GOVERNMENT POLYTECHNIC, DHENKANAL

Programme: Diploma in Mechanical Engineering

Course:Fluid Mechanics(Theory)

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Semester:4th

Chapter-1

PropertiesofFluid

Fluid

Definition:

Afluidisasubstancewhichiscapableofflowingorasubstancewhich deforms continuously when subjected to external shearing force.

Characteristics:

- Ithasnodefiniteshapeofitsownbutwilltaketheshapeofthecontainerin which it is stored.
- Asmallamountofshearforcewillcauseadeformation.

Classification:

Afluidcanbeclassifiedasfollows:

- Liquid
- Gas

Liquid:
Itisafluidwhichpossessesadefinitevolumeandassumedasincompressible

GAS:
Itpossessesnodefinitevolumeandiscompressible.

Fluidsarebroadlyclassifiedintotwotypes.

- Idealfluids
- Realfluids

Idealfluid:

Anidealfluidisonewhichhasnoviscosityandsurfacetensionand is incompressible actually no ideal fluid exists.

Realfluids:

Arealfluidisonewhichhasviscosity,surfacetensionandcompressibility in addition to the density.

PROPERTIESOFFLUIDS:

1. **densityormassdensity:(S)**

   Density of a fluid is defined as the ratio of the mass of a fluid to its vacuum. It is denoted by δ The density of liquidsare considered as constant while that of gases changes with pressure & temperature variations.

Mathematically

$$\rho = \frac{mass}{volume}$$

Unit= $\frac{kg}{m^3}$

$$\rho_{water} = 1000 \frac{kg}{m^3}$$

$$\text{or } \frac{gm}{cm^3}$$

2. **Specificweightorweightdensity((W):**

   Specificweightofafluidisdefinedastheratiobetween theweights of a fluid to its volume. It is denoted by W.

MathematicallyW= $\frac{weightoffluid}{volumeoffluid}$

=mg/v

W=ρg

Unit- $\frac{N}{m^3}$

3. **Specificvolume:**

   Specificvolumeofafluidisdefinedasthevolumeofafluid occupiedbyaunitmassorvolumeperunitmassofafluidiscalled specific volume.

Mathematically

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

4. **Specificgravity:**

Specificgravityisdefinedastheratiooftheweightdensityofafluidtothe density or when density standard fluid.

Forliquidsthestandardfluidiswater.

Forgasesthestandardfluidisair.It

is denoted by the symbol S

Mathematically, $S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus weight density of a liquid $= S \times \text{Weight density of water}$

$= S \times 1000 \times 9.81 \text{ N/m}^3$

The density of a liquid $= S \times \text{Density of water}$

$= S \times 1000 \text{ kg/m}^3$.

Unit: $\frac{m^3}{kg}$

SimpleProblems:

Problem:-1

Calculatethespecificweight,densityandspecificgravityofonelitreofa liquid which weighs 7N.

Solution. Given :

Volume $= 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$

Weight = 7 N

(*i*) Specific weight (w) $= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = $ **7000 N/m³. Ans.**

(*ii*) Density (ρ) $= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 =$ **.713.5 kg/m³. Ans.**

(*iii*) Specific gravity $= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \ \text{Density of water} = 1000 \text{ kg/m}^3\}$

= **0.7135. Ans.**

Problem:-2

Calculatethedensity,specificweightandspecificgravityofonelitreof petrol of specific gravity = 0.7

Solution. Given : Volume $= 1 \text{ litre} = 1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Using equation (1.1.A),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(*ii*) *Specific weight* (w)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m³. Ans.**

(*iii*) *Weight* (*W*)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

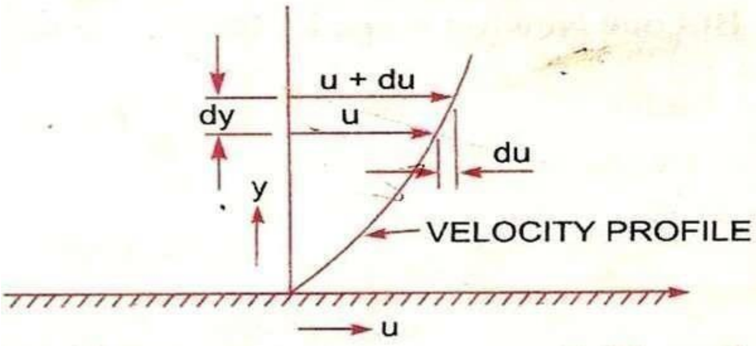
$$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

$\therefore$ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

<u>Viscosity</u>

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Let two layers of a fluid at a distance dy apart, move one over the other at different velocities u and u + du.



Velocity variation near a solid boundary.

The viscosity together with the with therelative velocity between the two layers while causes a shear stress acting between the fluid layers, the top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

Thisshearstressisproportionaltotherateofchangeofvelocitywith respect to y. It is denoted by r.

.

Mathematically $r \alpha \frac{du}{dy}$

$$r = \mu \frac{du}{dy}$$

Whereμ=co-efficientofdynamic viscosityorconstantof proportionality or viscosity

$\frac{du}{dy}$ =rateofshearstrainorvelocitygradient

$$\mu = \frac{c}{\frac{du}{dy}}$$

If $\frac{du}{dy}$=1,

thenμ=r

Viscosityisdefinedastheshearstressrequiredtoproduceunitrate of shear strain.

UnitofviscosityinS.Isystem- $\frac{Ns}{m^2}$

inC.G.S–$\frac{Dynes}{cm^2}$

InMKS-$kgfs/cm^2$

$\frac{Dynes}{cm^2}$ =1 Poise

$1\,\frac{Ns}{m^2}$ =10poise

1Centipoise= $\frac{1}{100}$ poise

KinematicViscocity:

Itisdefinedastheratiobetweenthedynamicviscosityanddensity of fluid.

Itisdenotedbyϑ.

Mathematically

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \qquad \text{...(1.4)}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{\because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}\right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4}\ \text{m}^2/\text{s}$

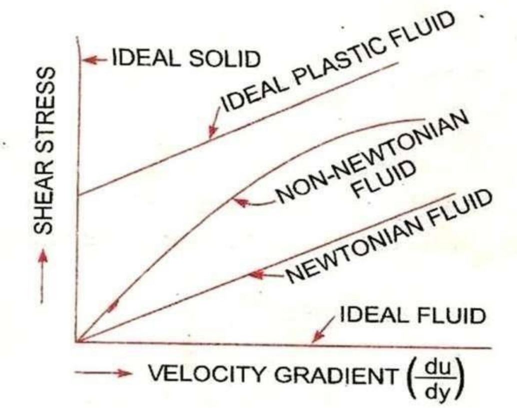
Centistoke means $= \frac{1}{100}$ stoke.

Newton'slawofviscosity:

Itstatesthattheshearstressonafluidelementlayerisdirectly proportionaltotherateofshearstearstrain.Theconstantofproportionality is called the co-efficient of viscosity.

