LECTURE NOTES ON

# CONTROL SYSTEM AND COMPONENTS

## 6TH SEMESTER ETC

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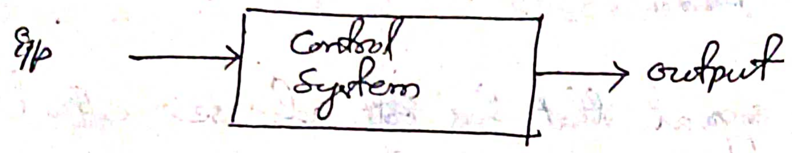
# 1. FUNDAMENTALS OF CONTROL SYSTEM

## 1.1 : Classifications of Control System :

A control system is a system, which provides the desired response by controlling the output.



Traffic light control system is an example of control system. Here a sequence of input signal is applied to this control system & the output is one of the three lights that will be on for some duration of time, the other two lights will be off. During this study at a particular junction, the on and off times of the lights can be determined. Accordingly, the input signal controls the output. So, the traffic lights control system operates on time basis.

### Basic Terminologies in Control System :-

**System** : A combination or arrangement of a number of different physical components to form a whole unit such that the combining unit performs to achieve a certain goal.

**Control** :- The action to command, direct or regulate a system.

**Plant or process** :- The part or component of a system that is required to be controlled.

**Input** :- It is the signal or excitation supplied to the control system.

**Output** :- It is the actual response obtained from the control system.

**Controller** :- The part or component of a system that controls the plant.

**Disturbances** :- The signal that has adverse effect on the performance of a control system.

**Control System** :- An interconnection of components forming a system configuration that will provide a desired response.

**Actuator** :- It is the device that causes the process to provide the output. It is the device that provides the motive power to process

**Design** :- The process of interventing the forms, parts, and details of systems to achieve a specified purpose.

**Simulation** ;- A model of a system that is used to investigate the behavior of a system by utilizing actual input signal.

**Negative feedback** :- The output signal is feedback so that it subtracts from the input signal.

**Block diagram** :- Unidirectional, operational blocks that represents the transfer functions of the elements of the system.

**Signal Flow Graph (SFG) :-** A diagram that consists of na connected by several directed ~~bracd~~ braches and that is a graphical representation of a set of linear relations.

**Specifications :-** Statements that explicitly state what the device or product is to be and to do. It is also defined as a set of prescribed performance crieteria.

## Classifications :-

### ① Natural Control System & Manmade Control System.

Natural Control System :- It is a control system that is created by nature, i.e :- Solar system, digestive system of any animal.

Man-made Control system :- A control system that is created by human, i.e :- automobile, power plants etc.

### ② Linear Control system & Non-linear Control System :-

Linear control system :- That follows the properties of homogenetive & additive.

Homogeneous property :- $f(x+y) = f(x) + f(y)$

Additive property :- $f(ax) = af(x)$

OR Law of superposition :-

Non-linear Control System :- It is a control system that does not obey the law of superposition.

## ③ Single Input Single Output (SISO) & Multiple Input Multiple Output

# Difference Between Open loop & Closed loop Control System

| Open loop Control System | Closed loop Control System |
|---|---|
| 1. The openloop systems are simple & economical | 1. The closed loop systems are complex and costly. |
| 2. They consume less power | 2. They consume more power. |
| 3. Easy to construct due to less number of components | 3. Diffecult to construct due to more number of components |
| 4. Inaccurate & unrelaible | 4. More accurate & ~~a~~ reliable. |
| 5. The changes in the output due to external disturbances can not be corrected automatically | 5. The changes in the output due to external disturbances are corrected automatically. |

## Effect of Feedback

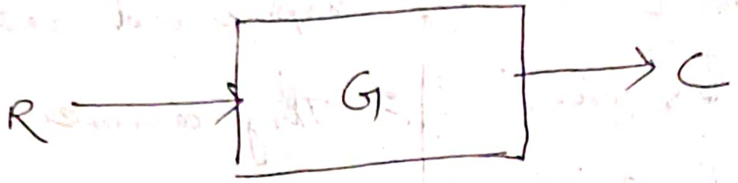
When a part ~~of~~ or the ~~wh~~ whole of the output signal is feedback to the input of the system, then it is called feedback system.

~~The~~ Feedback may be of 2 types +ve & -ve .

Output = C

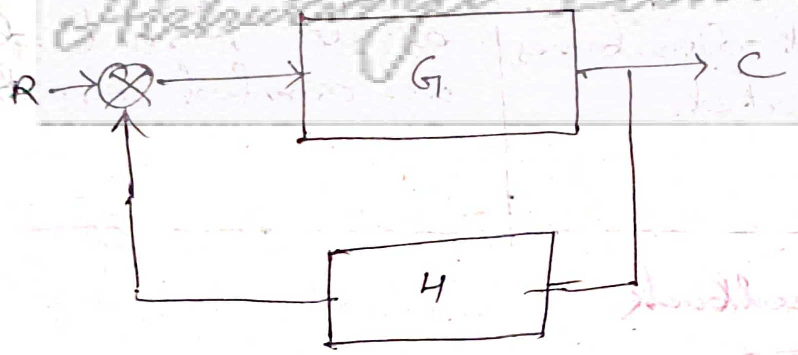
New input = CH + R

## ① Effect of feedback on overall gain



System gain without feedback

$$G = \frac{C}{R} \quad ---- (i)$$



New Input with feedback = R + CH

$$\text{Gain with feedback} = G_f = \frac{C}{R+CH}$$

$$= \frac{C \cdot 1}{C\left(\frac{R}{C}+H\right)} = \frac{1}{\frac{R}{C}+H}$$

$$= \frac{1}{\frac{1}{G}+H} = \frac{1}{\frac{1+GH}{G}} = \frac{G}{1+GH}$$

$$\boxed{G_f = \frac{G}{1+GH}} \quad ---- (ii)$$

The system of the following figure may have -ve or +ve feedback. Depending on the sign of GH the overall gain may increas or decrease. In practical control system G & H are the functions of frequency. So the magnitude of 1+GH may be greater than 1 ~~or~~ in ~~less than 1r~~ one frequency range but less than 1 in another.

## ② Effect of feedback on Stability :-

From eq-(ii) we can see that feedback changes the gain of non-feedback system by a factor of $\frac{1}{1+GH}$

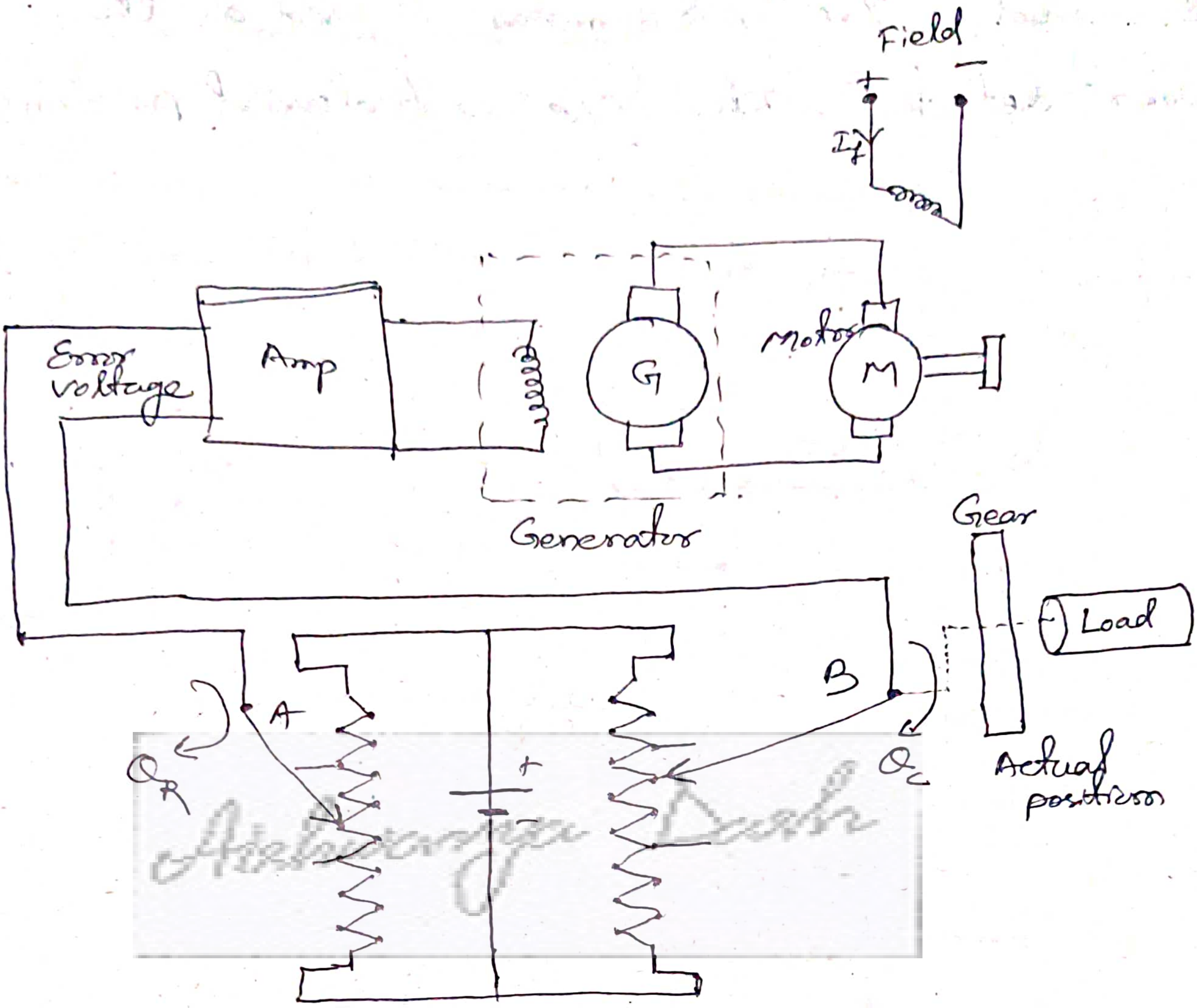
If the factor GH becoms $-1$, then the gain becomes

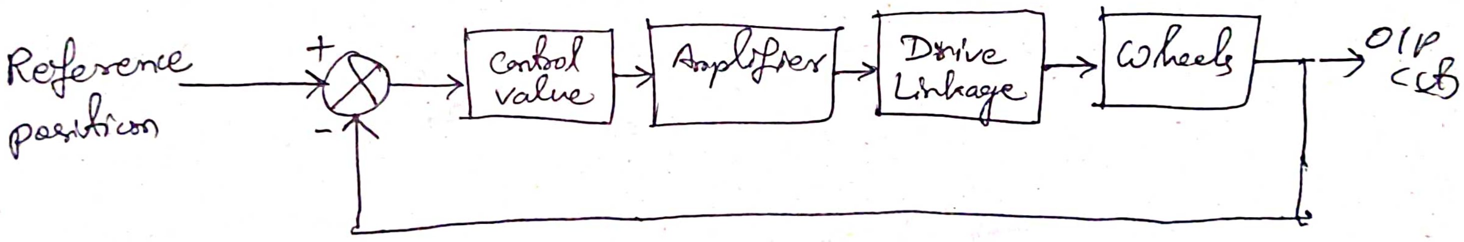
$$G_f = \frac{G}{1-1} = \frac{G}{0} = \infty$$

Hence the system is said to be unstable.
Therefore we can say that feedback can cause a stable system to become unstable.
But if it is used properly then it can stabilize an unstable system.

## Servomechanism

It is the feedback unit used in a control system the control variable is a mechanical signal, such as position, velocity or acceleration. Here the o/p signal is directly fed to the comparator as a feedback signal, b(t) of the closed loop control system. This type of system is used where both the command & o/p signal are mechanical in nature. A position control system as shown in figure is a simple example of this type of mechanism.





~~Of is the plant~~

Examples :-

Missile launcher

Machine tool position control

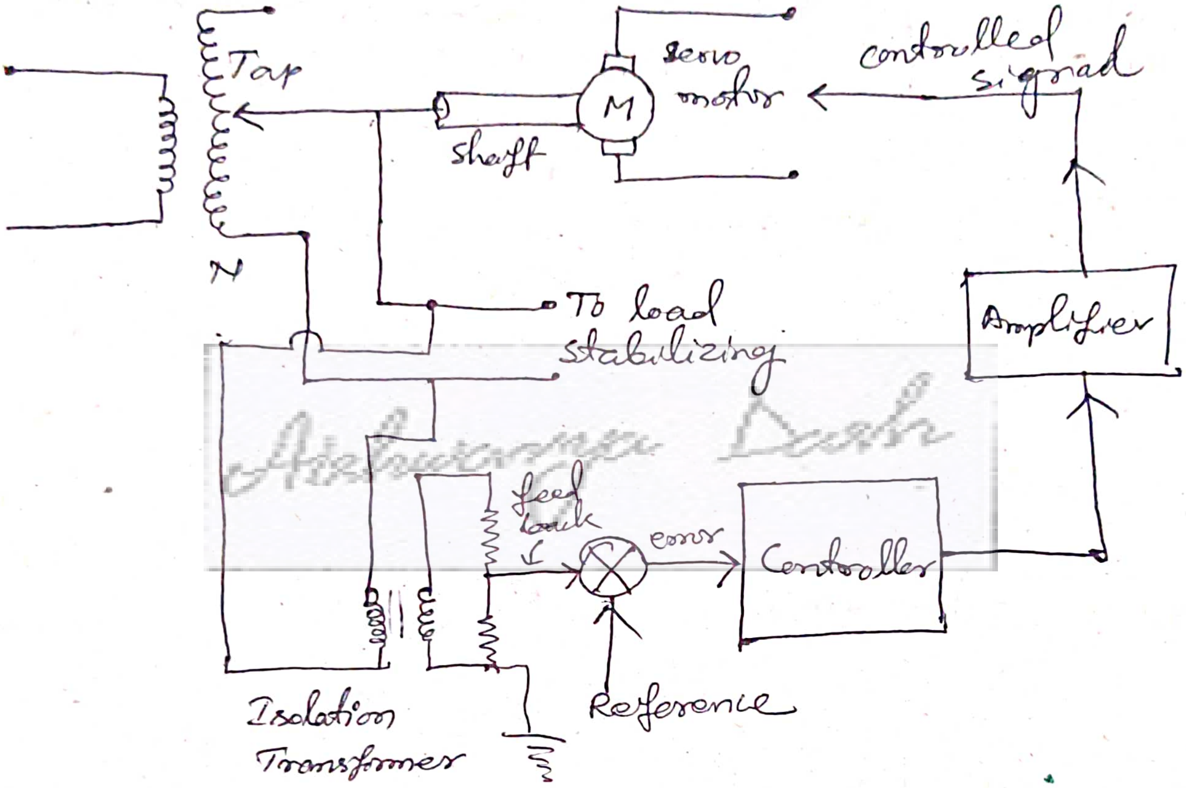
Power steering for an automobile

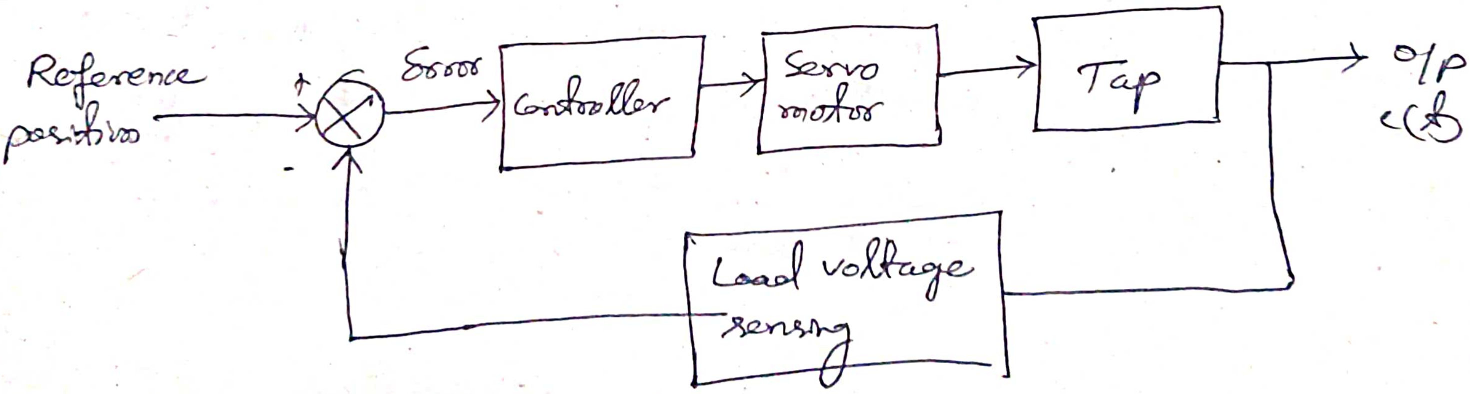
Roll stabilization

Here the driving motor is geared to the load to be moved. The potentiometer is used as the error detector. The o/p and desired positions

