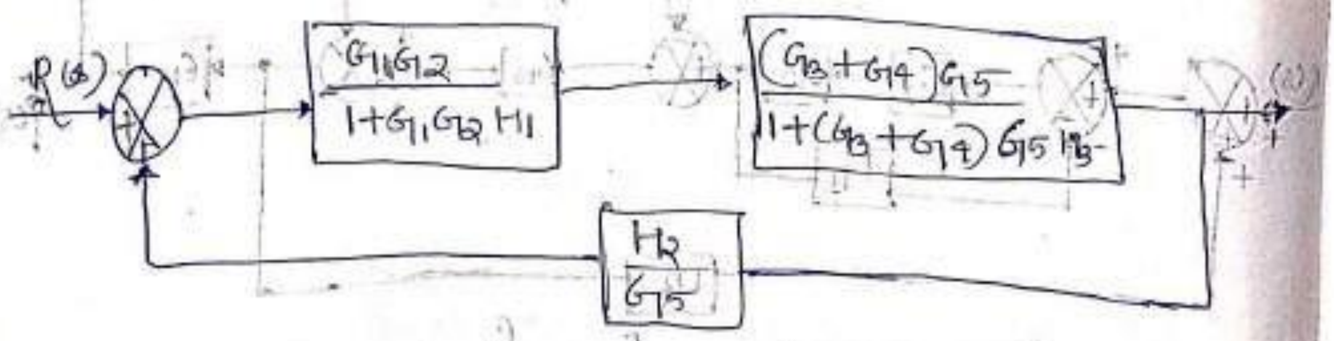
Step-2



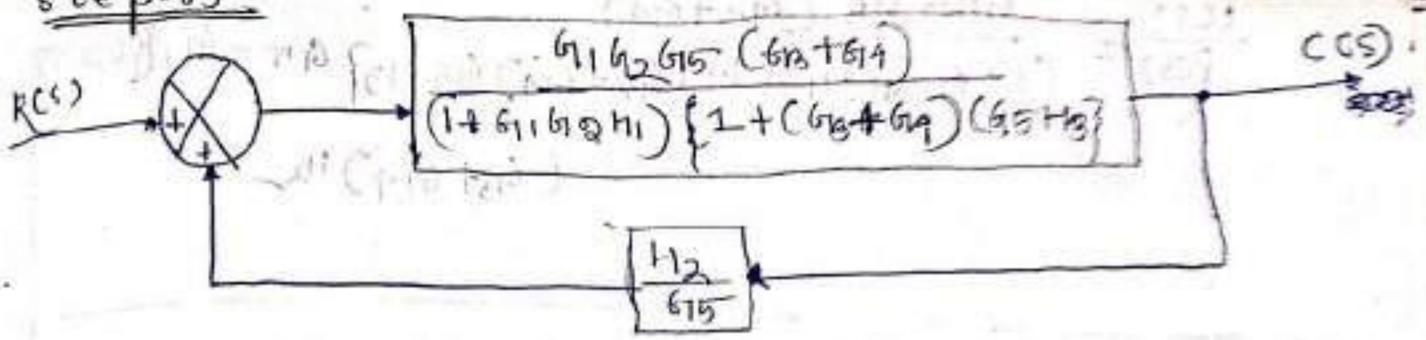
Step-03



Step-04



Step-05



Step-06

$$T.f = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)\cdot H(S)} \quad (\text{+ve feedback})$$

$$\Rightarrow G(S) = \frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\}}$$

$$H(S) = \frac{H_2}{G_5}$$

$$\Rightarrow \frac{\dfrac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\}}}{1 - \dfrac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\}} \times \dfrac{H_2}{G_5}}$$

$$= \frac{G_1G_2G_5(G_3+G_4) \Big/ (1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\}}{\dfrac{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\} - G_1G_2G_5(G_3+G_4) \times \frac{H_2}{G_5}}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\}}}$$

$$= \frac{G_1G_2G_5(G_3+G_4)}{G_5(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\} - G_1G_2G_5(G_3+G_4)H_2}$$

$$= \frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)(G_5+H_3)\}G_5 - G_1G_2G_5(G_3+G_4)H_2}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5 - G_1G_2G_5(G_3+G_4)H_2}$$

$$R(s) \rightarrow \left[\frac{G_1G_2G_3(G_3+G_4)}{(1+G_1G_2H_1)(1+(G_3+G_4)G_5H_3\}G_5 - G_1G_2G_5(G_3+G_4)H_2} \right]$$

Signal Flow Graphs

Node - A node represents a system variable which is equal to the sum of all the incoming signals at the node.

Input Node :- An input node is a node with only outgoing branches. It does not have any incoming branches.

Output Node :- An output node is a node with only incomings branches. It doesnot have any outgoing branches.

Forward path :→
A forward path is a path that starts at an input node and ends at an output node is called as forward path.