Mathematically $r{=}\mu \frac{du}{dy}$

Fluids which obey the above equation or law are known as Newtonian fluids & the fluids which do not obey the law are called Non-Newtonian fluids.



Surfacetension:

Surfacetensionisdefinedasthetensileforceactingonthesurfaceof a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a stretched membrane under tension. The magnitude of this force per unit length of the free will has the same value as the surface energy per unit area.

Itisdenotedbyσ

Mathematically $\sigma{=}\frac{F}{L}$

UnitinsisystemisN/m

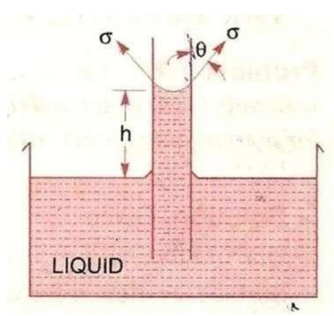
CGSsystemisDyne/cm

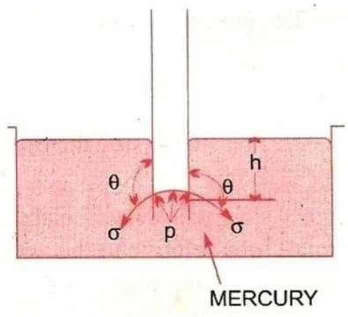
MKS system iskgf/m

Capillarity:

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is heldvertically in the liquid.The riseof liquid surfaceis is known as capillary rise while the fall of the liquid surface is known as capillary depression.

Itisexpressedintermsofcmormmofliquid





Its valuedependsuponthespecificweightoftheliquid,diameterof the tube and surface tension of the liquid.

Chapter-2

FluidPressure And It'sMeasurements

PressureofaFluid:

Whenafluidiscontainedinavessel,itexertsforceatallpointsonthesides&bottoms of the container. Theforce exertedper unit area is calledpressure.

If P=Pressureatanypoint

F=TotalforceuniformlydistributedoveranareaA=

unit area

P=F/A

Unitofpressure- $\frac{kgf}{m^2}$ inM.K.S.

- $\frac{N}{m^2}$ inS.I.

- $\frac{Dyne}{cm^2}$

1pascal=1N/m^2

1 kpa=1000N/m^2

Pressureheadofaliquid:

Aliquidissubjectedtopressureduetopressureduetoitsownweight,this pressure increases as the depth of the liquid increases.

Letabottomlesscylinderstandintheliquid

Let w=specificweightoftheliquid.

H=heightoftheliquidinthecylinder.A=

Area of the cylinder.

$$P=\frac{F}{A}=\frac{weight of the liquid in the cylinder}{Area of the cylinder}$$

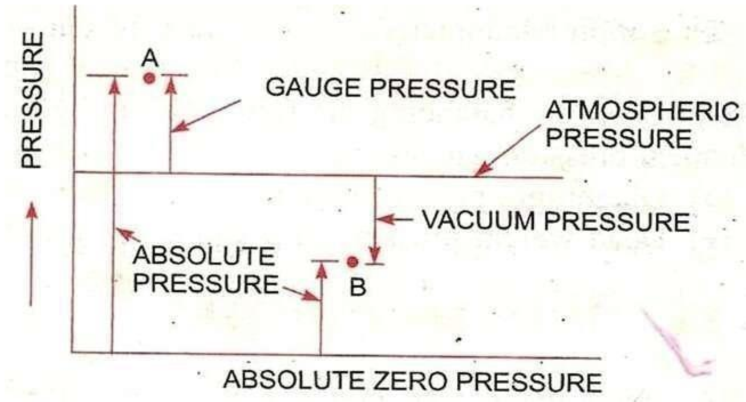
$$=\frac{W\times Ah}{A}$$

$$=Wh$$

$$=\rho gh$$

Sointensityofpressureatanypointinaliquidisproportionalto it depth.

ABSOLUTE,GAGUE,ATOMOSPHERIC,ANDVACCUMEPRESSURES:



AtmosphericPressure:

Theatmosphericairexertsanormalpressureuponallsurfaceswithwhich It is in contact & known as atmospheric pressure.

Absolutepressure:

Itisdefinedasthepressurewhichismeasuredwithreferencetoabsolute vacuum pressure or absolute zero pressure.

Gaugepressure:

It is defined as the pressure which is measured with the help of a pressure measuringinstrumentin which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Vacuumpressure:

Itisdefinedasthepressurebelowtheatmosphericpressure.Mathematically:

Absolutepressure=Atmosphericpressure+gaugepressureOr $P_{abs} = P_{atm} + P_{gauge}$

Vacuumpressure=Atmosphericpressure–Absolutepressure $P_{vacuum} = P_{atm} - P_{abs}$

<u>PressureMeasuringInstruments:</u>

Thepressureofafluidismeasuredbythefollowingdevices:

1. **Manometers**
2. **MechanicalGauges**.

<u>Manometers:</u>

Manometers are defined as the device used for measuring the pressure at a point in a fluid by balancing the collomn of fluid by the same another column of the fluid. They are classified as:

(a) **Simplemanometers.**
(b) **DifferentialManometers**.

<u>MechanicalGauges:</u>

Mechanical gauges are defined as the device used for measuring the pressure by balancing the fluid column by the spring or dead weight. Commonly used mechanical pressure gauges are :

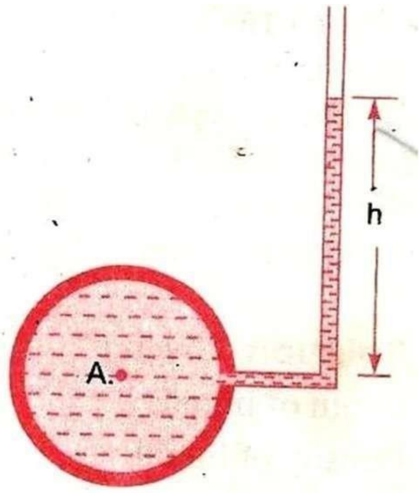
- **Diaphragmpressuregauge**
- **Bourdontubepressuregauge**
- **Dead–weightpressuregauge**
- **Bellowpressuregauge**

<u>SimpleManometres:</u>

Asimple manometerofaglasstubehavingoneofitsendsconnected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

- **Piezometer**
- **U-tubeManometer**
- **SingleColumnManometer**

Piezometer:

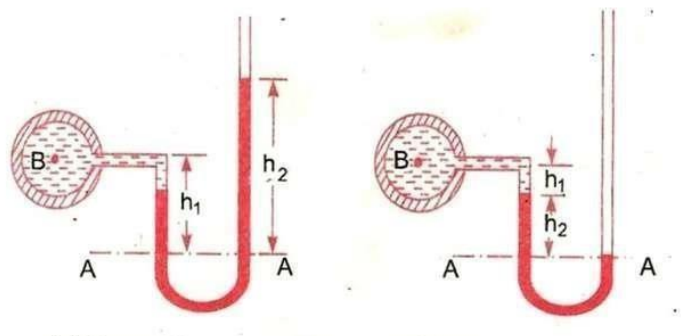


It is the simple form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is tobemeasured and other endisopen tothe atmosphere as shown in Figure. The rise of liquid gives the pressure head at that point A. Then pressure at A

$$P_A = \rho g h$$

U–tubeManometer:

It consist of glass tube bent in U- shape , one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure. The tube generally contains mercury.