## REGULATORS

It is also a feedback unit used in a control system like servomechanism. But the output is kept constant at its desired value. The schematic diagram of a regulating system is shown below with its corresponding simplified block diagram :-



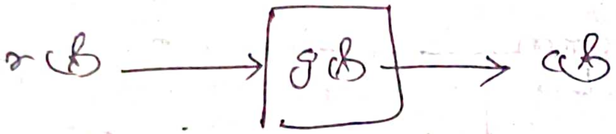


# 2. TRANSFER FUNCTION

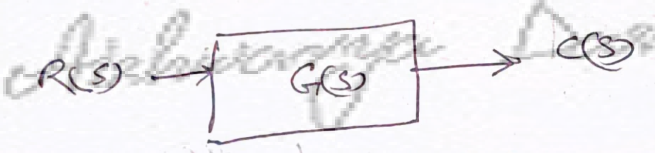
## Transfer Function

It is the ratio of Laplace Transform of output signal to Laplace transform of input signal assuming all the initial condition to be zero, i.e

Let there is a given system with input r(t) & output c(t) as shown in figure



Then its Laplace domain is



Here the T. F. G(S) can be represented as

$$G(S) = \left.\frac{C(S)}{R(S)}\right|_{\text{zero initial condition.}}$$

L. T.
~~T.F.~~ of electrical parameters :-

$i(t) \rightarrow I(S)$

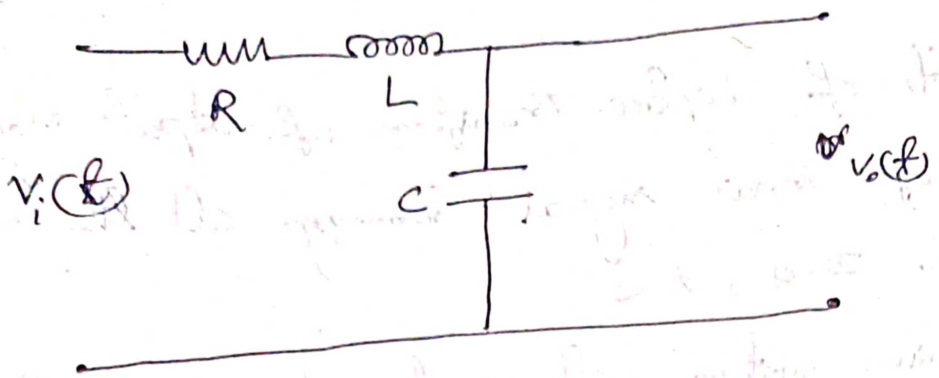
$v(t) \rightarrow V(S)$

$R \rightarrow R$

$L \rightarrow LS$

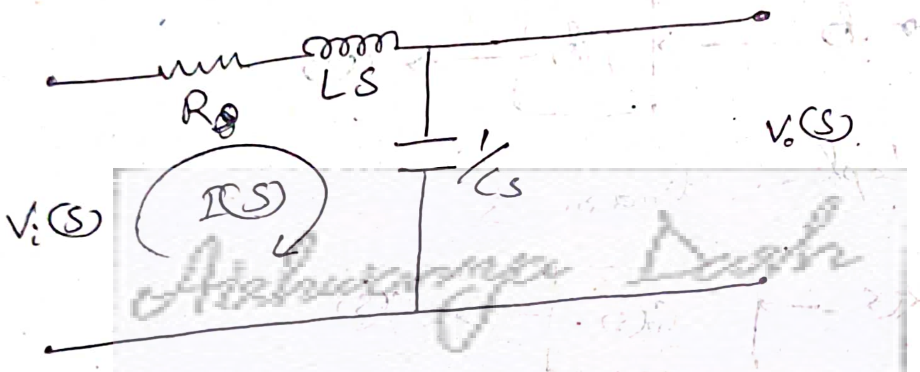
$C \rightarrow \frac{1}{CS}$

Ex :- Find the T. F. of the following :-



Sol$^n$ :-

In frequency domain :-



$$V_i(S) = R\,I(S) + LS\,I(S) + \frac{1}{CS} I(S)$$

$$= I(S)\left[R + LS + \frac{1}{CS}\right]$$

$$V_o(S) = I(S)\frac{1}{CS}$$

$$\therefore \; T.F. = \frac{V_o(S)}{V_i(S)} = \frac{\cancel{I(S)}\frac{1}{CS}}{\cancel{I(S)}\left[R + LS + \frac{1}{CS}\right]}$$

$$= \frac{1/cs}{R + LS + \frac{1}{cs}}$$

$$= \frac{1/cs}{\frac{RCS + LCS^2 + 1}{CS}} = \frac{1}{\cancel{CS}} \times \frac{\cancel{CS}}{RCS + LCS^2 + 1}$$

$$= \frac{1}{RCS + LCS^2 + 1}$$

$$\therefore \boxed{G(S) = \frac{1}{LCs^2 + RCs + 1}}$$

## Properties of T. F. :-

- → Zero initial condition
- → It is same as Laplace Transform of impulse response.
- → Replacing 'S' by $\frac{d}{dt}$ in the transfer function, the differential equation can be obtained
- → Poles & zeros can be obtained from the T. F.
- → Stability can be known
- → Can be applicable to linear system only.

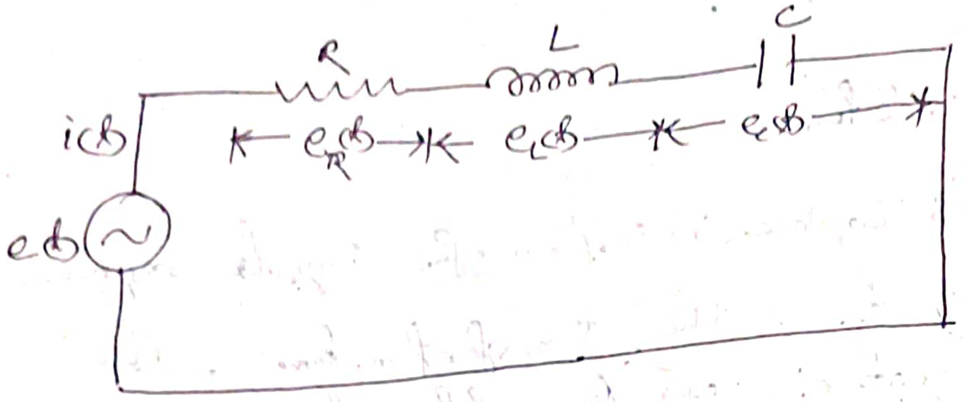
## Advantages of T. F.

- → It is a mathematical model & gain of the system.
- → Replacing 's' – –
- → Poles & zeros
- → stability can be known
- → Impulse response can be found

## Disadvantages

- → Applicable only to linear system
- → Not applicable if ~~old~~ initial ~~old~~ condition can not be neglected.
- → It gives no information about the actual structure of physical system.

Find the system T.F between the capacitance voltage to the source voltage in the following RLC circuit :-



# Control System Components & Mathematical Modelling of Physical System

## 3.1 Components of Control System

Components of mechanical systems : Mechanical systems are of two types :-

(i) Translational mechanical systems

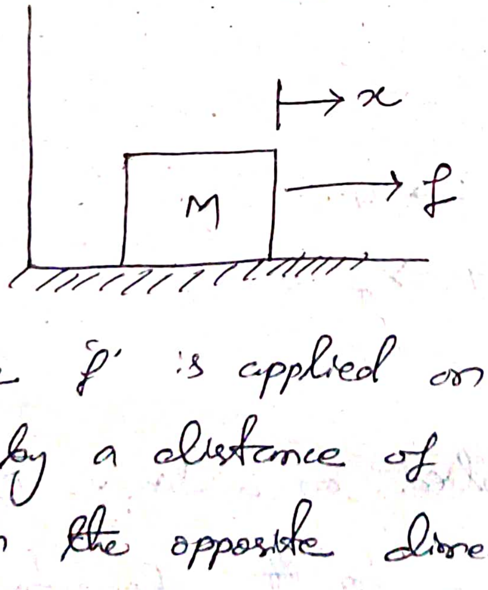
(ii) Rotational mechanical systems

### Translational Mechanical Systems

There are three basic elements in a translatemal mechanical system. i.e (i) Mass, (ii) Spring, (iii) Damper

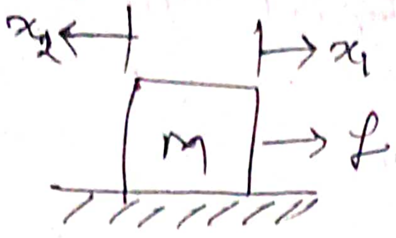
(i) Mass :- A mass is denoted by M. If a force 'f' is applied on it and it is displaced to a distance 'x' then

$$f = M\frac{d^2x}{dt^2}$$

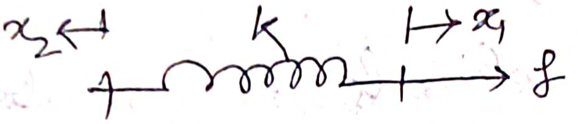


- If a force 'f' is applied on a mass M and it is displaced by a distance of $x_1$ in the direction of 'f' and '$x_2$' in the opposite direction, then

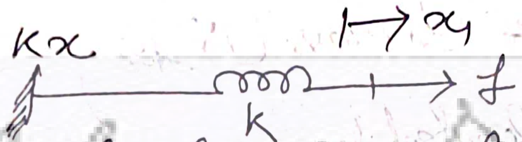
$$f = M\left[\frac{d^2x_1}{dt^2} - \frac{d^2x_2}{dt^2}\right]$$



(ii) Spring :- A spring is denoted by K and it displays a distance $x_1$ in the directⁿ of f and distance $x_2$ in the opposite direction, then $f = K(x_1 - x_2)$

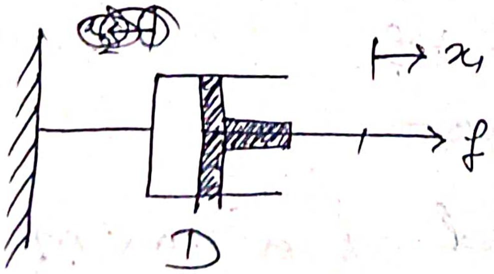


$f = Kx$



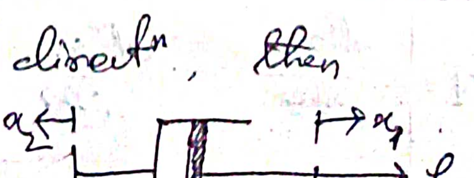
(iii) Damper :- A damper is denoted by D. If a force f is applied on it and it displays distance 'x' then

$$f = D\frac{dx}{dt}$$

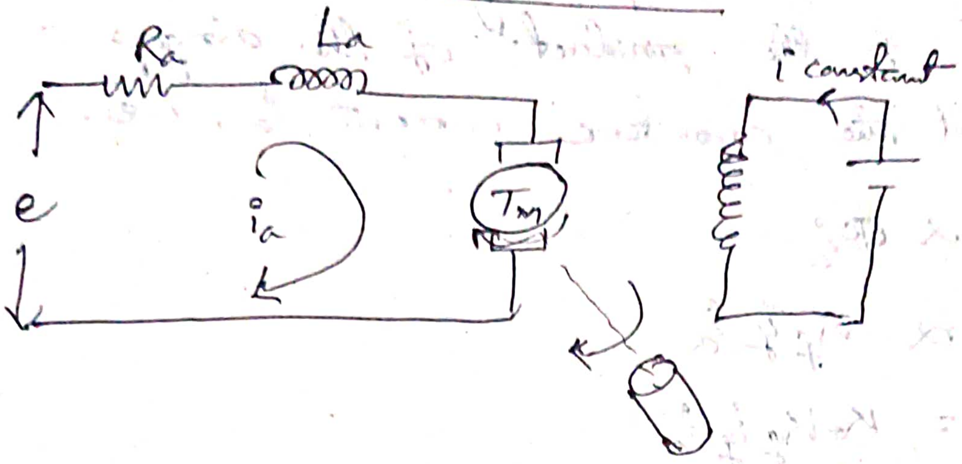


If a force is applied on a damper D and it displays distance $x_1$ in the direction of f $x_2$ in the opposite directⁿ, then

$$f = D\frac{dx}{dt}$$



# ARMATURE CONTROLLED DC SERVOMOTOR



An armature controlled dc motor is a dc shunt motor designed to satisfy the requirement of servomotor. If the field current is constant, then speed is directly proportional to armature voltage & torque is directly proportional to armature current. Hence torque & speed can be controlled by armature voltage.

In servo applications, the dc motors are generally used in the linear range of the magnetization curve. Therefore, the airgap flux $\phi$ is proportional to the field current, i.e.

$$\phi \propto i_f \Rightarrow \phi = K_f i_f$$

The Torque $T_m$ developed by the motor is proportional to the product of the airgap flux $\phi$ and the armature current $i_a$, i.e.

$$T_m \propto \phi i_a$$

$$T_m \propto K_f i_f i_a$$

$$T_m = K_I K_f i_f i_a$$

where $K_I$ is a constant

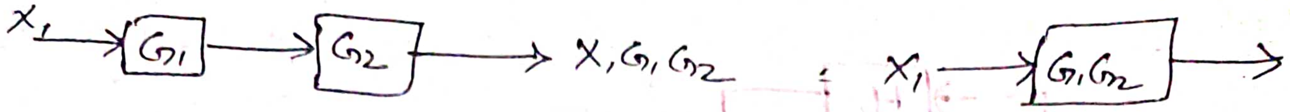
In armature control dc motor, field current is kept constant, so the equation for $T_m$ can be written as

$$T_m = K_T i_a$$

where $K_T$ is motor torque constant.

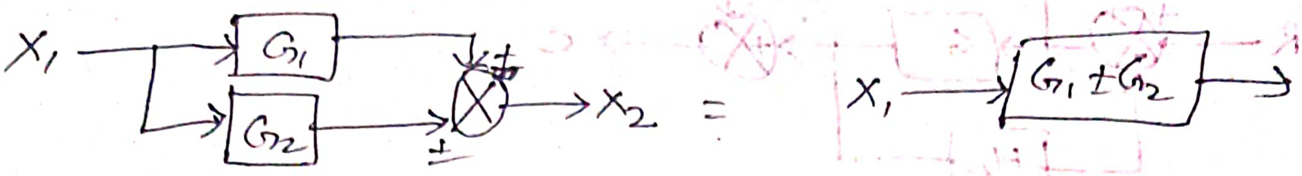
The

①



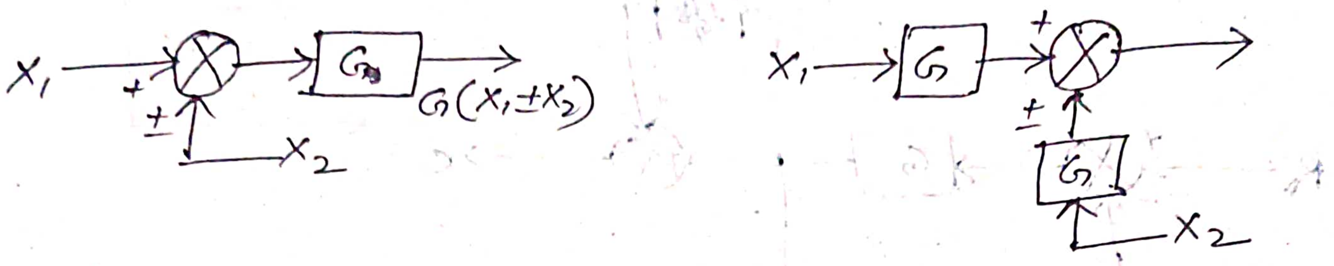
$X_1$
$G_1$
$G_2$
$X_1G_1G_2$
$X_1$
$G_1G_2$