Mixed Node :→
A mixed node is a node that has both incoming and outgoing branch.

Loop :→
A loop is a path which originates and terminates at the same node and along which no node is traversed more than once.

Nontouching loop :→
Non Touching loops are loops which do not possess any common node.

Loop gain → The product of the branch gains encountered in traversing the loop is called the loop gain.

Self loop :- A self loop is a loop consisting of a single branch.

Self Node :- A loop that consist of only one Node is called as Self Node.

Mason's gain Formula:→

$$T.f = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta}$$

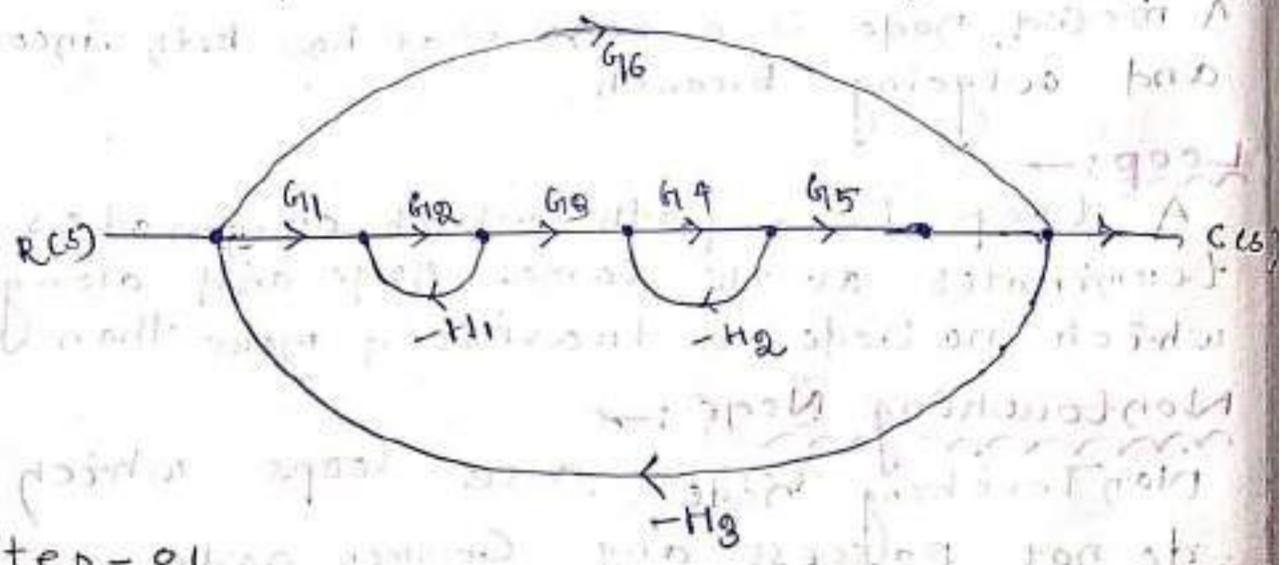
where,

N - Total number of forward path

P_i - gain of forward path.

Δ = 1-(Σ Loop gain) + (Σ gain off product of all possible Combinations of non-touching l + (Σ gain product of all possible Combinations of Two non-touching loop) + ... etc

Δ_i = The value of after elementing all loops that touch its forward path



Step-01

[forward path gain]

$P_1 = G_1 G_2 G_3 G_4 G_5$

$P_2 = G_6$

Step-02 (Gain of Two non-touching loop)

$L_{12} = L_1 \times L_2 = -G_2H_1 \times -G_4H_2$

$= G_2H_1G_4H_2 = G_2G_4H_1H_2$.

$L_{14} = L_1 \times L_4 = -G_2H_1 \times -G_6H_3$

$= G_2G_6H_1H_3$

$L_{24} = -G_4H_2 \cdot -G_6H_3$

$= G_4G_6H_2H_3$

Step-03 (Loop gain) :

$L_1 = -G_2H_1$

$L_2 = -G_4H_2$

$L_3 = G_1 \cdot G_2 \cdot G_3 \cdot G_4 \, G_5 \cdot H_3$

$L_4 = -G_6H_3$

Step-04 (Gain of three non-touching loop)

$L_{124} = L_1 \times L_2 \times L_4$

$= -G_2H_1 \times -G_4H_2 \times -G_6H_3$

$= -G_2G_4G_6H_1H_2H_3$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_{12} + L_{14} + L_{24}) + L_{124}$

$= 1 - (-G_2H_1 - G_4H_2 + G_1G_2G_3G_4G_5H_3 - G_6H_3)$

$+ (G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3)$

$- G_2G_4G_6H_1H_2H_3$

$= 1 + G_2H_1 + G_4H_2 - G_1G_2G_3G_4G_5H_3 + G_6H_3 +$

$G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3$

$- G_2G_4G_6H_1H_2H_3$