(a) For gauge pressure (b) For vacuum pressure

(a) ForGaugePressure:

Letbeisthepointwhichistobemeasured,whosevalueisp.The datum line is A-A.

Leth_1= Height of light liquid above the datumline

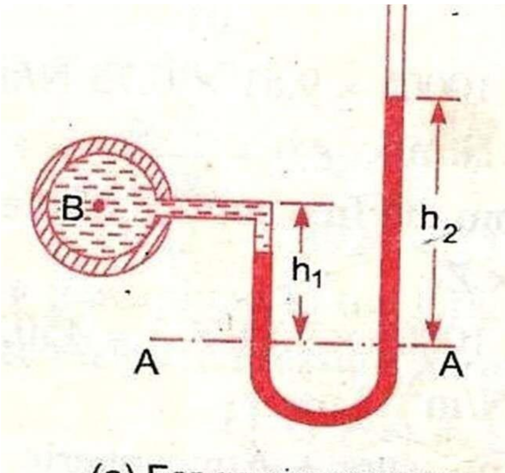
h_2=HeightofheavyliquidabovethedatumlineS$_1$=

Sp. gr. of light liquid

ρ_1=Densityoflightliquid=1000×S_1S_2=

Sp. Gr. Of heavy weight

ρ_2=densityofheavyweight=1000×S_2



(a) For gauge pressure

Pressure is same in a horizontal surface. Hence pressure above the horizontal datumsurface line A-Ain the left column and in theright column of U-tube manometer should be same pressure aboveA-Ain the left column

$$=p_A+\rho_1\times g\times h_1$$

PressureaboveA-Aintherightcolumn

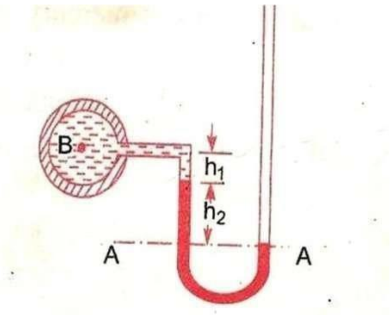
$=\rho_2 \times g \times h_2$Henceequat

ingthetwopressures

$$p_A+\rho_1 g h_1=\rho_2 g h_2$$

$$p_A=(\rho_2 g h_2-\rho_1 g h_1).$$

(b) ForVacuumPressure:



For measuring vacuum pressure, the level of the heavy liquid inthemanometerwillbeasshowninfigure.ThenPressureabove A- A in the left column

$$=\rho_2 g h_2+\rho_1 g h_1+p_A$$

PressureheadintherightcolumnaboveA-A=0

$$\rho_2 g h_2+\rho_1 g h_1+p_A=0$$

$$p_A=-(\rho_2 g h_2+\rho_1 g h_1)$$

SingleColumnManometer:

Single column Manometer is modified form of a U- tube manometer in which a reservoir,having a large cross- sectional area (about 100 timesas compared to thearea of the tube) is connected to one of the limbs (say left limb)of the manometer as shown in figure. Due to large cross- sectionalarea of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid inthe other limb.The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

- **VerticalSingleColumnManometer**
- **InclinedSingleColumnManometer**

1.VerticalSingleColumnManometer:

Let X-X be the datumline in the reservoir and in the right limb ofthe manometer, when it is not connected to the pipe.When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

LetΔh=Fallofheavyliquidinreservoir H_2

=riseofheavyliquidinrightlimb

H_1=heightofcenterofpipeaboveX-X

P_A=PressureatA,whichistobemeasuredA=

Cross – sectional area of the reservoira =

Cross sectional area of the right limb

S_1=Sp.gr.ofliquidinpipe

S_2=Sp.gr.ofheavyweightliquidinreservoirandrightlimbP_1=

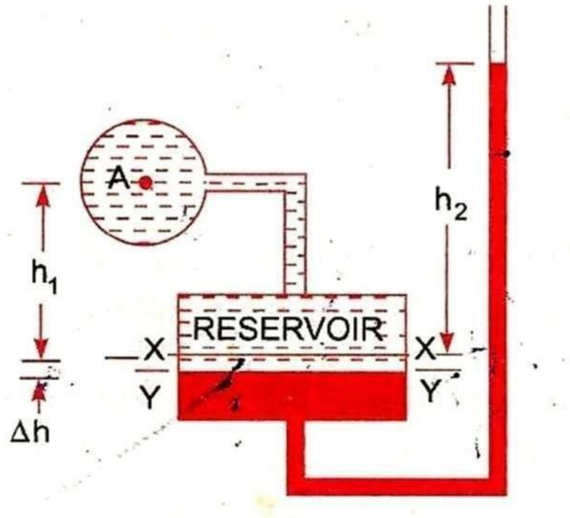
Density in liquid in pipe

P_2=Densityofliquidinthereservoir

Fall of heavy liquid in the reservoir will cause a rise of heavy liquid level in the right limb.

∴ $A \times \Delta h = a \times h_2$

∴ $\Delta h = \frac{a \times h}{A}$ .... (i)



NowconsiderthedatumlineY-YasshowninFig2.15.Then pressure in the right limb above Y-Y.

$$=\rho_2\times g\times(\Delta h+h_2)$$

Pressure in left limb above Y-Y $\quad =\rho_1\times g\times(\Delta h+h_1)+p_A$

Equating the pressure, we have

$$\rho_2\times g\times(\Delta h+h_2)=\rho_1\times g\times(\Delta h+h_1)+P_A$$

$$P_A=\rho_2 g(\Delta h+h_1)-\rho_1 g(\Delta h+h_1)$$

$$=\Delta h[\rho_2 g-\rho_1 g]+h_2\rho_2 g-h_1\rho_1 g$$

Butfromequation(i), $\quad \Delta h=\frac{a\times h}{A}$

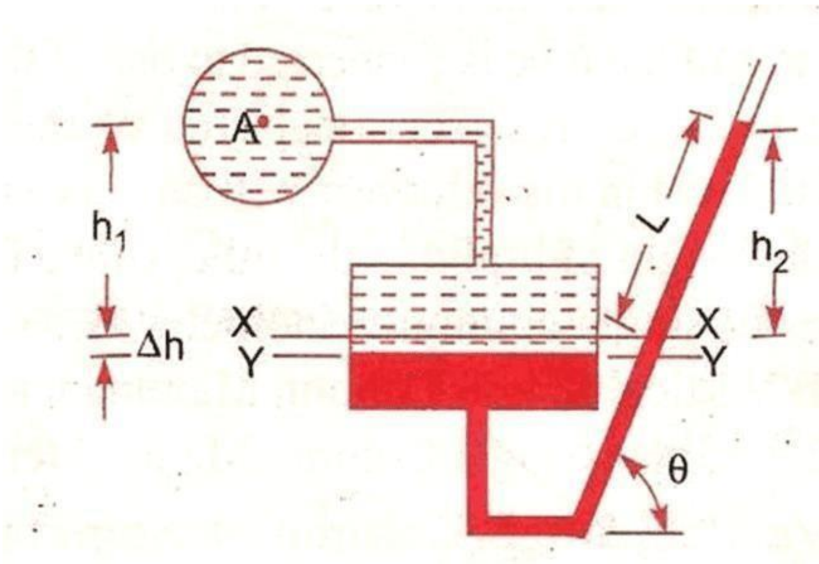
So,$P_A=\frac{a\times h}{A}[\rho_2 g-\rho_1 g]+h_2\rho_2 g-h_1\rho_1 g$