②



$X_1$
$G_1$
$G_2$
±
±
$X_2$
=
$X_1$
$G_1 \pm G_2$

③

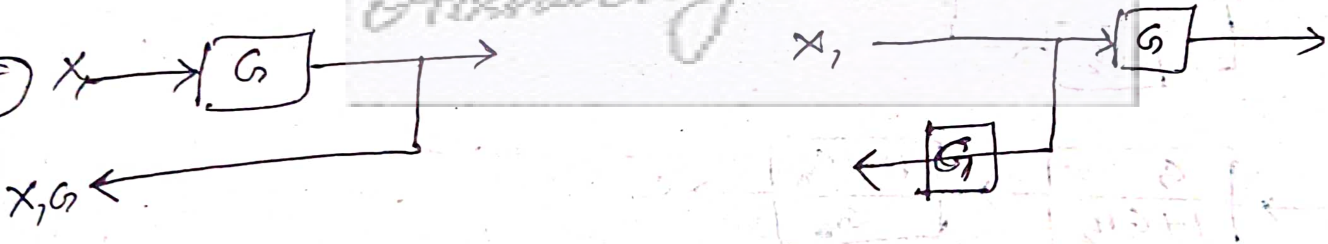


$X_1$
+
±
$X_2$
G
$G(X_1 \pm X_2)$
$X_1$
G
+
±
G
$X_2$

④

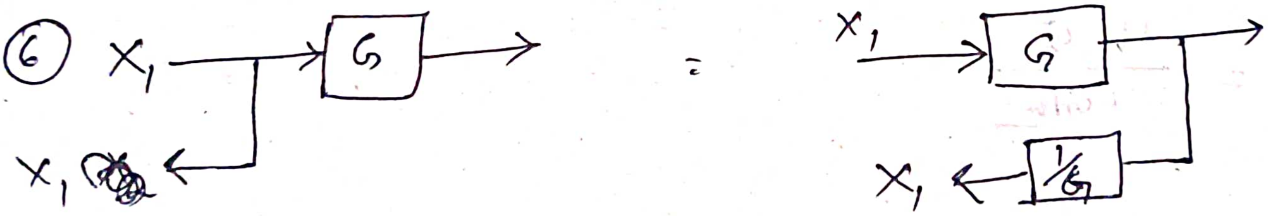
$X_1$
G
+
±
$X_2$
$X_1$
±
G
$\frac{1}{G}$
$X_2$

⑤



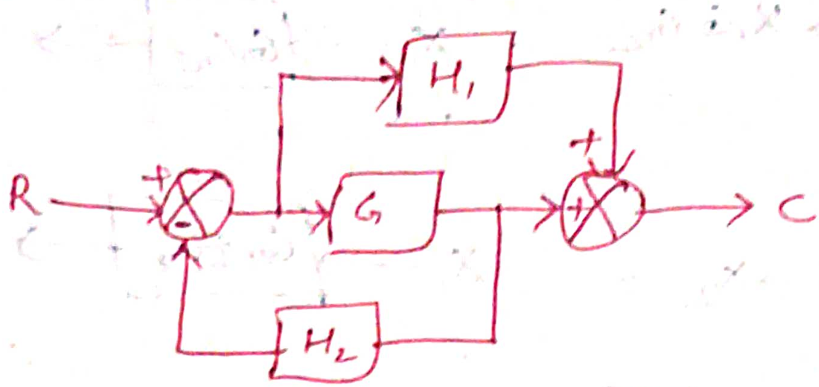
$X_1$
G
$X_1G$
$X_1$
G
G

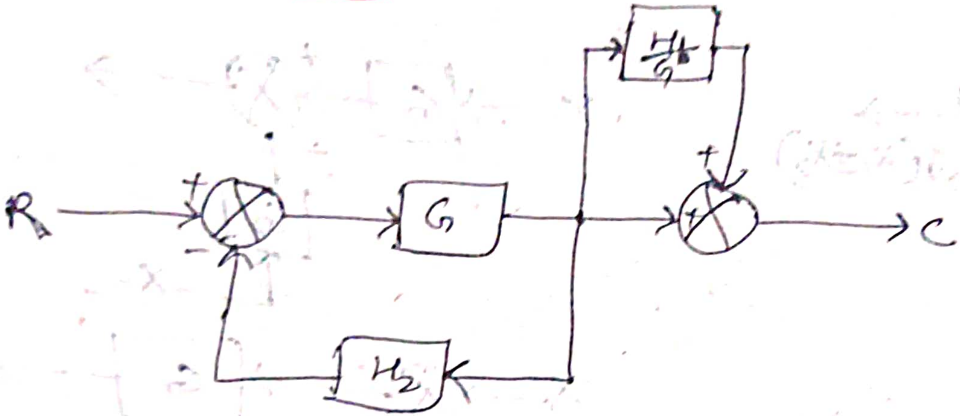
⑥

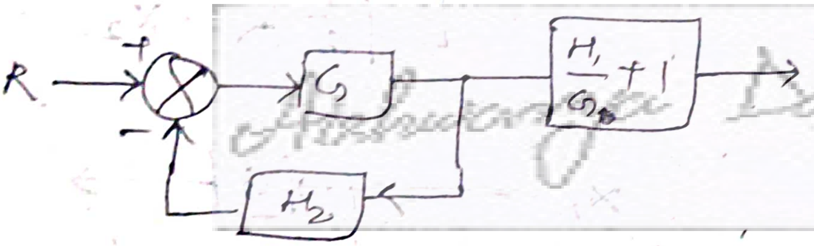


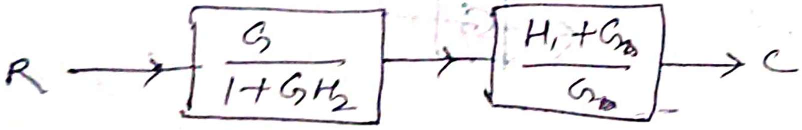
$X_1$
G
$X_1$
=
$X_1$
G
$1/G$
$X_1$

Q:-



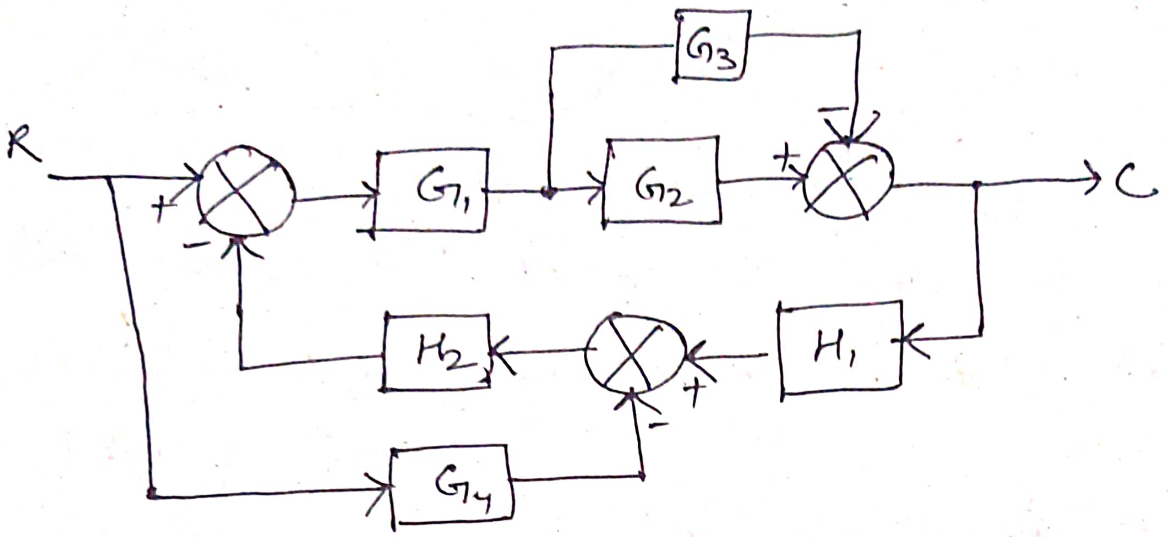




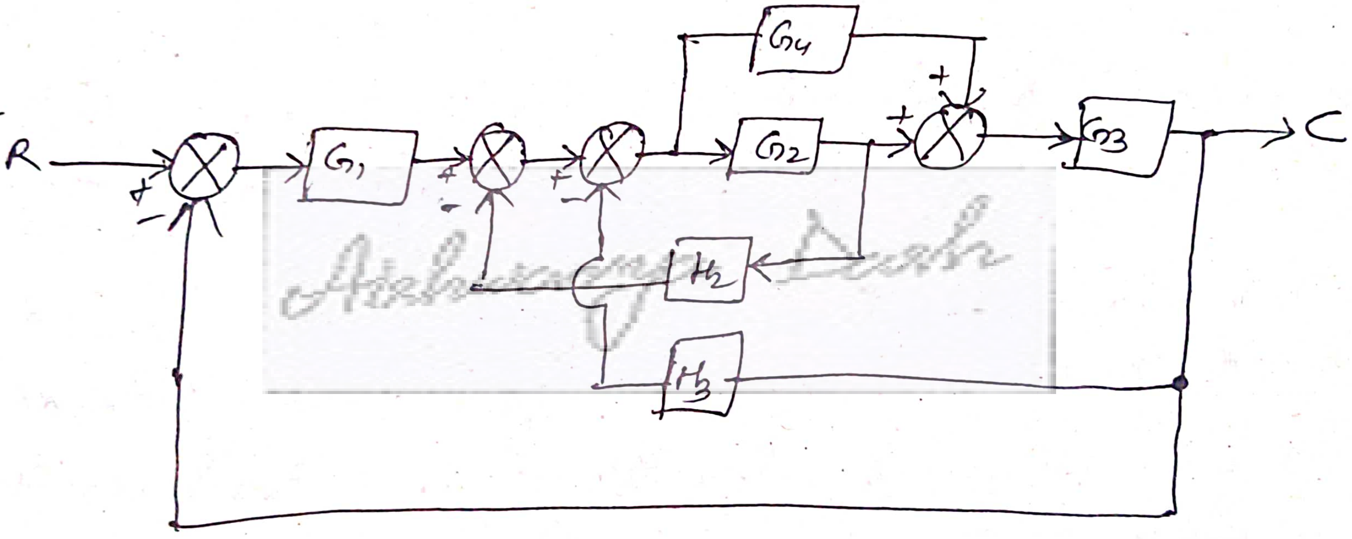


$$\frac{C}{R} = \frac{H_1 + G}{1 + GH_2}$$

Q:- Obtain the T.F



Q:-



# Signal Flow Graph (SFG)

## 4.6. Basic Definitions of SFG

SFG is a pictorial representation of a system that graphically displays the signal transmission in it.

→ Input or Source Node : It is a node that has only outgoing branches.

→ Output or Sink Node :- It is a node that has only incoming branches.

→ Chain Node : It is a node that has both incoming and outgoing ~~nodes~~ branches.

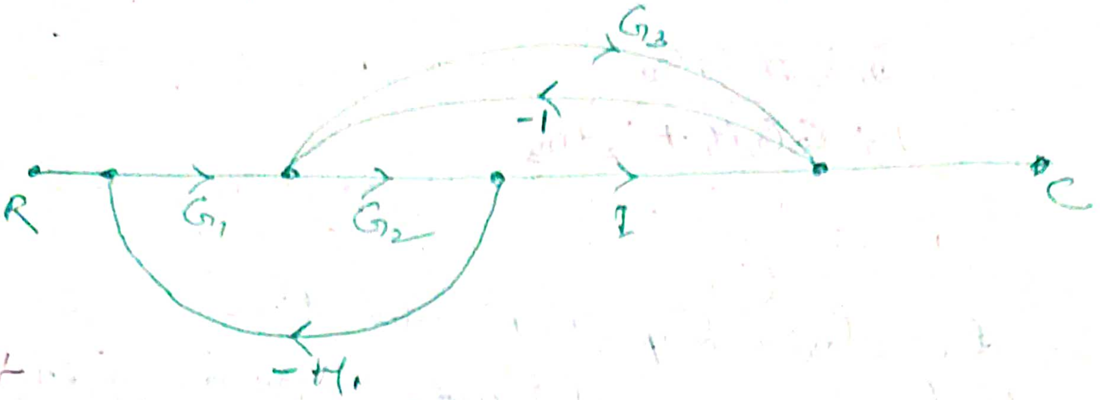
→ Gain or Transmittance :- It is the relationship between variables denoted by two nodes or value of branches.

→ Forward path :- It is the path from input node to output node without repeating any of the nodes in between them.

→ Feedback path :- It is a path from output node or a node near the output to the input ~~or~~ node or a node near the input node without repeating any of the nodes in between them.

→ Loop :-

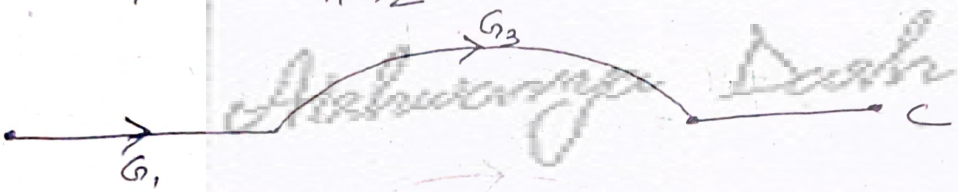
Q:- Obtain the closed loop T.F. by using Mason's gain formula.



Solⁿ:-

No. of forward path = 2

Gain $P_1 = G_1G_2$

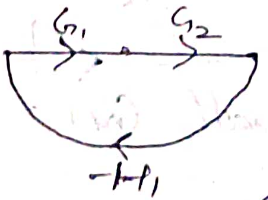


Gain $P_2 = G_1G_3$

$\Delta_1 = 1 - 0 = 1$

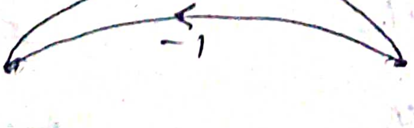
$\Delta_2 = 1 - 0 = 1$

$L_1 = -G_1 G_2 H_1$



$L_2 = -G_2$

$L_3 = -G_3$



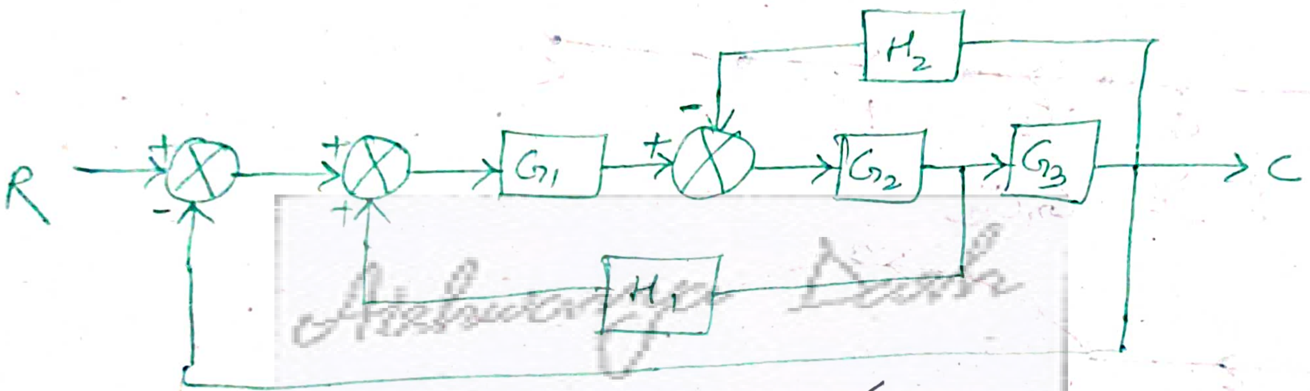
$\Delta = 1 - (L_1 + L_2 + L_3) + 0$

$= 1 - (-G_1G_2H_1 - G_2 - G_3) = 1 + G_1G_2H_1 + G_2 + G_3$

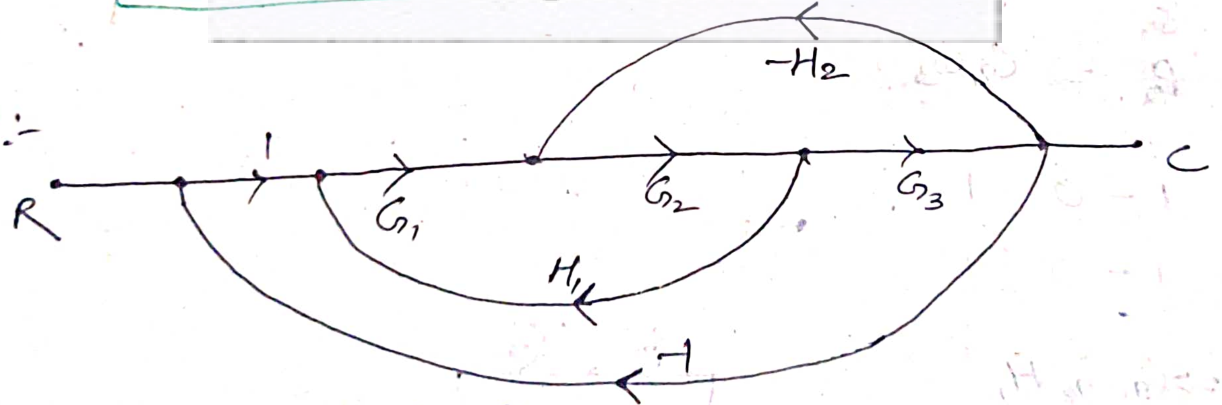
$$\therefore \; T.F. = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_1 + G_2 + G_3}$$

Q :- Obtain the closed loop T.F. for the given block diagram by using Mason's gain formula.