AsthereaAisverylargeascomparedtoa,henceratio$\frac{a}{A}$becomes verysmallandcanbeneglected.Then

$$P_A= h_2\rho_2 g- h_1\rho_1 g$$

2. InclinedSingleColumnManometer:

The given figure shows the inclined single column manometer which is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.



Let L=lengthofheavyliquidmovedinrightlimbfromX-X

θ=Inclinationofrightlimbwithhorizontal

h_2=VerticalriseofheavyliquidinrightlimbfromX-X

$= L \times \sin\theta$

Fromtheaboveequationforthepressureinthesinglecolumnmanometer the pressure at A is

$P_A = h_2 \rho_2 g - h_1 \rho_1 g.$

Substitutingthevalueofh_2,weget

$P_A = \sin\theta \rho_2 g L - h_1 \rho_1 g.$

DIFFERENTIALMANOMETERS:

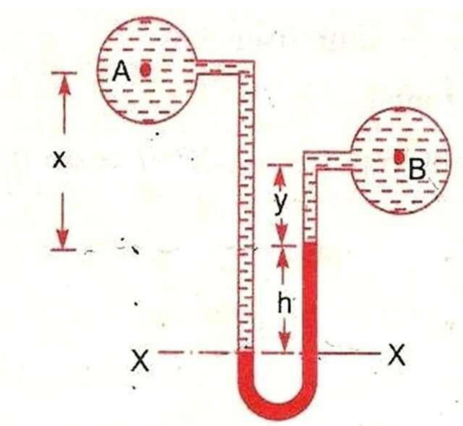
Differential manometers are the device use for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U- tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly used differential manometers are :

1. **U-tubedifferentialmanometer**
2. **InvertedU-tubedifferentialmanometer**

U-tube differential

manometer:TwopointsAandBareatdifferentlevel

ThegivenfigureshowsthedifferentialmanometersofU-tubetype.



Let the two points Aand B are at different level also contains liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at Aand B are P_A and P_B.

Let h=DifferenceofmercurylevelintheU-tube.

y=DistanceofthecenterofB,fromthemercurylevelinthe rightlimb.

ρ_1=DensityofliquidatA.

ρ_2=DensityofliquidatB.

ρ_g=Densityofheavyliquidormercury.

Taking datum line at X-X .

PressureaboveX-Xinthelimb

$=\rho_1 g(h+x)+P_A$

WherepressureP$_A$=PressureatA.

PressureaboveX-Xintherightlimb

$=\rho_g \times g \times h+\rho_2 \times g \times y+p_B$

Where pressure p_B = pressure at B.

Equating the two pressure, we have

$P_1 g(h+x) \quad +P_A = p_g \times g \times h + p_2 gy + p_B$

∴ $P_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h+x)$

$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

∴ DifferentofpressureatAandB

$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

TwopointsAandBareatsamelevel

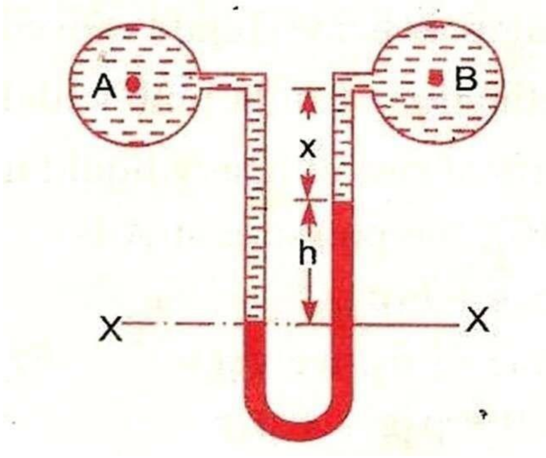
InthegivenfigureAandBarethesamelevelandcontainsthesame liquid of density ρ_1, then

PressureaboveX-Xinrightlimb

$=\rho_g \times g \times h+\rho_1 \times g \times X+p_B$

PressureaboveX-Xinleftlimb

$=P_1 \times g \times (h+x)+P_A$



Equatingthetwopressure

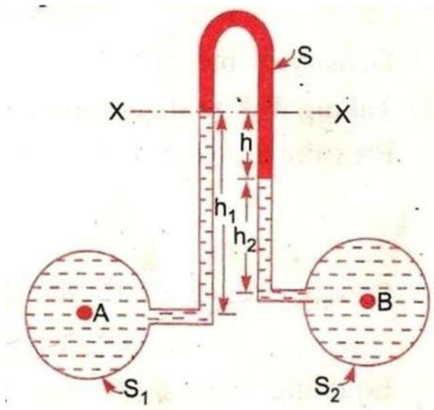
$p_g \times g \times h + P_1 \times g \times X + p_B = P_1 \times g \times (h+x) + P_A$

$\therefore \quad P_A - p_B = P_g \times g \times h + P_1 g x - P_1 g \times (h+x)$

$= g \times h(P_g - P_1)$

InvertedU-tubeDifferentialManometer:

It consists of an inverted U-tube, containing a light liquid. The two ends of the U-tube are connected to the points whose difference of pressure istobemeasured.Itisusedformeasuringdifferenceoflowpressures.Fig 2.21 shows an inverted U-tube differential manometer connected to the points Aand B. Let the pressure at Ais more than the pressure at B.



Let h_1=Heightofliquidintheleftlimbbellowthedatumline X-X

h_2=Heightofliquidintherightlimb h=

Differenceoflightliquid

p_1=DensityofliquidatA

p_2=DensityofliquidatB

p_s=Densityoflightliquid

p_A=*PressureatA*

p_B=PressureatB.

TakingX-Xdatumline.

Thenpressureintheleftlimbbelow X-X

$$=P_A-\rho_1\times g\times h_1.$$

Pressuresintherightlimbbelow X-X

$$=P_B-\rho_2\times g\times h_2-\rho_S\times g\times h$$

Equatingthetwopressure

$$P_A-\rho_1\times g\times h_1=P_B-\rho_2\times g\times h_2-\rho_S\times g\times h$$
$$P_A-P_B=\rho_1\times g\times h_1-\rho_2\times g\times h_2-\rho_S\times g\times h$$

Bourdon'sTubePressureGauge:

- The pressure above or below the atmospheric pressure may be easily measured with the help of Burdon tube pressure gauge.
- It consists of an elliptical tube ABC bent into an arc of a circle. This bent up tube is called Burdon tube.
- When the gauge tube is connected to the C, the fluid under pressure flows into the tube the bourdon tube as a result of the increased pressure tends to straighten itself.
- Since the tube is encased in a circular cover therefore.it tends to become circular instrad of straight.
- Theelasticbeforemationofthebourdonrotatesthepointer.
- Thepointermovesoveracalibrateswhichdirectlygivesthe pressure.

Chapter-3

Hydrostatics

Hydrostatics:

Hydrostatics means the study of pressure exerted by thye liquid at rest &the direction of such a pressure isalways right angle to the surfaceon which it acts.

Totalpressureandcenterofpressure:

Totalpressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane orcurved when thefluidcomesincontactwith surfaces. This force always acts normal to the surface.

Centerofpressure:

Center of pressure is defined as the point of application of the total pressure on the surface.

There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined. The submerged surfaces may be:

1. **Verticalplanesurface**

2. **Horizontalplanesurface**
3. **Inclinedplanesurface**
4. **Curvedsurface.**

<u>Verticalplanesurfacesubmergedinliquid</u>

Consideraplaneverticalsurfaceofarbitraryshapeimmersedina liquid as shown in figure

Let A=totalareaofthe surface

H=distancedofC.G.oftheareafromfreesurfaceofliquidG=

center of gravity ofplane surface

P=centerofpressure

h*=distanceofcenterofpressurefromfreesurfaceofliquid.

<u>Totalpressure(F)</u>

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on surface is then calculated by integrating the force on small strip.

Consider a strip of thickness dh & width b at a depth of h form free surface of liquid.

Pressureintensityonthestrip

$$p=\rho gh$$

Areaofthestrip,dA=b× dh

Totalpressureforceonstrip,dF=ρdA

=ρgh×b×dhTotal

pressure force on thge whole surface

$$F=\int dF=\int \rho gh\times b\times dh$$

$$=\rho g\int h\times b\times dh$$

$\int$h×dA=momentofsurfaceareaaboutthefreesurfaceofliquid

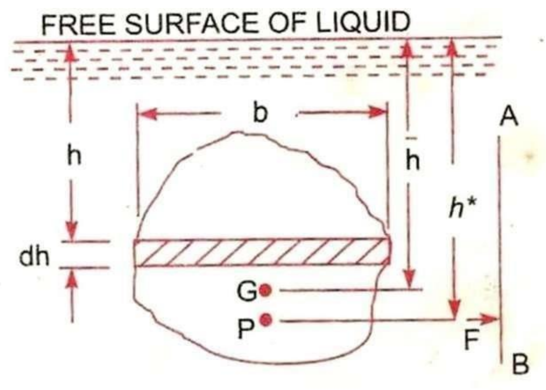
=Areaofsurface×distanceofC.G.fromthefreesurface

$=A\times\bar{h}$

So, $F=\rho gA\bar{h}$

<u>Centreofthepressure:(</u>h*)

Centre of pressure is calculated by using the principle of moments which states that the moment of resultant force about an axis is equal to the sumof moments of the components about the same axis.



The resultant force F is acting at P, at a distance **h*** from the free surface of liquid.

HencemomentofforceFaboutfreesurfaceofliquid=F×h*

But moment forcedF actingon astrip aboutthe freesurfaceofliquid =dF × h

Sumofmomentsofallsuchforcesaboutfreesurfaceofliquid

$=\int\rho gh\times b\times dh\times h$

$=\rho g\int h\times b\times dh\times h$

$=\rho g\int bh^2dh$

$=\rho g\int h^2dA$

$\int h^2dA$=momentofinertiaofthesurfaceareaaboutthefree surface of liquid = Io

Sumofthemomentsaboutfreesurface

$= \rho g I_o$

$F \times h^* = \rho g I_o$

$\rho g A \bar{h} \times h^* = \rho g I_o$

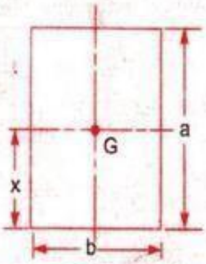
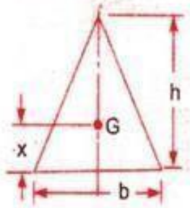
$$h^* = \frac{\rho g I_o}{\rho g A \bar{h}}$$

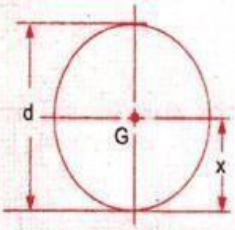
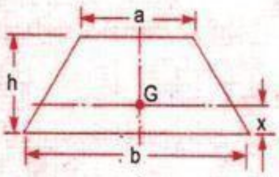
$$= \frac{I_o}{A \bar{h}}$$

Bytheparallelaxistheorem,wehave

$I_o = I_G + A \times (\bar{h})^2$

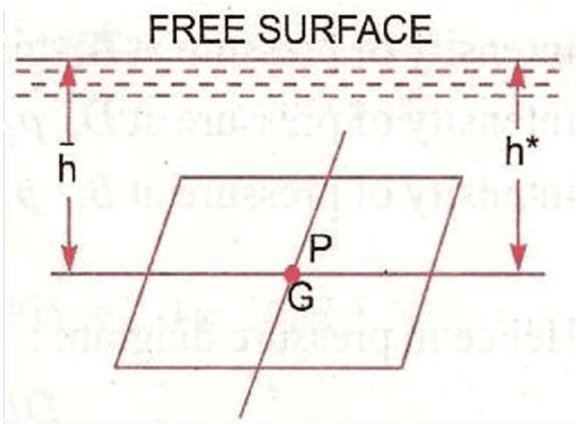
$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Horizontalplanesurfacesubmergedinliquid:

Consider a plane horizontal surface immersed in a static fluid as every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface.



$P = \rho g h$

A=totalarea

F = P ×A

$= \rho g A \bar{h}$

Archimedesprinciple:

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of fluid displaced by the body.

Buoyancy:

Whenever a body is immersed wholly or partially in a fluid it is subjected to an upword force which tends to lift itup. This tendency for an immersedbodytobeliftedupinthefluidduetoanupwardforceoppositeto action of gravity is known as buoyancy this upward force is known as force of buoyancy.

CentreofBuoyancy:

It is defined as the point through which the forced of buoyancy is supposedtoact.The forceofbuoyancyisavertical forceandisequaltothe weightofthefluiddisplacedbythebody.

Canter of buoyancy will be the centre of gravity of the fluid displaced.

Problem-1:

Find the volume of the water displaced & position of centre of duoyancy for a wooden block of width 2.5m & of depth 1.5m when it flats horizontally in water. The density of wooden block is 6540 kg/m3.& its length 6.0m.