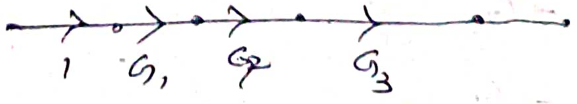


Sol$^n$ :-

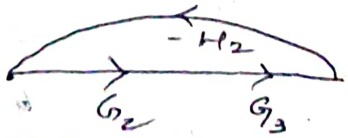


No. of forward path (N) = 1

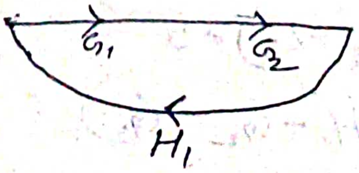
$P_1 = G_1G_2G_3$



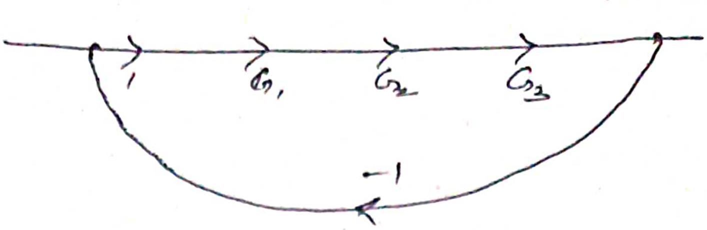
$L_1 = -G_2G_3H_2$



$L_2 = G_1G_2H_1$



$L_3 = -G_1 G_2 G_3$



$\Delta_1 = 1 - 0 = 1$

$\Delta = 1 - (L_1 + L_2 + L_3)$

$= 1 - (-G_2G_3H_2 + G_1G_2H_1 - G_1G_2G_3)$

$= 1 + G_2G_3H_2 + G_1G_2G_3 - G_1G_2H_1$

Applying Mason's gain formula :-

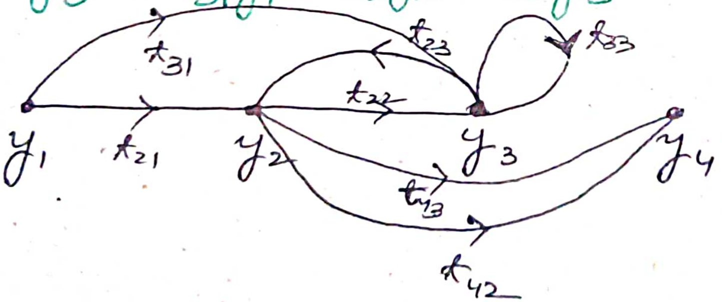
$$\therefore \frac{C(S)}{R(S)} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1 + G_2G_3H_2 + G_1G_2G_3 - G_1G_2H_1}$$

Q:- Construct the SFG for the following equations :-

$y_2 = t_{21}y_1 + t_{23}y_3$

$y_4 = t_{42}y_2 + t_{43}y_2$

$y_3 = t_{31}y_1 + t_{22}y_2 + t_{33}y_3$



# 5. TIME DOMAIN ANALYSIS OF CONTROL SYSTEM

5.1

**Time Response** :- It is the output of the system as a function of time, when subjected to a known input.

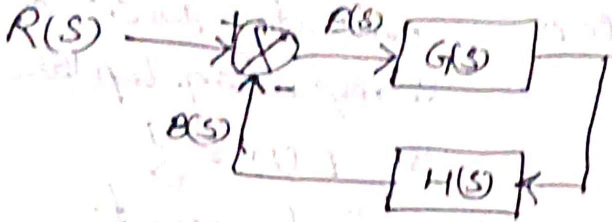
**Steady State Response** :- This response is obtained during the post interval of transient part. Theoritically this response means a state of the output of a control system as the time approaches infinity after initiation of the input.

**Transient Response** :- It is the response that occurs in the initial part of the time response of a control system. This part of time response which goes to zero after large interval of time is known as transient response.

**Accuracy** :-

## Steady State Error

It is the difference between the actual output and desired output



$E(S)$ = Error signal

$B(S)$ = Feedback signal

$$E(S) = R(S) - B(S)$$

& $B(S) = C(S)\,H(S)$

$$\Rightarrow E(S) = R(S) - C(S)\,H(S)$$

$$C(S) = E(S)\,G(S)$$

$$\Rightarrow E(S) = \frac{R(S)}{1 + G(S)H(S)}$$

Applying final value theorem

$$e_{ss} = \lim_{S \to 0} S\,E(S) = \lim_{S \to 0} S\,R(S)\,\frac{1}{1 + G(S)H(S)}$$

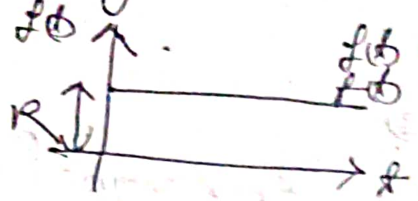
The actual output of control system may be in any physical form, it is called position or displacement. The 1st derivative of actual output is called 'velocity' and the 2nd derivative is acceleration.

### Types of Inputs

Some specified input test signals are applied for time response analysis of a control system are described below :-

### ① Step Function :-

It is defined as sudden application of input signal



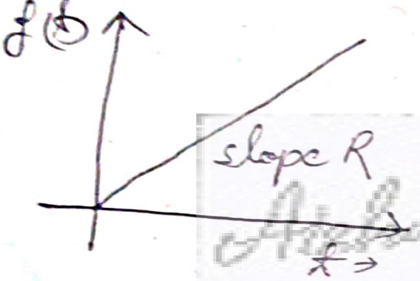
$f(t) = R$ for $t \geq 0$
$f(t) = 0$ for $t < 0$

* If $R = 1$ unit, the step functⁿ is called unit step functⁿ & the corresponding Laplace transform is $L(1) = \frac{1}{s}$

Step function is also called as displacement function.

### ② Ramp Function :-

It is described as gradual application of input signal :-



If $R = 1$, ~~then~~ $f(t) = t$.
then ramp functⁿ is called unit ramp functⁿ & also the corresponding Laplace transform is $L(t) = \frac{1}{s^2}$

Ramp function is also know as velocity function.

### ③ Parabolic Function :-

It is described as more gradual application of input in comparision with ramp function.

If $R = 1$ then $f(t) = \frac{t^2}{2}$ and the parabolic function is called unit parabolic function & Laplace transform of parabolic functⁿ is

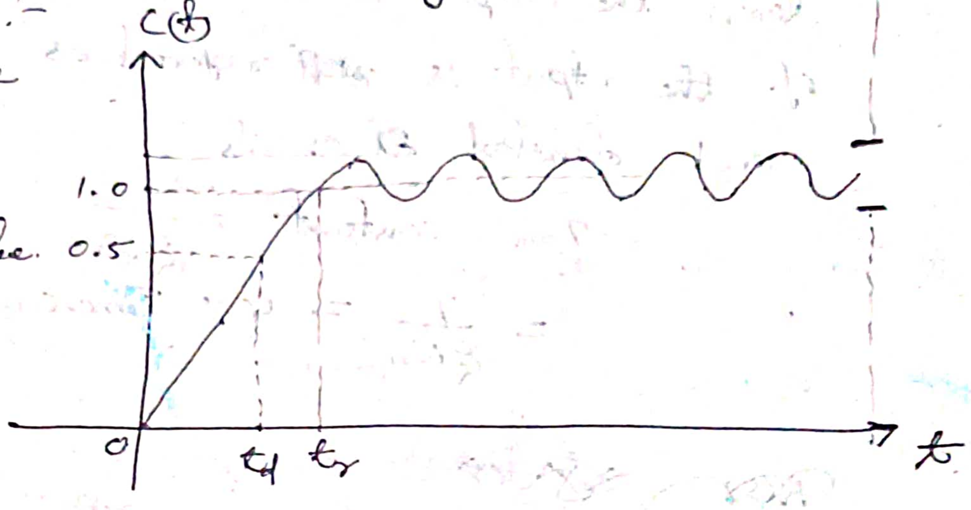
$$L\left(\frac{t^2}{2}\right) = \frac{1}{s^3}$$

The function is also known as acceleration functⁿ

## Time Domain Specifications

The ~~time domain~~ desired performance characteristics of control systems are specifed ~~by~~ in terms of time domain specifications:-

① Delay time:- The time taken for the system to reach 50% of its final value.



② Rise time($t_r$) :- It is the time required for the response to rise from 10% to 90% of the final value for overdamped system, and 0 to 100% of the final value for under damped systems.

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

③ Peak time ($t_p$) :- It is the time required for the response to reach the peak of time response or the peak overshoot.

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

④ Peak overshoot ($M_p$)=

$$M_p = e^{-\pi\xi/\sqrt{(1-\xi^2)}}$$

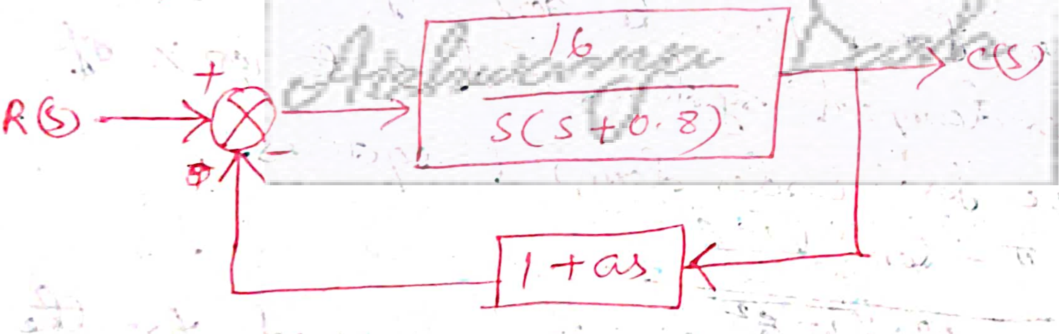
## ④ Settling time ($t_s$)

$t_s$ = Time required to settle down the output within 2% of desired value of the output is refered as settling time and denoted as $t_s$

$$\text{Time constant} = \frac{1}{\xi\omega_n}$$

$$= \frac{4}{\xi\omega_n} = 4 \times \text{Time constant}$$

Q:- Consider the system as shown in the figure below.



Determine the values of rise time and maximum overshoot Mp in the step response.

# STABILITY CONCEPT
# &
# ROOT LOCUS METHOD

Stability of a system is determined by its response to inputs or disturbances.

A stable system is that which will remain at rest unless excited by an external source & will return to rest if all excitations are removed.

## Effect of Location of Poles on Stability :-

# Routh Hurwitz Stability Criterion

In order to determine the existence of a root having +ve real part for a polynomial equation given by :-

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_n = 0$$

How the arrays are formed is illustrated below

$$a_0 s^6 + a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s^1 + a_6 = 0$$

$$\begin{array}{c|lllll}
s^6 & a_0 & a_2 & a_4 & a_6 & \text{(Even terms coefficients)} \\
s^5 & a_1 & a_3 & a_5 & & \text{(Odd coefficients)} \\
s^4 & b_1 & b_3 & b_5 & & \\
s^3 & c_1 & c_3 & & & \\
s^2 & d_1 & d_3 & & & \\
s^1 & e_1 & & & & \\
s^0 & f_1 & & & &
\end{array}$$

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad c_1 = \frac{b_1 a_3 - a_1 b_3}{b_1}$$

$$b_3 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad d_1 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

$$b_5 = \frac{a_1 a_6 - a_0 \times 0}{a_1} = a_6$$

→ The number of changes of sign in the 1st column elements of Routh array gives the number of +ve real part roots of the polynomial.

→ Therefore, for a stable system there should be no change of sign in the 1st column of Routh array.

Ex - 1 : A closed loop control system has the characteristics equation given by $s^3 + 4.5s^2 + 3.5s + 1.5 = 0$ Find out whether the system is stable or not.

Sol$^n$ :-

| | | |
|---|---|---|
| $s^3$ | 1 | 8.5 |
| $s^2$ | 4.5 | 1.5 |
| $s^1$ | 3.16 | 0 |
| $s^0$ | 1.5 | 0 |

$$\frac{4.5\times3.5 - 1\times1.5}{4.5} = 3.16$$

$$\frac{3.16\times1.5 - 4.5\times0}{3.16}$$

No sign change occured in the 1st column, Hence all poles lies in the left half of the imaginary axis.

∴ There is no root of characteristic equation with +ve real part. Hence the system is stable.

## Same Special Cases

① When 1st column term in any row is zero.

It leads two ~~cas~~ conclusions :-

(i) Equal roots with opposite signs. As one of the root is +ve, the system is ~~stablas~~ unstable as indicated by the sign change in the 1st column.

(ii) Pair of conjugate root on imaginary axis. This gives marginally stable system providing there is no sign change in the 1st column.

**N. B.**

In Routh-Hurwitz application no power of 'S' is absent.

(i) Any absence of such power indicates the presence of at least one +ve real part root and confirms system unstable.

(ii) If characteristic equation has either only odd powers of 's' or even powers of 's', this indicates that, roots have no real parts & posseses only imaginary part.

**Ex-2** : Consider the following equations :-

$$s^4+3s^3+s^2-3s-2$$

Solⁿ :-

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 1 | −2 |
| $s^3$ | 3 | −3 | |
| $s^2$ | 2 | −2 | |
| $s^1$ | 0 | | |
| $s^0$ | | | |

For this case we suppose this as very small +ve value ε where ε → 0 , then the array becomes:-

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 1 | −2 |
| $s^3$ | 3 | −3 | |
| $s^2$ | 2 | −2 | |
| $s^1$ | $\varepsilon$ | | |
| $s^0$ | $\frac{-2\varepsilon-2}{\varepsilon}$ | | |

$\therefore \frac{-2\varepsilon-2}{\varepsilon}$ negative

Since, ε is very small positive value.

There is one sign change, hence the system is unstable.

**Ex-3** : $s^4+3s^3+3s^2+3s+2=0$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 3 | 2 |
| $s^3$ | 3 | 3 | |
| $s^2$ | 2 | 2 | |
| $s^1$ | 0 $(\varepsilon)$ | | |
| $s^0$ | 2 | | |

No sign change occurs. It indicates that, the system has pair of conjugate root on imaginary axis. The system is marginally stable.

## ② When all elements of any row becomes zero :-

In this case, the row having all elements zero. We form an equation taking elements of just above of the row containing zero elements. This equation is called auxilliary equation. Now differentiating this equation we can find the elements of next row and proceed.

There are two cases arised that shows :-

(i) If all elements If a row in Routh's table are zero, it indicates a pair of conjugate root on imaginary axis and when we apply coefficients $\frac{dA(s)}{ds}$ in the zero elements. Now we proceed, if no sign changes, it means marginally stable.

(ii) If any two row of Routh table becomes zero, it indicates repeated root on imaginary axis, means system is unstable, & even if it has no sign change.

Ex - 4 : $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 8 | 20 | 16 |
| $s^5$ | 2 | 12 | 16 | 0 |
| $s^4$ | 2 | 12 | 16 | 0 |
| $s^3$ | 0 | 0 | 0 | 0 |
| $s^2$ | | | | |
| $s^1$ | | | | |
| $s^0$ | | | | |

→ Auxillary equation

$A(s) = 2s^4 + 12s^2 + 16$

~~$dA(s)$~~

→ All zero element row

Since the order of auxillary equation is 4 and order of characteristic equation is 6, Hence $(6-4) = 2$ roots lie in the left half of s plane.

Now we formed again Routh - Hurwitz table :-

| | | | | | |
|---|---|---|---|---|---|
| $s^6$ | 1 | 8 | 20 | 16 | |
| $s^5$ | 2 | 12 | 16 | 0 | |
| $s^4$ | 2 | 12 | 16 | 0 | |
| $s^3$ | 8 | 24 | | | $\longrightarrow$ coefficients of $\frac{dA(S)}{ds}$ |
| $s^2$ | 6 | 16 | | | |
| $s^1$ | 2.67 | | | | |
| $s^0$ | 16 | | | | |

There is no sign change, this indicates that, system is marginally stable. Here auxillary equation has order 4 and after this no sign change Hence 4 roots lie on imaginary axis, no right side pole.

**Ex - 5** :- $B(S) = s^6+3s^5+6s^4+12s^3+12s^2+12s+8 = 0$

Find the stability of the system.

Sol$^n$

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 6 | 12 | 8 |
| $s^5$ | 3 | 12 | 12 | |
| $s^4$ | 2 | 8 | 8 | |
| $s^3$ | 0 | 0 | 0 | |
| $s^2$ | | | | |
| $s^1$ | | | | |
| $s^0$ | | | | |

→ Auxillary equation

$A(s) = 2s^4 + 8s^2 + 8 = 0$

→ all zero row

$$\frac{dA(s)}{ds} = 8s^3 + 16s = 0$$

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 6 | 12 | 8 |
| $s^5$ | 3 | 12 | 12 | |
| $s^4$ | 2 | 8 | 8 | |
| $s^3$ | 8 | 16 | 0 | |
| $s^2$ | 4 | 8 | | |
| $s^1$ | 0(8) | 0 | | |
| $s^0$ | 8 | | | |

→ Auxillary equation

$A(s) = 4s^2 + 8$

$\frac{dA(s)}{ds} = 8s$

→ all zero row

There is no sign change in first ~~of~~ column but there are two rows which have zero elements. Hence system is unstable with repeated root on imaginary axis. 1st auxillary equation has order 4 and after & before the auxillary equation, there is no sign change. Hence no Right hand poles (RHP) & 4 pole on imaginary axis and 2 left hand pole (LHP).

**Ex-6** : The open-loop T.F. of a unity feedback control system is given by:-

$$G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$$

Determine the range of K for which system is stable.

**Solⁿ** :-

Characteristics equation is given by :-

$$1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0$$

$$(s+2)(s+4)(s^2+6s+25) + k = 0$$

$$\Rightarrow (s+2)(s^3+6s^2+25s+4s^2+24s+100)+k=0$$

$$\Rightarrow (s+2)(s^3+10s^2+49s+100)+k=0$$

$$\Rightarrow s^4+6s^3+49s^2+100s+2s^3+12s^2+98s+200+k=0$$

$$\Rightarrow s^4+8s^3+61s^2+198s+200+k$$

① For unity -ve feedback system

$$G(S) = \frac{K(S+1)(S+2)}{(S+0.1)(S-1)}$$

Determine the range of value of K for which the closed loop system has 0, 1, or 2 poles in Right half of s-plane.

② Apply Routh-Hurwitz criteria to

$$3s^4+10s^3+5s^2+5s+2=0$$

③ $s^4+8s^3+18s^2+16s+5=0$

④ $G(S)H(S) = \frac{1}{(S+2)(S+4)}$

⑤ $G(S)H(S) = \frac{1(S+3)}{s(S+3)(S+8)}$

⑥ $G(S)H(S) = \frac{9}{s^2(s+2)}$

⑦ Find the range of K for which the system is stable

ⓐ $s^3+2ks^2+(k+2)s+4=0$

ⓑ $s^4+4s^3+13s^2+36s+k=0$

ⓒ $s^4+20ks^3+5s^2+10s+15=0.$

# ROOT LOCUS

The locus of roots of characteristics equation when gain is varied from zero to infinity is called root locus. Since it is the plot of roots of characteristics equation means poles of closed loop system. Hence, from characteristics equation:-

$$1 + G(S)H(S) = 0$$

$$\Rightarrow G(S)H(S) = -1$$ ~~$\angle G(S)H(S) = \pm(2K+1)180$~~

Here we can find magnitude and angle

$$|G(S)H(S)| = 1$$

$$\angle G(S)H(S) = (2K+1)\,180°.$$

**Ex:-** $G(S)H(S) = \dfrac{K}{s(s+4)(s+5)}$, Find whether $s=-1$ is on root locus or not.

$$\frac{\angle K + j0}{\angle(-1+j0)\,\angle(3+j0)\,\angle(4+j0)} = \frac{0°}{180°\ 0°\ 0°} = -180°$$

Since $\angle G(S)H(S) = -180°$ at $s=-1$, this satisfies the angle criterion. Hence point $s=-1$ is on root locus.

Ex - 2 : For the above equation find the value of 'K' at $s = -1$.

Solⁿ : Since $s = -1$ satisfies angle criterion. Hence we can use magnitude criterion.

$$|G(s)\,H(s)| = 1$$

$$\frac{K}{(-1|\,|4-1||5-1|)} = 1$$

$$\Rightarrow \frac{K}{12} = 1$$

$$\Rightarrow K = 12$$

Here we can conclude two points :-

① Angle criterion is used to find whether any point in plane lies on root locus or not. Any point satisfying angle criterion must be the root of characteristics equation.

② When it is confirmed the point is lying on root locus, then using magnitude criterion one can find value of K at that point.

# Rules for Construction of Root Locus :-

→ Root locus is symmetrical about the real axis.

→ Root locus starts from an open loop pole with $K=0$.

→ The root locus terminates ($K = \infty$) either on open loop zero or at infinity.

**Ex :-**

$$G(S)H(S) = \frac{K(S+3)}{(S+2)}$$

. Find the starting & ending points of the root loci.

**Soln**

There is one open loop pole $s = -2$

Hence root locus starts from $s = -2$ with $K=0$

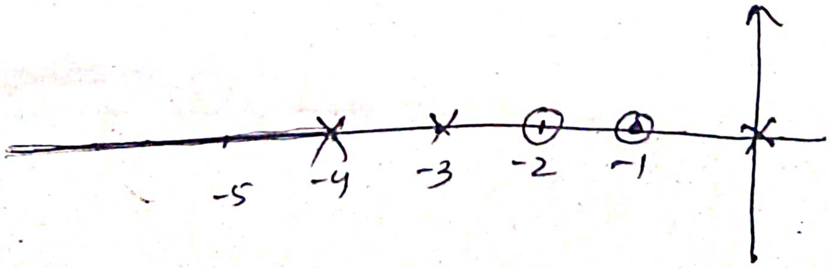
And there is one open loop zero at $s = -3$

Hence, root locus ends at $s = -3$ with $K = \infty$

→ A point on the real axis lies on the locus or if the number of open loop poles plus zeros on the real axis to the right of this points is odd.

**Ex :-**

$$G(S)H(S) = \frac{K(S+1)(S+2)}{S(S+3)(S+4)}$$



Q:- For a unity feedback system

$$G(s) = \frac{K}{s(s+4)(s+5)}$$

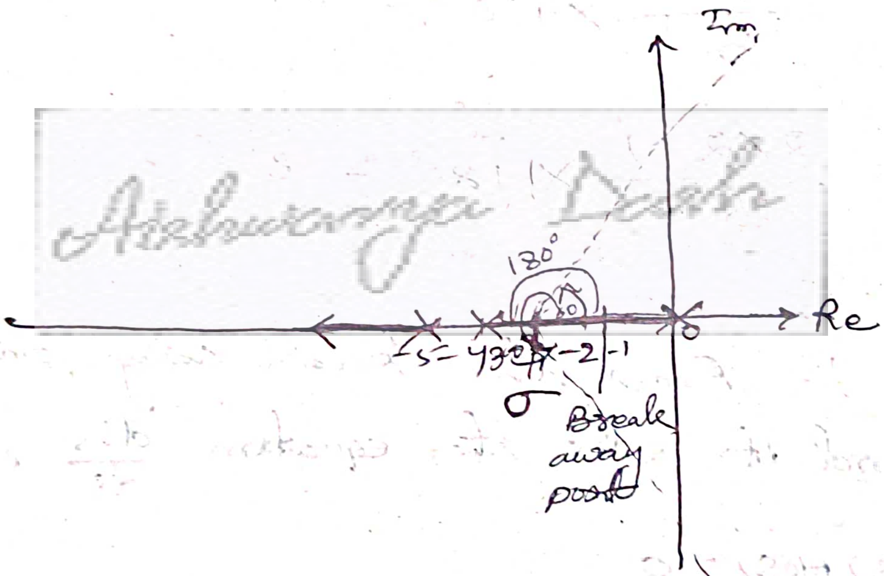
step - I :- Roots of the equation :-

Number of poles = (n) = 3

Pole locations = 0, -4, -5

Number of zeros = (m) = 0

step : II :- Plotting of pole - zero on s - plane.



Step - III : Find the existence of root locus

RL lies between -5 to -∞

and 0 to -4

step - IV : Find the centroid :-

$$\sigma = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{n - m} = \frac{0 + (-4) + (-5) - 0}{3 - 0} = \frac{-9}{3} = -3$$

Step-V : Angle of asymptotes :-

Number of asymptotes :- $n - m = 3$

Hence, there ~~is~~ would be 3 angles of asymptotes.

$$\phi = \cancel{\frac{2k+1}{n-m}} \; \frac{2k+1}{n-m} \times 180^\circ$$

$$\phi_1 = \frac{2\times 0+1}{\cancel{3}} \times \overset{60^\circ}{\cancel{180^\circ}} = 60^\circ$$

$$\phi_2 = \frac{2\times 1+1}{3} \times 180^\circ = 180^\circ$$

$$\phi_3 = \frac{2\times 2+1}{\cancel{3}} \times \overset{60^\circ}{\cancel{180^\circ}} = 300^\circ$$

Step-VI :- Find the break away point.

To find it solve the equation $\frac{dk}{ds} = 0$

$1 + G(S)H(S) = 0$

$\Rightarrow s(s+4)(s+5) + k = 0$

$\Rightarrow (s^2+4s)(s+5) + k = 0$

$\Rightarrow s^3 + 5s^2 + 4s^2 + 20s + k = 0$

$\Rightarrow K = -s^3 - 9s^2 - 20s$

$\Rightarrow \frac{dk}{ds} = 0$

$\Rightarrow 3s^2 + 9s + 20 = 0$

$\Rightarrow s_1 = -4.47$ (This point is out of the root locus)

$-1.47$ (Exists on root locus.

**Step - VII** : Find the point of intersection on imaginary axis.

Apply Routh Hurwitz criterion

|  |  |  |
|---|---|---|
| $s^3$ | 1 | 20 |
| $s^2$ | 9 | K |
| $s^1$ | $\frac{180-K}{9}$ |  |
| $s^0$ | K |  |

* When root is on imaginary axis, the system is marginally stable. That means any one row of the R-H array must be zero.

Equating $\frac{180-K}{9} = 0$

$\Rightarrow 180 - K = 0$

$\Rightarrow K = 180$

Auxillary equation = $A(s) = 9s^2 + K = 0$

$\Rightarrow 9s^2 + 180 = 0$

$\Rightarrow s^2 = \frac{-180}{9} = -20$

$\Rightarrow s = \pm j4.47$

Draw the Root Locus plot for the below T.F s.

① $G(s) = \dfrac{K}{s(s+2)(s+4)}$

② $G(s) = \dfrac{10}{s(s+1)(s+2)}$

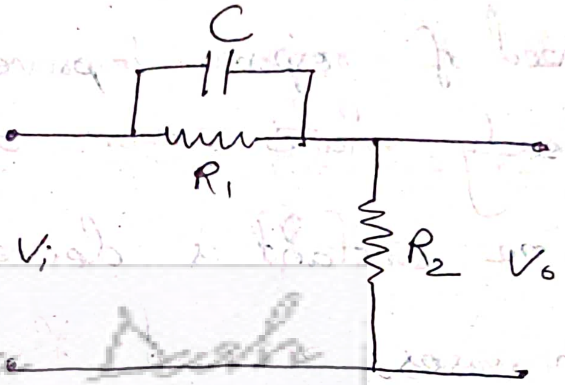
③ $G(s) = \dfrac{k}{s(s+4)(s+5)}$

## Compensator

The additional device added in control system to obtain the performance as per desired specification is known as compensator.

## Phase Lead Compensator

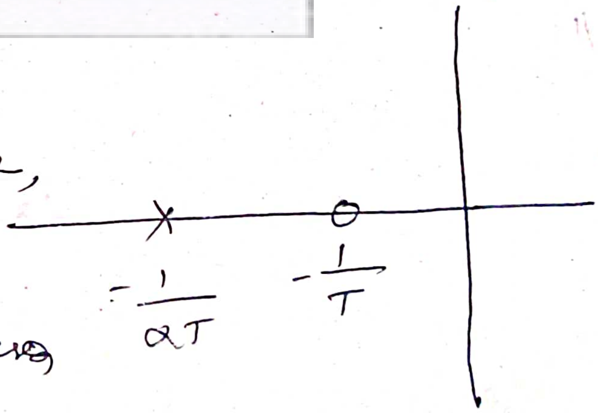
$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+sT)}{1+s\alpha T}$$



$$\alpha = \frac{R_2}{R_1+R_2} \quad \& \quad T = R_1C$$

where $\alpha < 1$

This is evident from figure, phase lead compensator is zero dominant, thereby increases phase shift.



Two corner frequency

$$\omega_1 = \frac{1}{T} \text{ (Lower)}$$

$$\omega_2 = \frac{1}{\alpha T} \text{ (upper)}$$

Minimum phase lead occurs at mid corner frequency $\omega_m$.

$$\omega_m = \frac{1}{2}\left[\log_{10}\left(\frac{1}{T}\right) + \log_{10}\left(\frac{1}{\alpha T}\right)\right]$$

Phase angle :

$$\tan \phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

or

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

## Properties

→ It shifts gain cross over frequency to a higher value. Thus bandwidth is increased.

→ Speed of response improved

→ Steady state error does not show much improvement.

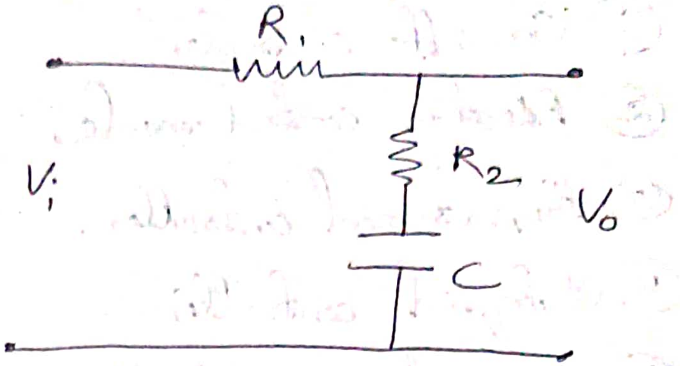
→ Time constant is decreased.

→ Increases resonant frequency.

//

# Phase Lag Compensator

$$\frac{V_o(S)}{V_i(S)} = \frac{1+ST}{1+S\beta T}, \beta > 1$$

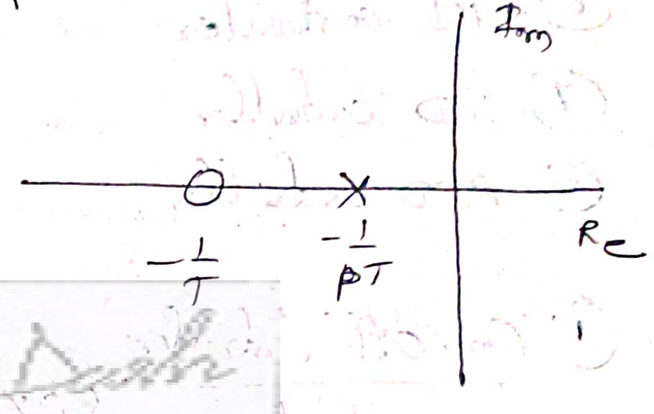


$$\beta = \frac{R_1+R_2}{R_2} \quad \& \quad T = R_2C$$

It is a pole-dominant network

$$\omega_1 = \frac{1}{T} \text{ (upper)}$$

$$\omega_2 = \frac{1}{\beta T} \text{ (lower)}$$



Maximum phase-lag occurs at mid frequency,

$$\omega_m = \frac{1}{\sqrt{\beta} T}$$

$$\tan \phi_m = \frac{1-\beta}{2\sqrt{\beta}} \quad \text{or} \quad \sin \phi_m = \frac{1-\beta}{1+\beta}$$

## Properties :

→ It drops the magnitude curve down to 0 dB at the gain crossover frequency.

→ Bandwidth is somewhat reduced

→ Improvement in steady state error is observed.

→ Speed of response is reduced.

→ Time constant increases.

# Controllers

① On-off controller
② Floating control mode
③ Proportional controller.
④ Integral controller
⑤ Derivative controller
⑥ PI controller
⑦ PD controller
⑧ PID controller,

## ① On-Off controller

$$\text{Output } (P) = \begin{cases} 0\% & \text{for error} < 0 \\ 100\% & \text{for error} > 0 \end{cases}$$

It is best adapted for relatively slow process rates.

## ② Floating control Mode

The specific output of controller is not uniquely determined by the error.

Output floats when error comes to zero, when error occurs output changes

Controller output

$$P = \pm K_F t + P(0)$$

$K_F$ = Rate constant

$P(0)$ = controller output at $t = 0$

### ③ Proportional Controller

$$P = K_p e_0 + P_0$$

$K_p$ = Proportional gain

$P_0$ = Controller output with no error

Properties :

→ Sluggish overdamped response can be made faster

→ Maximum overshoot can be reduced without sacrificing steady state accuracy.

→ Zero error controller output can never be achieved offset is introduced due to load changes.

### ④ Derivative Controller

$$\text{output}(P) = K_p \frac{de_p}{dt}$$

It is not used alone because it can not produced output when error is constant, output returns to its nominal value.

### ⑤ Integral Controller

$$P(\cancel{t}) = \int_0^t e_p dt + P(0)$$

P is output , P = P(0) when error = 0

if error exists P = ramps up or down and finally saturated at 100% output.

## ⑤ Properties

- → Reset gain $\frac{1}{K_I}$, also known as reset controller.
- → For too large process lag - error oscillates about zero
- → Can be used alone only with small process lag.

## ⑥ Proportional Derivative Controller (PD)

Output $P = K_p e_p + K_p . K_D \frac{de_p}{dt} + P_0$

$e_p$ = % error

### Properties

- → Effective damping is increased.
- → Maximum overshoot is reduced
- → Natural frequency remains unchanged
- → Rise time is reduced.