Solution:

Width=2.5 m

Densityofwoodenblock=650kg/m^{3}Depth

=1.5m

Length=6m

Volumeoftheblock

$=2.5\times1.5\times6$

$=22.50m^3$

Volumeoftheblock=Wtofwaterdisplaced

$=W\times V$

$=\rho g\times V$

$=650\times9.81\times6$

=143471N

Volumeofwaterdisplaced

$$=\frac{weight}{\rho w\times g}$$

$$= \frac{143471}{1000 \times 9.81}$$

$=14.625m^3$

Positionofcentreofbuoyancy

Volumeofwoodenblockinwater=volumeofwaterdisplaced

$$2.5 \times 6 \times h = 14.625$$

$$\Rightarrow h = \frac{14.625}{2.5 \times 6}$$

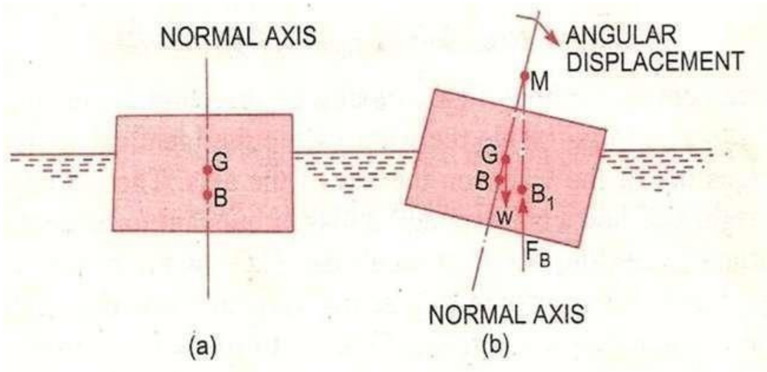
$=0.975m$

Centreofbuoyancy= $\frac{0.975}{2}$

=0.4875mfrombase.

Meta-centre:

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The mate centre may also be defined as the point at which the lme of action of the force of buoyancy will melt the normal axis. Of the body when the body is given a small angular displacement.



Matecentreheight:

The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height i.e the distance MG.

Conceptofflotation:

Flotation:

When a body is immersed in any fluid, it experiences two forces. First one is theweight of body W actingvertically downwards,second is the buoyancy force $F_\beta$acting vertically upwards in case W is greater than F_β, the weight will cause the body to sink in the fluid. In case W = F_β the body will remain in equilibrium at any level. In case W is small than F_β the body will move upwards in fluid. The body moving up will come to rest or top moving up in fluid when the fluid displaced by it's submerged part is equal to itsweight W, thebody inthissituation issaidto befloatingand this phenomenon is known as flotation.

Chapter-4

KinamaticsofFlow

Introduction

This chapter includes the study of forces causing fluid flow. The dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

TYPESOFFLOW

Thefluidflowisclassifiedasfollows:

- **STEADYANDUNSTEADYFLOW**
- **UNIFORMANDNON-UNIFORMFLOWS**
- **LAMINARANDTURBULANTFLOWS**
- **COMPRESSIBLEANDINCOMPRESSIBLEFLOWS**
- **ROTATIONALANDIRROTATIONALFLOWS**
- **ONE,TWO,THREEDIMENSIONALFLOW**

➢ **STEADYANDUNSTEADYFLOW**

1. **Steadyflow**:-

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density at a point do not change with time.

Thus, mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0}=0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0}=0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0}=0$$

Where x_0,y_0,z_0 is a point in fluid flow.

2. **Unsteady flow:-**

Unsteady flow is defined as that type of flow in which the velocity, pressure, and density at a point changes w.r.t time.

Thus, mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0}\neq 0,$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0,y_0,z_0}\neq 0,$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0}\neq 0$$

➢ UNIFORM AND NON-UNIFORM FLOWS:-

1. **Uniform flow:-**

It is defined as the flow in which velocity of flow at any given time does not change w.r.t length of flow or space.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{t=constant}=0$$

where ∂v = velocity of flow,

∂s=lengthofflow,T

=time

2. **<u>Non-uniformflows</u>:-**

Itisdefinedastheflowinwhichvelocityofflowatanygiventime changes w.r.t length of flow.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{t=constant} \neq 0$$

LAMINARANDTURBULANTFLOWS:-

1. **Laminarflow**:-
Laminar flow is that type of flow in which the fluid particles are moved in a well defined path called streamlines. The paths are parallel and straight to each other.

2. **Turbulentflow**:-
Turbulent flow is that type of flow in which the fluid particles are moved in a zig-zag manner.

ForapipeflowthetypeofflowisdeterminedbyReynoldsnumber (R_e)

Mathematically

$$R_e = \frac{VD}{v}$$

WhereV=meanvelocityofflowD

= diameter of pipe

V=kinematicviscosity

IfR_e<2000,thenflowislaminarflow. IfR_e>4000,

thenflowisturbulentflow.

If$R_e$liesinbetween2000and4000,theflowmaybelaminaror turbulent.

- **COMPRESSIBLEANDINCOMPRESSIBLEFLOWS**:-

1. **Compressibleflow**:-
Compressibleflowisthattypeofflowinwhichthedensityoffluid changes from point to point.

So,$\partial \neq$constant.

2. **Incompressibleflow:-**
Incompressibleflowisthattypeofflowinwhichthedensityisconstantfor the fluid flow.

So,$\partial =$constant

- **ROTATIONALANDIRROTATIONALFLOWS**:-

1. **Rotationalflow:-**

Rotationalflowisthatofflowinwhichthefluidparticleswhileflowing along stream lines also rotate about their own axis.

2. **Ir-rotationalflow**:-

Irrotationalflowisthattypeofflowinwhichthefluidparticleswhile flowing alongstreamlines do not rotate about their own axis.

- **ONE,TWO,THREEDIMENSIONALFLOW**:-

1. **Onedimensionalflow**:-

Onedimensionflowisdefinedasthattypeofflowinwhichvelocityisa function of time and one space co-ordinate only.

Forasteadyonedimensionalflow,thevelocity isafunctionofone space co-ordinate only.

So, $U=f(x)$,

$V = 0$,

$W=0$

U,V,Warevelocitycomponentsinx,y,zdirectionrespectively.

2. **Two-dimensionalflow**:-

Two-dimensional flow is the flow in which velocity is a function of time and 2- space co- ordinates only. For a steady 2- dimensional flow the velocity is a function of two – space co-ordinate only.

So, $U=f_1(x,y)$,

$V=f_2(x,y)$,

$W = 0$

3. **Three-dimensionalflow**:-

Three – dimensional flow is the flow in which velocity is a function of time and 3- space co-ordinates only. For steady three- dimensional flow, the velocity is a function of three space co-ordinates only.

So $U=f_1(x,y,z)$

$V=f_2(x,y,z)$

$W=f_3(x,y,z)$

RATEOFFLOWORDISCHARGE

Itisdefinedasthequantityofafluid flowingpersecondthrougha section of pipe.

Foranincompressiblefluidtherateofflowordischargeis expressedasthevolumeoffluidflowingacrossthesectionpersecond.