## ⑦ PI Controller

$$P = K_p e_p + K_p K_I \int_0^I e_p dt + P_I(0)$$

$P_I(0)$ = initial value.

## ⑧ Derivative Feedback Controller

Actuating signal = Proportional signal − derivative of O/P

**Properties**

① Damping ratio increased

② Maximum overshoot is reduced

③ Risetime is increased

④ steady state error is increased.

## ⑨ PID Controller

Most powerful but complex.

$$P = K_p e_p + K_p K_I \int_0^t e_p dt + K_p K_D \frac{de_p}{dt} + P_I(0)$$

# STATIC ERROR COEFFICIENTS

For evaluating steady state error, the input functions is specified as either unit step (displacement) or unit ramp (velocity) or unit parabolic (acceleration). Accordingly there are different types of static error coefficients.

## (i) Static Positional error coefficient :

When input applied is unit step, then static error is positional error coefficient $K_p$.

We know that

$$e_{ss} = \lim_{S \to 0} S\, R(S) \frac{1}{1 + G(S)\, H(S)}$$

As the input is $R(S) = \frac{1}{S}$

$$e_{ss} = \lim_{S \to 0} \cancel{S} \frac{1}{\cancel{S}} \frac{1}{1 + G(S)\, H(S)}$$

$$= \lim_{S \to 0} \frac{1}{1 + G(S)\, H(S)} = \frac{1}{1 + \lim_{S \to 0} H(S)\, G(S)}$$

Putting $K_p = \lim_{S \to 0} H(S)\, G(S)$

$$\boxed{e_{ss} = \frac{1}{1 + K_p}}$$ → Steady state error

$$\boxed{K_p = \lim_{S \to 0} G(S)\, H(S)}$$ → Positional error coefficient.

## (ii) Static Velocity Error Coefficient

When input applied is unit ramp, then the static error is ~~unit ramp~~ known as velocity error coefficient and denoted by $K_V$.

Steady state error with unit ramp is given by

$$e_{ss} = \lim_{S \to 0} S \cdot \frac{1}{S^2} \frac{1}{1 + G(S)\,H(S)}$$

$$= \lim_{S \to 0} \frac{1}{S + S\,G(S)\,H(S)}$$

$$e_{ss} = \frac{1}{\lim_{S \to 0} S\,G(S) + H(S)} = \frac{1}{K_V}$$

Where $\boxed{K_V = \lim_{S \to 0} S \cdot G(S)\,H(S)}$ → Velocity error coefficient.

## (iii) Static Acceleration Error Coefficient

When the applied input is unit parabolic the static error is known as static acceleration error coefficient and is denoted by $K_a$.

Steady state error is given by

As $R(S) = \frac{1}{S^3}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^3} \frac{1}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s) H(s)}$$

Putting $K_a = \lim_{s \to 0} s^2 G(s) H(s)$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 G(s) H(s)} = \frac{1}{K_a}$$

where $K_a$ = acceleration error coefficient.

## Type of a system

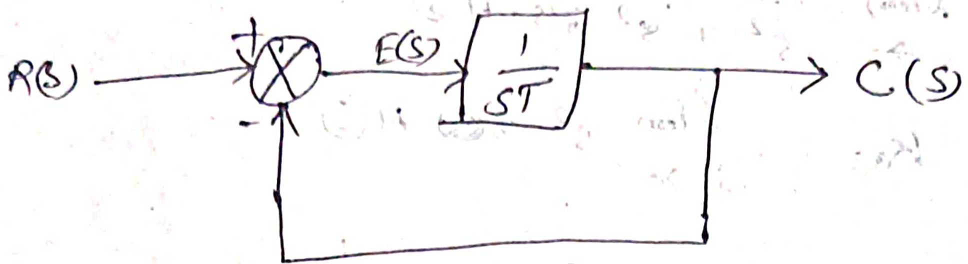
The number of poles existing on origin ~~decid~~ decides types of a system.

## Order of a system

The total number of poles of a system is the order of a system.

# Time Response of 1st Order Control System

For a 1st order system $G(S) = \frac{1}{ST}$



Let us take unity feedback $H(S) = 1$

∴ Overall transfer function =

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + H(S)\,G(S)}$$

$$= \frac{\frac{1}{ST}}{1 + 1 \times \frac{1}{ST}}$$

$$= \frac{1/ST}{(ST+1)/ST} = \frac{1}{ST+1}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{1}{1+ST}}$$

# Time Response of 1st order system with unit step Input

$$\frac{C(S)}{R(S)} = \frac{1}{1+ST}, \qquad R(S) = \frac{1}{S}$$

$$\Rightarrow C(S) = R(S)\frac{1}{1+ST} = \frac{1}{S} \times \frac{1}{1+ST}$$

$$C(S) = \frac{A}{S} + \frac{B}{1+ST}$$

$$= \left.\frac{A}{S} + \frac{B}{1+ST}\right]_{S \to \frac{-1}{T}}$$

[illegible]

For $S = 0$

$$A(1+0) + 0 = 1$$

For $S = \frac{-1}{T}$

$$A \times 0 + B\frac{-1}{T} = 1$$

$$B = -T$$

$$\therefore C(S) = \frac{1}{S} + \frac{-T}{1+ST}$$

$$\Rightarrow \boxed{c(t) = 1 - Te^{-t/T}}$$

$$\therefore \text{Error} = r(t) - c(t) = e(t)$$

$$= 1 - (1 - e^{-t/T}) = e^{-t/T}$$

$$\therefore e_{ss} = \lim_{t \to \infty} e^{-t/T} = 0$$

# Time Response of first order System with Unit Ramp Input.

The output for the system is expressed as

$$C(S) = R(S)\frac{1}{ST+1} , \quad R(S) = \frac{1}{S^2}$$

Hence output $c(t) = t - T + Te^{-t/T}$

and error is given by

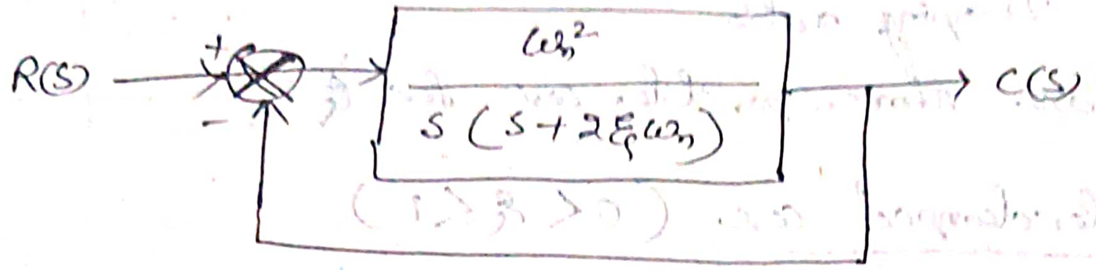
$$e(t) = r(t) - c(t) = T - Te^{-t/T}$$

Steady state error

$$e_{ss} = \lim_{t \to \infty}\left(T - Te^{-t/T}\right) = T$$

## Time Response of a Second Order System :-

The block diagram of a 2nd order system is given below :-



$$G(S) = \frac{\omega_n^2}{s(s+2\xi\omega_n)} \quad ; \quad H(S) = 1$$

$$\therefore \frac{C(S)}{R(S)} = \frac{G(S)}{1+H(S)\,G(S)} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+1\times\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}$$

$$= \frac{\omega_n^2}{s(s+2\xi\omega_n)+\omega_n^2} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore \boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

Taking input as unit step $r(t) = 1$, $R(S) = \frac{1}{s}$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left\{(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right\}$$

∴ Error for the system $= e(t) = r(t) - c(t)$

$$= \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left\{(\omega_n\sqrt{1-\xi})t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right\}$$

where $\omega_n$ = Natural frequency of oscillation

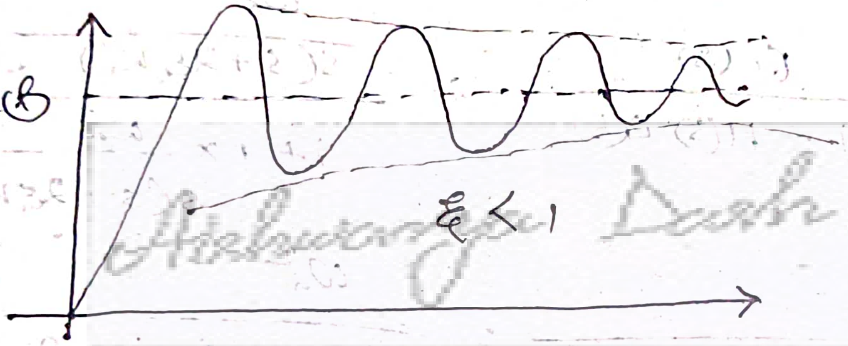
$\omega_d = \omega_n\sqrt{1-\xi^2}$ is called damped frequency of oscillation.

$\xi$ = Damping ratio

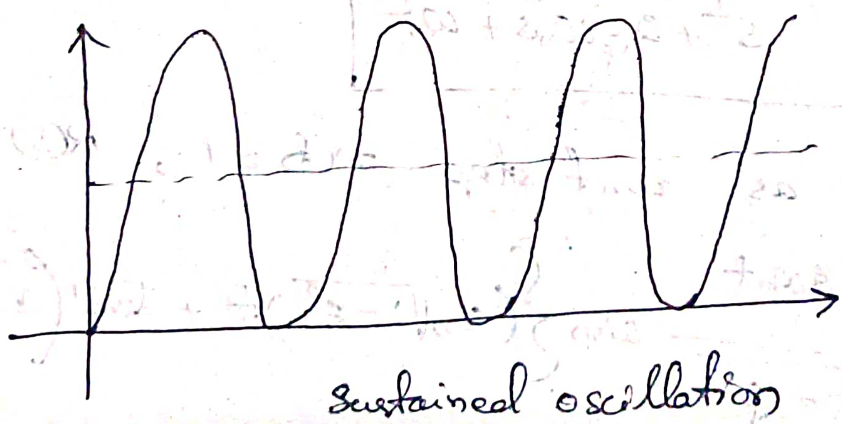
There are three possible cases for "$\xi$"

ⓐ Underdamped case $(0<\xi<1)$

For $\xi<1$, the response is called underdamped response

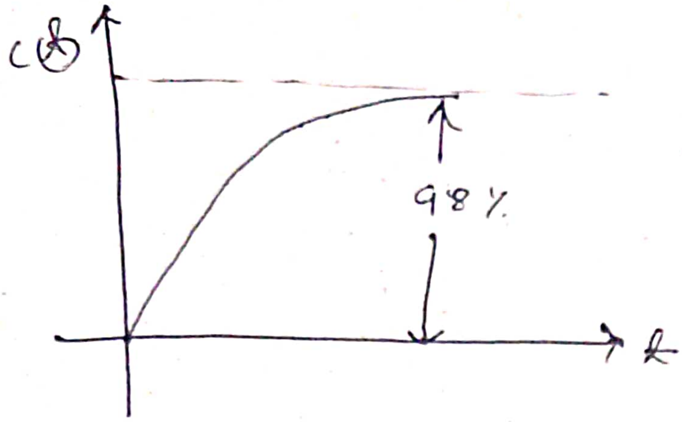


ⓑ When $\xi=0$ - Undamped Case

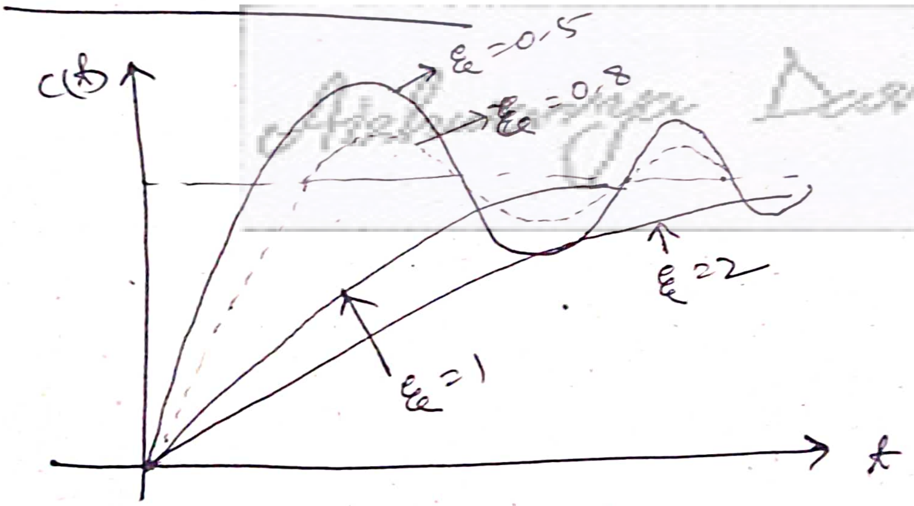


Sustained oscillation

## ⓒ $\xi = 1$ Critically damped ~~$\xi = 1$ case~~



## ⓓ $\xi > 1$ - Over damped case

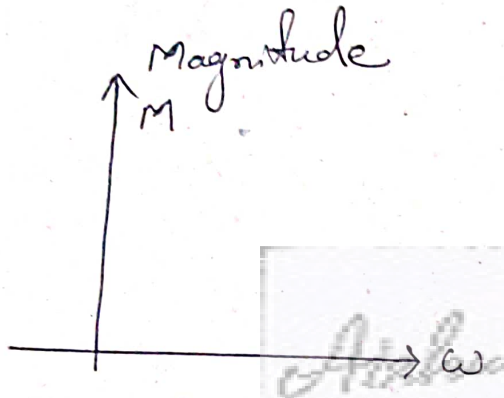


# Bode Plot

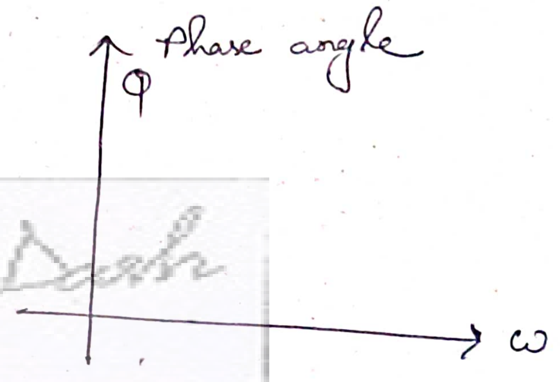
Bode plot is a graphical method to analyze the stability of a system in frequency domain.

Bode plot consists of two subplots :-

→ Magnitude plot
→ Frequency plot



$M = 20 \log |G(j\omega)H(j\omega)|$



$\phi = \angle G(j\omega) H(j\omega)$

→ It tests the relative stability of open loop transfer function & not related to closed loop transfer function.

→ For a system to be stable, both the gain margin & phase margin must be +ve.

## Basic Bode Plots

1. Constant K

2. Integral Term : $\frac{K}{j\omega}$ or $\frac{K}{(j\omega)^n}$

3. Derivative term : $K(j\omega)$, $K(j\omega)^n$

4. First order term in denominator (Pole on real axis)

$$\frac{1}{1+j\omega} \quad \text{or} \quad \frac{1}{(1+j\omega)^n}$$

5. First order term in numerator

$K(1+j\omega)$ or $K(1+j\omega)^n$ (zero on real axis)

6. Quadratic term in Denominator

$$\frac{1}{1+2\xi\frac{j\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2}$$

7. Quadratic term in numerator

$$1+2\xi\frac{j\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2$$

## Procedure to draw Bode Plot

1. Replace s by jω to convert the OLT to frequency domain.
2. Calculate magnitude in dB, $M = 20\log_{10}|G(j\omega)H(j\omega)|$
3. Find phase angle $\phi = \tan^{-1}\left[\frac{\text{Imaginary part}}{\text{Real part}}\right]$
4. Vary ω from minimum to maximum to find M & φ & draw magnitude & phase plot.