Forcompressiblefluids,therateofflowisusuallyexpressedasthe weight of fluid flowing across the section.

$$Q=A.V$$

WhereA=crosssectionalareaofthepipe

V=velocityoffluidacrossthesection

Unit:-

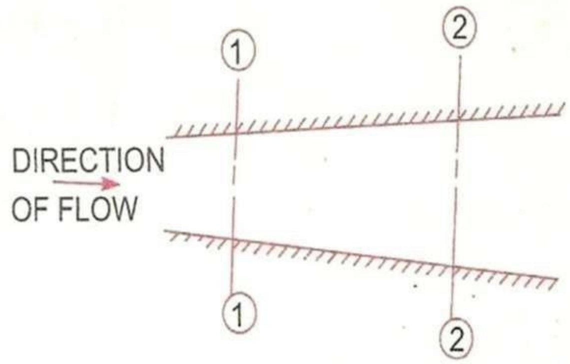
1. Forincompressiblefluid $\frac{m3}{sec}$ or $\frac{litre}{sec}$

2. Forcompressiblefluid:

$$\frac{newton}{sec}\text{ (S.Iunits)},\ \frac{kgf}{sec}\text{ (M.K.Sunits)}$$

EQUATIONOFCONTINUITY:-

It is based on the principle of conservation of mass. For a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.



Fluid flowing through a pipe.

Let V_1=averagevelocityatcross-section1-1.

ρ_1=densityatcross-section1-1

A_1=areaofpipeatsection1-1

V_2=averagevelocityatcross-section2-2

ρ_2=densityatcross-section2-2

A_2=areaofpipeatsection2-2

Therateofflowatsection1-1=$\rho_1 A_1 V_1$

Therateofflowatsection2-2=$\rho_2 A_2 V_2$

Accordingtolawsofconservationof massrateofflowatsection1-1 is equal to the rate of flow at section 2-2,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Thisiscalledcontinuityequation.

Ifthefluidiscompressible,then$\rho_1=\rho_2$,so

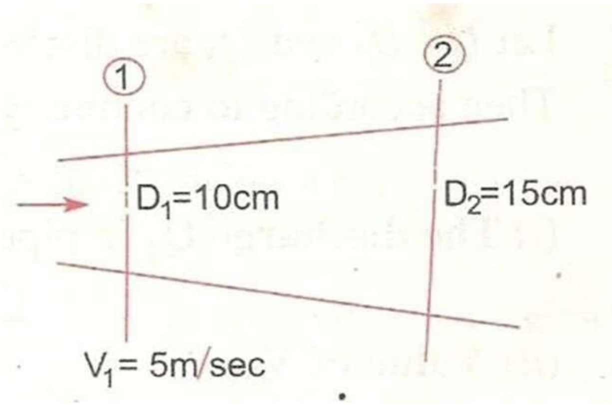
$$A_1 V_1 = A_2 V_2$$

“Ifnofluidisaddedremovedfromthepipeinanylengththen themass passing across different sections shall be same”

SimpleProblems

Problem:-1

The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of thewater flowingthrough the pipe at section 1 is 5m/s.Determine also the velocity at section 2.



Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

(*i*) Discharge through pipe is given by equation (5.1)

or $Q = A_1 \times V_1$

$= .007854 \times 5 =$ **0.03927 m³/s. Ans.**

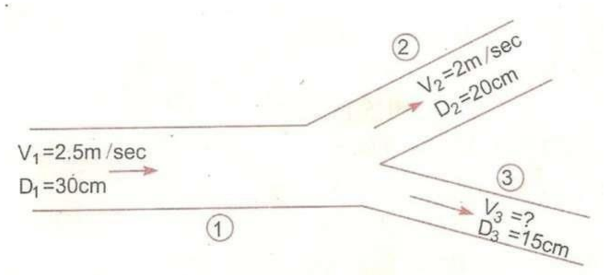
Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) ∴ $V_2 = \frac{A_1V_1}{A_1} = \frac{.007854}{.01767} \times 5.0 =$ **2.22 m/s.**

Problem:-2

A 30m diameterpipeconveying waterbranchesintotwopipesof diameter20cmand15cmrespectively.Iftheaveragevelocityinthe340cm diameterpipeis2.5m/s, findthedischargeinthispipe. Alsodeterminethe velocityin15cmpipeiftheaveragevelocityin20cmdiameterpipeis2m/s

Solution:



GivenData:

D_1=30cm=0.30m

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$

V_1=2.5m/s

D_2=20cm=0.2m

$A_2 = \frac{\pi}{4} 0.2^2 = 0.0314 m^2$

V_2=2m/s

D_3 =15cm=0.15m

$A_3 = \frac{\pi}{4} 0.15^2 = 0.01767 m^2$

LetQ_1,Q_2,Q_3aredischargesinpipe1,2,3respsctivelyQ_1=

Q_2+Q_3

ThedischargeQ_1inpipe1isgivenasQ_1= A_1

V_1

$=0.07068\times2.5 m^3/s$

$Q_2 = A_2V_2$

$=0.0314\times2.00.0628 m^3/s$

SubstitutingthevaluesofQ$_1$andQ$_2$ontheaboveequationwe get

$0.1767=0.0628+Q_3Q_3$

$= 0.1767–0.0628$

$=0.1139m3/s$

Again $Q_3=A_3V_3$

$=0.01767\times$

V_3Or$0.1139=0.01767\times V_3$

$$V_3=\frac{0.1139}{0.01767}$$

$=6.44m/s$

Problem:-3

Water through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter. At C, the pipe branches. Branch CD is 0.8 m in diameter and carrier one third of the flow in AB. The flow velocity in branch` CE is 2.5 m/s. Find the volume rate offlow in AB, the velocity inBC, the velocity inCD and the diameter of CE.

Solution:

GivenData:

Diameter of pipe AB, $D_{AB}=1.2m$

VelocityofflowthroughAB, $V_{AB}=3.0m/s$

Dia,ofpipeBC D_{BC}=1.5m

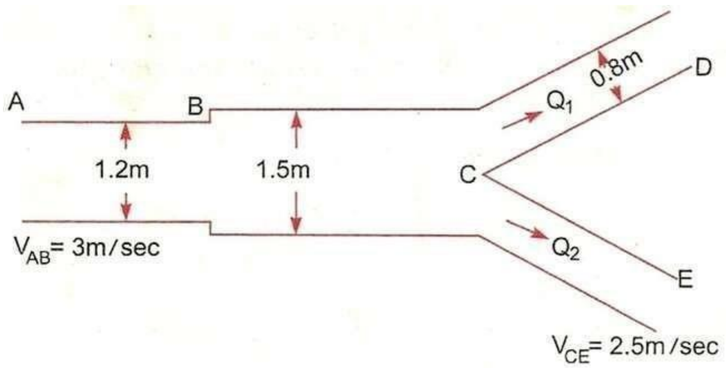
Dia ofbranched pipe CD = V_{CD}= 0.8m

Velocity of flow in pipe CE,V_{CE} =2.5m/s Let

flowrate inpipe AB=Qm^3/sVelocityof

flow in pipe BC = V_{BC} m/s Velocity of

flow in pipe CD = V_{CD}



Diameter of pipe $CE = D_{CE}$

Then flow rate through $CD = Q/3$

and flow rate through $CE = Q - Q/3 = \frac{2Q}{3}$

(*i*) Now volume flow rate through $AB = Q = V_{AB} \times$ Area of AB

$$\doteq 3.0 \times \frac{\pi}{4}(D_{AB})^2 = 3.0 \times \frac{\pi}{4}(1.2)^2$$

=3.394m3/s

(*ii*) Applying continuity equation to pipe *AB* and pipe *BC*,

$$V_{AB} \times \text{Area of pipe } AB = V_{BC} \times \text{Area of pipe } BC$$