## ① Bode plot for constant 'K' :-

$G(s) = K$

$\Rightarrow G(j\omega) = K$ (as there is no 's' term)

Magnitude in dB $= 20\log|G(j\omega)|$

$= 20\log_{10}|K|$

If $K = 1$

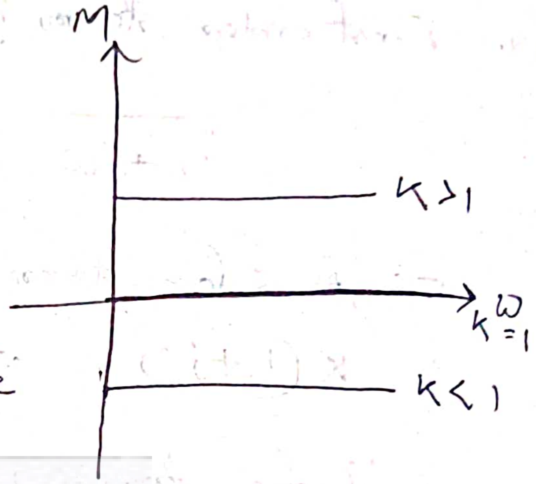
$M_{dB} = 20\log_{10}|1| = 0$

If $K > 1$

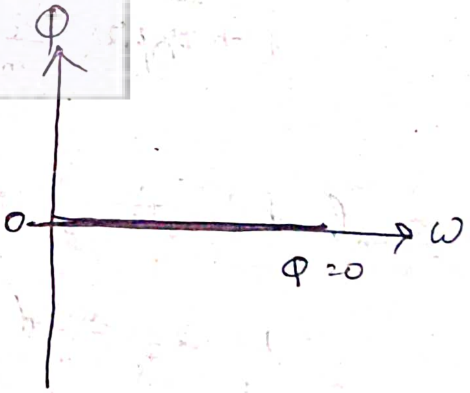
$M_{dB} = 20\log_{10}|K|$ = always +ve

If $K < 1$ ($K$ = -ve or $0<K<1$ fractional value)

$M_{dB} = 20\log_{10}|K|$ = always -ve

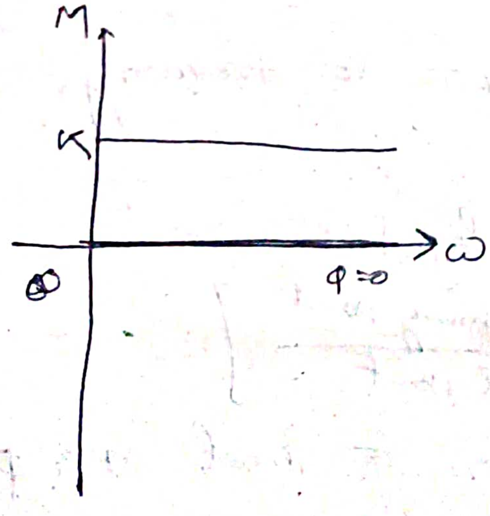


Phase angle $\phi = \tan^{-1}\left(\frac{0}{K}\right) = 0$

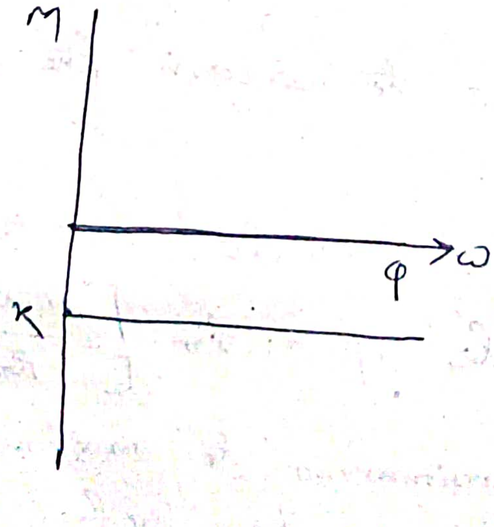


Combining these two plots :-

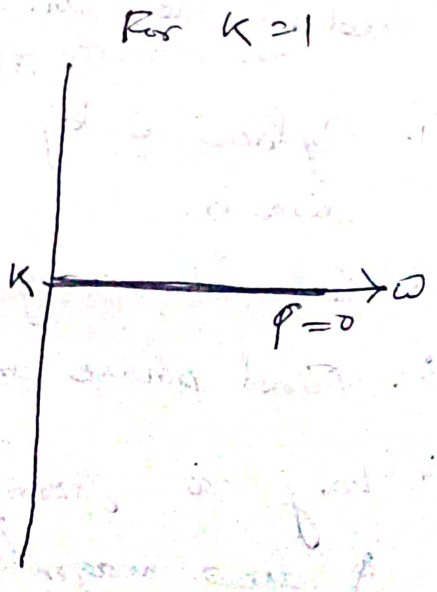
For K>1



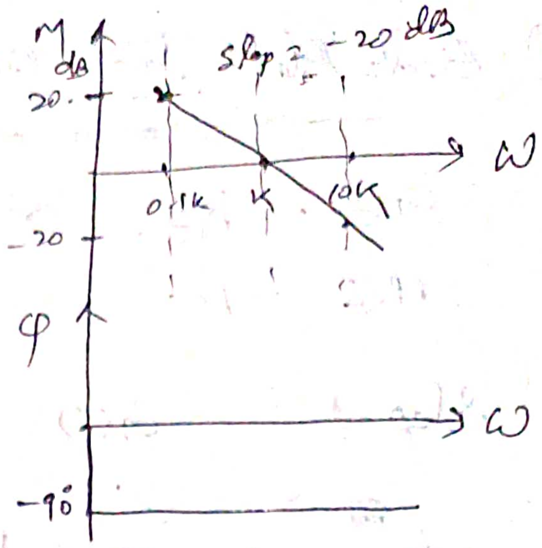
For K<1



For K=1



## ② Bode plot for Integral term :



$\frac{K}{s}$ or $\frac{K}{s^n}$

$G(s) = \frac{K}{s}$

Substitute $s = j\omega$

$G(j\omega) = \frac{K}{j\omega}$

$M_{dB} = 20\log|G(j\omega)| = 20\log_{10}\left(\frac{K}{\omega}\right)$

$\varphi = \angle G(j\omega) = \tan^{-1}\left(\frac{\text{Imaginary}}{0}\right) = -90^\circ$

| ω | 0.1K | K | 10K |
|---|---|---|---|
| $M_{dB}$ | 20 | 0 | -20 |

## ③ Bode Plot for Differential Term

$G(S) = KS$ or $KS^n$

$H(S) = 1$

Step-1 : $G(S) = G(j\omega) = jK\omega$

Step-2 : $M = |G(S)H(S)| = \sqrt{0^2 + (K\omega)^2}$

$= \sqrt{(K\omega)^2} = K\omega$

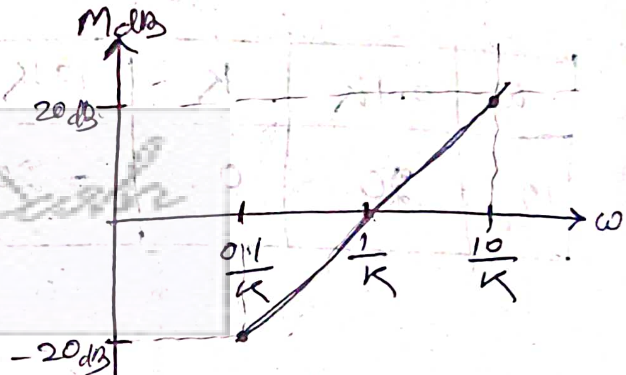
~~step-3:~~ $M_{dB} = 20 \log |G(j\omega)H(j\omega)|$

$= 20 \log K\omega$

For $\omega = \frac{0.1}{K}$

$M_{dB_1} = 20 \log \frac{0.1}{K} \times K$

$= 20 \log 10^{-1} = -20$

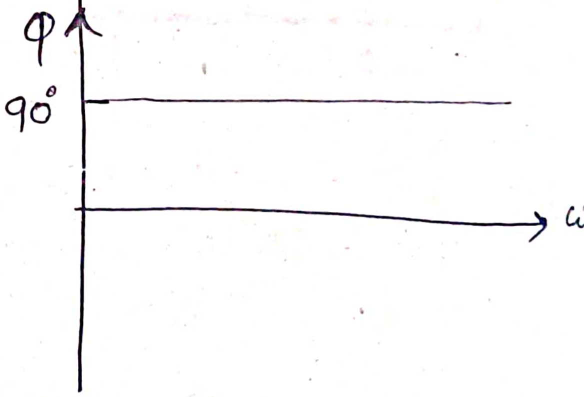


For $\omega = \frac{1}{K}$

$M_{dB_2} = 20 \log \frac{1}{K} \times K = 0$

For $\omega = \frac{10}{K}$

$M_{dB_3} = 20 \log \frac{10}{K} \times K = 20$



| | | | |
|---|---|---|---|
| $\omega$ | $\frac{0.1}{K}$ | $\frac{1}{K}$ | $\frac{10}{K}$ |
| $M_{dB}$ | $-20$ | 0 | 20 |

Step-3 : $\phi = \tan^{-1} \frac{\text{Imaginary}}{\text{Real}}$

$= \tan^{-1} \frac{K\omega}{0} = 90°$

## ④ Bode Plot for 1st order term in Denominator

$$G(S) = \frac{1}{1+ST} \quad \text{or} \quad \frac{1}{(1+ST)^n}$$

let $G(S) = \frac{1}{1+ST}$

$H(S) = 1$

step-1 :- $G(S) = G(j\omega) = \frac{1}{1+j\omega T}$

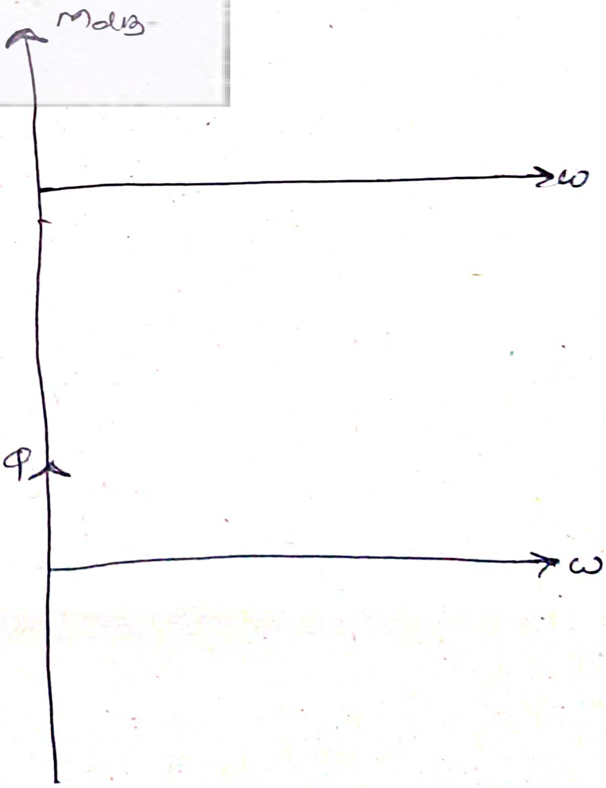
step-2 :- $M = \frac{1}{\sqrt{1+\omega^2T^2}} = (1+\omega^2T^2)^{-\frac{1}{2}}$

$$\Rightarrow M_{dB} = 20 \log (1+\omega^2T^2)^{-\frac{1}{2}}$$

$$= -10 \log (1+\omega^2T^2)$$

step-3 : $\phi = \tan^{-1}\frac{\text{Imaginary}}{\text{Real}} = \tan^{-1}\left(\frac{-\omega T}{1}\right)$

$= -\tan^{-1}\omega T$



$$G(s)H(s) = \frac{1}{s(s+2)(s+4)}$$

$$G(j\omega)H(j\omega) = \frac{1}{j\omega\left(\frac{j\omega}{2}+1\right)\left(\frac{j\omega}{4}+1\right)}$$

The corner frequencies corresponding to 1st order ~~system~~ integral factors are 2 rad/s and 4 rad/s.

Minimum frequeny is choosen 0.01 rad/s

Maximum " " 100 rad/s

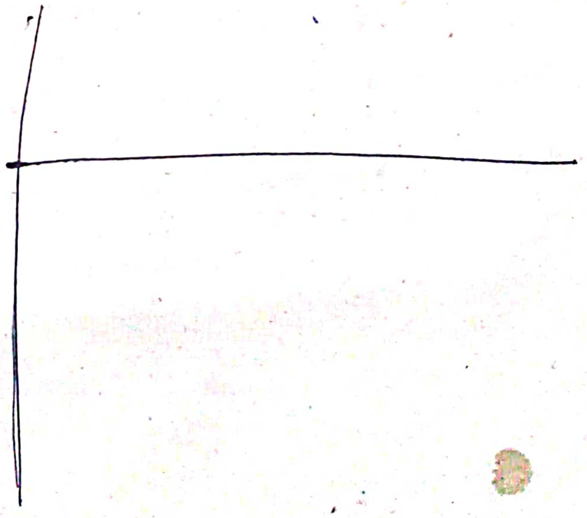
Bode magnitude using asymptotic property of integral ~~factor~~ first order term $\tau = \frac{1}{2}$

$$G(s)\,H(s) = \frac{1}{s^2(s+1)}$$

$$G(j\omega)\,H(j\omega) = \frac{1}{(j\omega)(j\omega)(j\omega+1)}$$

| | | | | | |
|---|---|---|---|---|---|
| | 0.01 | 0.1 | 1 | 10 | 100 |
| $20\log\frac{1}{j\omega}$ | 40 | 20 | 0 | -20 | -40 |
| $20\log\frac{1}{j\omega}$ | 40 | 20 | 0 | -20 | -40 |
| $20\log\frac{1}{j\omega+1}$ | 0 | 0 | -3 | -20 | -40 |
| Bode magnitude | 80 | 40 | -3 | -60 | -120 |

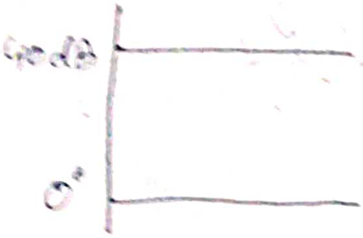
$\frac{1}{s}$



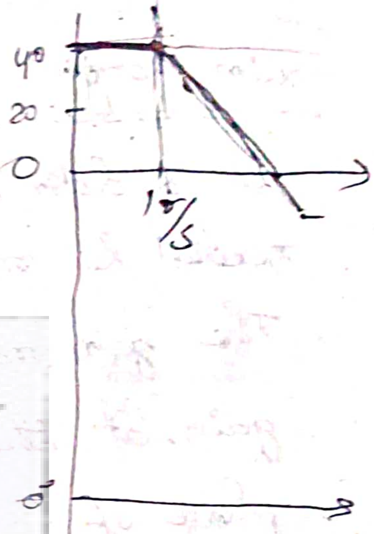
$$G(S)\,H(S) = \frac{300\,s}{S+3}$$

$$= 300\,s \times \frac{1}{3\left(\frac{s}{3}+1\right)} = 100 \times s \times \frac{1}{1+\frac{s}{3}}$$

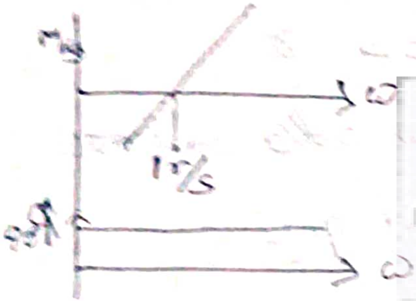
100 (Pure gain)



$M_{dB} = 20 \log 100 = 40$

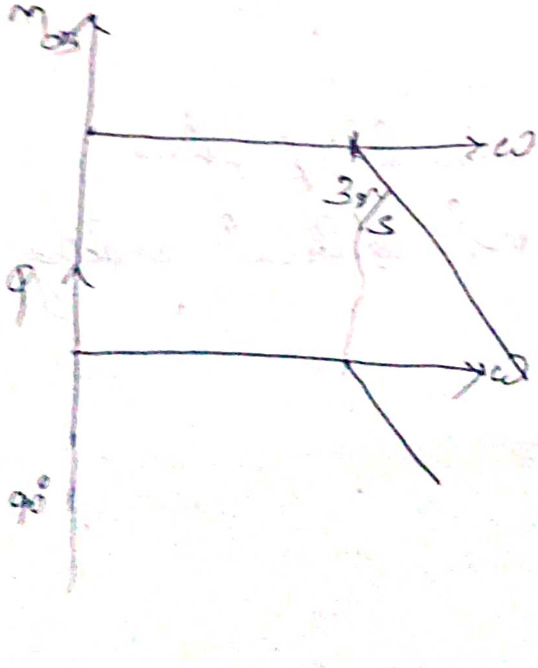


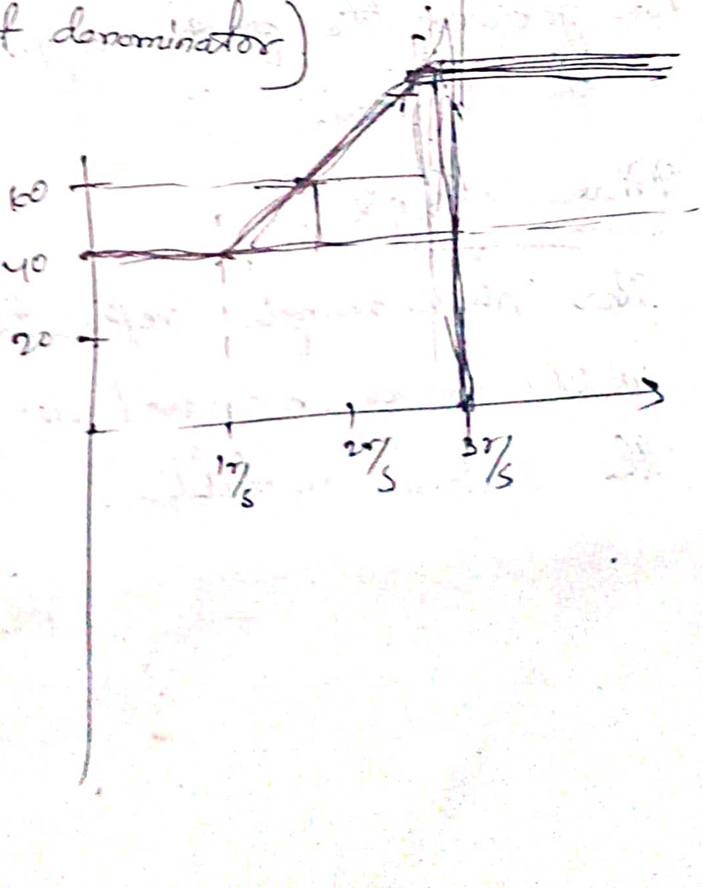
s (Derivative term)



+20dB / slope at all frequency

$\frac{s}{3}+1$ (1st order term at denominator)





# Stability in Frequency Domain

## Phase Crossover Frequency

The frequency at which the phase plot 1st crosses the −180°

## Gain Crossover frequency :-

The gain crossover frequency is the frequency at which the open loop gain first reaches the value 1.

## Gain Margin :

It is defined as the amount of change in open-loop gain needed to make closed loop system unstable.

The gain margin is the difference between 0dB and the gain at the phase cross over frequency that gives a phase of −180°.

The greater the gain margin, the greater the stability of the system.

## Phase Margin :

The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable.

## Nyquist Stability Criterion :

It is useful in frequency domain analysis to determine the relative stability of a closed loop system.

As per principle argument

$$N = Z - P$$

where N = number of encirclements of the origin by Q(S) plane locus.

Z = number of zeros of Q(S) encircles by the S-plane locus

P = number of poles of Q(S) encircles by the S-plane locus.

Q:-1. Sketch the Bodeplot for the T.F.

$$G(S) = \frac{1000}{(1+0.1S)(1+0.001S)}$$

Q-2:- Obtain the Bode plot for the following function

$$G(S) = \frac{10(s+10)}{s(s+100)}$$

Q-3:- Sketch the Bodeplot for

$$G(S)H(S) = \frac{10(s+5)^2}{s^3(s+100)}$$

Q:-4:- Draw the Bodeplot for

$$\frac{100(0.02S+1)}{(s+1)(0.1s+1)(0.01s+1)}$$

Q-5:- Sketch the Bodeplot for

$$G(S) = \frac{10}{s(1+0.5s)(1+0.01s)}$$

## Nyquist Contour

In order to investigate the presence of any zero of $q(S) = 1 + G(S)\,H(S)$ ( characteristic eq$^{n}$) in the right half s-plane, let us choose a contour which completely encloses

$$E_b(S) = s K_b \theta(S)$$

$$(sL_a+R_a) I_a(S) = E_a(S) - E_b(S)$$

$$(s^2 J + sf)\,\theta(S) = T_m(S) = K_t I_a$$

$$\theta(S) = \frac{E_b(S)}{sK_b} \Rightarrow E_b = sK_b\,\theta(S)$$

$$\theta(S) = \frac{K_t I_a(S)}{s^2 J + sf}$$

$$E_a(S) = (sL_a+R_a)\,I_a(S) + E_b(S)$$

$$= (sL_a+R_a)\,I_a(S) + sK_b\,\theta(S)$$

$$\therefore \frac{\theta(S)}{E_a(S)} = \frac{\dfrac{K_t I_a(S)}{s^2J+sf}}{(sL_a+R_a)\,I_a(S) + sK_b\,\theta(S)}$$

$$= \frac{K_t I_a(S)/s^2J+sf}{(sL_a+R_a)\,I_a(S) + sK_b \dfrac{K_t I_a(S)}{s^2J+sf}}$$

$$= \frac{K_t I_a(S)/\cancel{s^2J+sf}}{\dfrac{(sL_a+R_a)(s^2J+sf)\,I_a(S) + sK_b K_t I_a(S)}{\cancel{s^2J+sf}}}$$

$$= \frac{K_t \cancel{I_a(S)}}{\left[(sL_a+R_a)(s^2J+sf) + sK_b K_t\right]\cancel{I_a(S)}}$$

$$\Rightarrow \frac{\theta(s)}{E_a(s)} = \frac{K_B}{s\left[(R_a + sL_a)(sJ + f) + K_e K_b\right]}$$

# STATE VARIABLE ANALYSIS

A state space representation is a mathematical model of a physical system as set of inputs, outputs and state variable related by 1st order differential equation.

General representation is :-

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$

where $\dot{x}(t)$ is called 'velocity vector' $(n \times 1)$

$x(t)$ is called 'state vector' $(n \times 1)$

$y(t)$ is called 'output vector' $(p \times 1)$

$u(t)$ is called 'Input vector' $(m \times 1)$

$A$ = State matrix $(n \times n)$

$B$ = Input matrix $(n \times m)$

$C$ = Output matrix $(p \times n)$

$D$ = feed forward matrix $(q \times p)$

State variable can be ~~representation form~~ represented in both continuous and discrete part.

**Ex** :- Express the following differential equation is state-space representation form :-

$$a\frac{d^2x(t)}{dt^2} + b\frac{dx(t)}{dt} + c\,x(t) = u(t)$$

with initial condition given as $\frac{d\,x(t)}{dt} = 0$ & $x(t) = 0$ at $t = 0$.

Sol$^n$ :- Equation can be rearranged as:-

$$\frac{d^2x(t)}{dt^2} = -\frac{b}{a}\frac{dx(t)}{dt} - \frac{c}{a}x(t) + u(t)$$

Now, write equation such that,

[Algebraic variable comprising of only 1st order derivative]

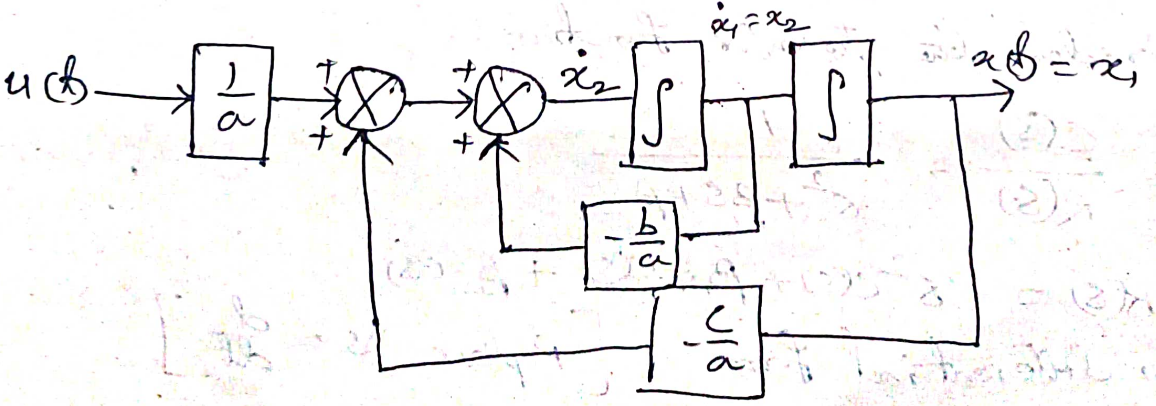
= [Algebraic variables free from any derivatives]

Let $x(t) = x_1$ & $\dot{x}(t) = x_2$

$$\dot{x}_1 = x_2 \text{ ——— (i)}$$

$$\dot{x}_2 = -\frac{b}{a}x_2 - \frac{c}{a}x_1 + \frac{1}{a}u(t) \text{ ——— (ii)}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{c}{a} & -\frac{b}{a}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{a}\end{bmatrix}u(t)$$



## Some Definations Related to State-space :-

① **State Variables :-** The smallest possible subset of system variables that can represent the entire state of the system at a given time.

② **State :-** The ~~state~~ of a system at any time $t = t$ (Initial condition) together with input determines th behaviour of system at any time $t > t_0$.

## State Space Representation for a Transfer Function :-

The transfer function of a control system can be represented in :-

(i) Controllable ~~form~~ Canonical form

(ii) Observable canonical form

### Controllable Cannonical Form

Consider the transfer function :-

$$\frac{C(S)}{R(S)} = \frac{1}{s^2 + \beta_1 s + \beta_2}$$

$$R(S) = s^2 C(S) + \beta_1 s\, C(S) + \beta_2\, C(S)$$

In differential form $\left[ \text{Replace } s = \frac{d}{dt} \right]$

$$r(t) = \frac{d^2}{dt^2} c(t) + \beta_1 \frac{dc(t)}{dt} + \beta_2\, c(t)$$

~~$\frac{d^2c}{dt} = -\beta \frac{dc}{dt} - \beta_2 c(t)$~~ -

$$\Rightarrow \frac{d^2c(t)}{dt} = -\beta_1 \frac{dc(t)}{dt} - \beta_2 c(t) + r(t)$$

Let $\frac{dc(t)}{dt} = \dot{x}_1 = x_2$

$$\frac{d^2c(t)}{dt^2} = \dot{x}_2 = \dot{x}_1$$

$$\therefore \dot{x}_2 = -\beta_1 x_2 - \beta_2 x_1 + r(t)$$

| $f(t)$ | $F(s)$ |
|---|---|
| $1$ | $1/s$ |
| $K\,u(t)$ | $\frac{K}{s}$ |
| $u(t-a)$ | $\frac{e^{-as}}{s}$ |
| $u(t+a)$ | $\frac{e^{as}}{s}$ |
| $r(t) = t$ | $1/s^2$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ |
| $\sinh \omega t$ | $\frac{\omega}{s^2-\omega^2}$ |
| $\cosh \omega t$ | $\frac{s}{s^2-\omega^2}$ |
| $r(t-a)$ | $\frac{e^{-as}}{s^2}$ |
| $e^{-at} \sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| $e^{-at} \cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |
| $e^{at} \sin \omega t$ | $\frac{\omega}{(s-a)^2+\omega^2}$ |
| $e^{at} \cos \omega t$ | $\frac{s-a}{(s-a)^2+\omega^2}$ |

| $f(t)$ | $F(s)$ |
|---|---|
| $e^{-at}\sinh\omega t$ | $\frac{\omega}{(s+a)^2-\omega^2}$ |
| $e^{-at}\cosh\omega t$ | $\frac{s+a}{(s+a)^2-\omega^2}$ |
| $e^{at}\sinh\omega t$ | $\frac{\omega}{(s-a)^2-\omega^2}$ |
| $e^{at}\cos h\omega t$ | $\frac{\cancel{s+a}\ s-a}{(s-a)^2-\omega^2}$ |
| $\delta(t)$ | $1$ |
| $\delta(t-a)$ | $e^{-as}$ |
| $t e^{at}$ | $\frac{1}{(s-a)^2}$ |
| $t e^{-at}$ | $\frac{1}{(s+a)^2}$ |
| $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ |

# THEOREMS OF LAPLACE TRANSFORM

① **Linearity** :- If $F_1(s) = \mathcal{L}[f_1(t)]$

$F_2(s) = \mathcal{L}[f_2(t)]$

Then $\mathcal{L}[a f_1(t) \pm b f_2(t)]$

$= aF_1(s) \pm bF_2(s)$

② **Scaling Property** :- If $F(s) = \mathcal{L}[f(t)]$

Then $\mathcal{L}\, f(at) = \frac{1}{a} F\left(\frac{s}{a}\right)$

③ **Frequency shifting** Property

If $F(s) = \mathcal{L}\, f(t)$

Then $\mathcal{L}[e^{\pm at} f(t)] = F(s \mp a)$

④ Time shifting property

# Laplace Transfrom

$$\mathcal{L}[f(t)] = F(S) = \int_{-\infty}^{\infty} e^{-st} f(t)\, u(t)\, dt = \int_{0}^{\infty} f(t)\, e^{-st}\, dt$$

The complex variable 'S' is generally

$$S = \sigma + j\omega$$

$$\mathcal{L}^{-1}[F(S)] = f(t) = \frac{1}{2\pi j} \int_{-j}^{+j} F(S)\, e^{st}\, ds$$

## Impulse

$$\mathcal{L}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)\, e^{-st}\, dt$$

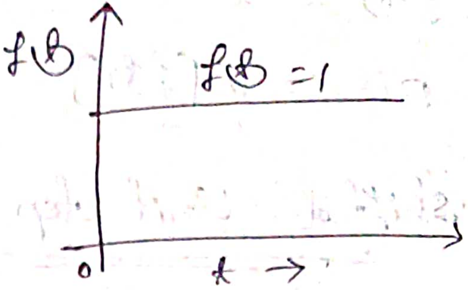
$$= \int_{0}^{\infty} 1 \cdot e^{-st}\, dt$$

$$= \left.\frac{e^{-st}}{S}\right]_{t=0} = 1$$

## Unit step Function :-

$f(t) = u(t) = 1 \quad t \geq 0$

$\quad\quad\quad\quad = 0 \quad t < 0$



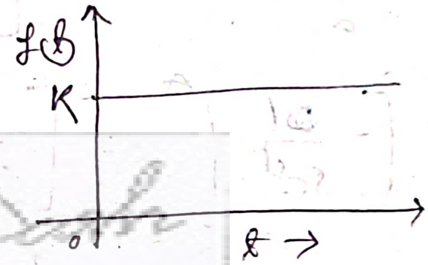
$\mathcal{L}[f(t)] = \int_0^\infty f(t) e^{-st} dt$

$= \int_0^\infty 1 . e^{-st} dt = \left[ \frac{e^{-st}}{-s} \right]_0^\infty$

$= \frac{e^{-\infty t}}{-\infty} - \frac{e^{-s \times 0}}{-s}$

$= 0 + \frac{1}{s} = \frac{1}{s}$

## Step Function :



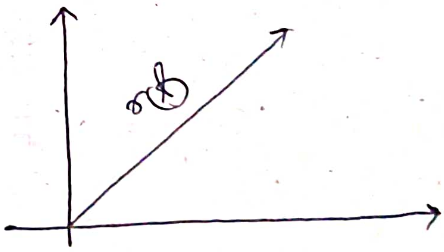
$f(t) = K u(t) = K \quad t \geq 0$

$\quad\quad\quad\quad = 0 \quad t < 0$

$\therefore \mathcal{L}[f(t)] = \int_0^\infty K e^{-st} dt$

$= K \int_0^\infty e^{-st} dt = K \cdot \frac{1}{s} = K/s$

## Unit Ramp Function :-



$f(t) = r(t) = t \quad t \geq 0$

$\quad\quad\quad\quad\quad 0 \quad t < 0$

$F(s) = \int_0^\infty t e^{-st} dt$

$= t \int_0^\infty e^{-st} - \int_0^\infty \frac{d}{dt} t \int_0^\infty e^{-st} dt \; dt$

$= t \frac{e^{-st}}{-s} - \int_0^\infty \frac{e^{-st}}{-s} dt = \frac{-te^{-st}}{s} - \frac{e^{-st}}{s^2}$

$= \left[ - \left[ \frac{te^{-st}}{s} + \frac{e^{-st}}{s^2} \right] \right]_0^\infty = -0 + \frac{1}{s^2} = \frac{1}{s^2}$

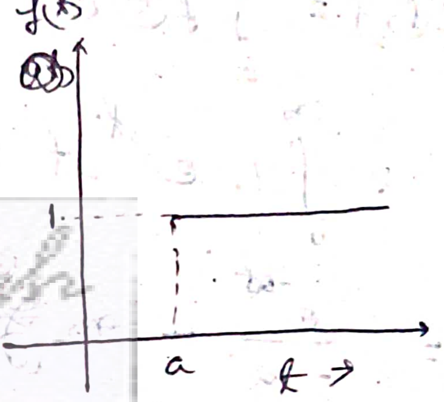
$\because f(t) = t^n$

$F(s) = \mathcal{L}[f(t)] = \dfrac{n!}{s^{n+1}}$

## Shifted Unit step Function

$f(t) = u(t-a) = 1 \quad t \geq a$

$\qquad\qquad\quad = 0 \quad t < a$



$\mathcal{L}[u(t-a)] = \int_{-\infty}^{\infty} u(t-a)\, e^{-st}\, dt$

$= \int_{a}^{\infty} e^{-st}\, dt = \left[-\dfrac{e^{-st}}{s}\right]_a^{\infty}$

$= \left[-\dfrac{1}{se^{st}}\right]_a^{\infty} = \overset{0}{\cancel{\left[-\dfrac{1}{se^{\infty}}\right]}} - \left[-\dfrac{1}{se^{as}}\right]$

$= \dfrac{1}{se^{as}} = \dfrac{e^{-as}}{s} = \dfrac{1}{s}e^{-as}$