or $$3.0 \times \frac{\pi}{4}(D_{AB})^2 = V_{BC} \times \frac{\pi}{4}(D_{BC})^2$$

or $$3.0 \times (1.2)^2 = V_{BC} \times (1.5)^2$$

or $$V_{BC} = \frac{3 \times 1.2^2}{1.5^2} = \mathbf{1.92\ m/s.\ Ans.}$$

(*iii*) The flow rate through pipe

$$C_D = Q_1 = \frac{Q}{3} = \frac{3.393}{3} = 1.131\ \text{m}^3/\text{s}$$

∴ $$Q_1 = V_{CD} \times \text{Area of pipe } C_D \times \frac{\pi}{4}(C_{CD})^2$$

or $$1.131 = V_{CD} \times \frac{\pi}{4} \times .8^2 = 0.5026\ V_{CD}$$

∴ $$V_{CD} = \frac{1.131}{0.5026} = \mathbf{2.25\ m/s.\ Ans.}$$

(*iv*) Flow rate through *CE*,

$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262\ \text{m}^3/\text{s}$$

∴ $$Q_2 = V_{CE} \times \text{Area of pipe } CE = V_{CE}\frac{\pi}{4}(D_{CE})^2$$

or $$2.263 = 2.5 \times \frac{\pi}{4} \times (D_{CE})^2$$

or $$D_{CE} = \sqrt{\frac{2.263 \times 4}{2.5 \times \pi}} = \sqrt{1.152} = 1.0735\ \text{m}$$

∴ Diameter of pipe *CE* = **1.0735 m. Ans.**

Problem:-4

A 25 cmdiameter pipe carries oil of sp. Gr. 0.9 at a velocity of 3m/s.Atanothersectionthediameteris20cm. Findthe velocityatthis section and also mass rater of flow of oil.

Solution. Given :

at section 1, $D_1 = 25 \text{ cm} = 0.25 \text{ m}$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .25^2 = 0.049 \text{ m}^3$$

$V_1 = 3 \text{ m/s}$

at section 2, $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$V_2 = ?$

Mass rate of flow of oil = ?

Applying continuity equation at sections 1 and 2,

$$A_1 V_1 = A_2 V_2$$

or $$0.049 \times 3.0 = 0.0314 \times V_2$$

$\therefore$ $$V_2 = \frac{0.049 \times 3.0}{.0314} = \mathbf{4.68 \text{ m/s. Ans.}}$$

Mass rate of flow of oil $= \text{Mass density} \times Q = \rho \times A_1 \times V_1$

Sp. gr. of oil $= \dfrac{\text{Densit of oil}}{\text{Densit of water}}$

$\therefore$ Density of oil = Sp. gr. of oil $\times$ Density of water

$$= 0.9 \times 1000 \text{ kg/m}^3 = \frac{900 \text{ kg}}{\text{m}^3}$$

$\therefore$ Mass rate of flow $= 900 \times 0.049 \times 3.0 \text{ kg/s} = \mathbf{132.23 \text{ kg/s. Ans.}}$

Bernoulli'sequation:

Statement: Itstatesthatinasteadyidealflowofanincompressible fluid, the total energy at any point of flow is constant.

Thetotalenergyconsistsofpressureenergy, kineticenergy& potential energy or datumenergy. These energies per unit weight are

Pressureenergy= $\frac{P}{\rho g}$

Kineticenergy= $\frac{v^2}{\rho g}$

Datumenergy=z

Mathematically

$$\frac{P}{\rho g}+\frac{v^2}{\rho g}+z=\text{Constant}$$

Derivation:

Consideraperfectincompressibleliquid,flowingthroughanonuniform pipe the pipe is running full & there

LetusconsidertwosectionsAA&BBofthepipe

Nowassumethatthepipeisrunningfull&thereisacontinuityofflowbetweenthe two sections

Let Z_1 = Height of AA

P_1=PressureofAA

V_1=VelocityofliquidofAA

Q_1=CrosssectionalareaofthepipeofAA&Z_2,

P_2 , V_2 , Q_2 are the corresponding values at BB.

LettheliquidbetweenthetwosectionsAA&BBmovetoAA’&BB’ through very small lenth dl_1& dl_2

LetWistheweightoftheliquidbetweenAA&A_1A_1&BB&B_1B_1asthe flow is continuous

$W=wa_1dl_1=wa_2dl_2$

$=a_1dl_1=\frac{W}{\omega}=a_2dl_2$

TheworkdonebypressureofAAinmovingtheliquid$A'A'$

=ForceXdistance

$= P_1Q_1dl_1$

Similarly

WorkdonebypressureatBB

$=-P_2Q_2dl_2$

Totalworkdonebypressure

$=P_1A_1dl_1-P_2Q_2dl_2$

$=P_1A_1dl_1-P_2Q_1dl_1$

$=a_1dl_1(P_1–P_2)$

$=\frac{W}{\omega}(P_1–P_2)$

LossofPotentialenergy

$=w(Z_1–Z_2)$

GaininKineticenergy

$=\frac{W}{2g}(V_2^2-V_1^2)$

Lossofpotenteialenergy+workdonebypressure

=Gaininkineticenergy

$$w(Z_1 - Z_2) + \frac{w(P_1 - P_2)}{\omega} = \frac{w(V_2^2 - V_1^2)}{2g}$$

$$Z_1 - Z_2 + \frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2$$

Limitations:

1. the velocity of the liquid particle at the center ofcross section is maximum. And the velocity gradually decreases towards the periphery of the pipe due to friction offered by the walls of the pipe line but in Bernoulli's equation it has been assumed that the velocity of liquid particle at any pointacross section is uniform.
2. Lassofenergyduetopipefrictionduringflowofliquid,from one section to another are neglected in Bernoulli's equation.
3. Bernoulli's equation does not take into consideration loss of energy due to turbulent flow.
4. Bernoulli's equation does not take into consideration the loss of energy due to change of direction.

Problem:-5

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5mabove the datum line.

Solution. Given :

Diameter of pipe	$= 5\text{ cm} = 0.5\text{ m}$
Pressure,	$p = 29.43\text{ N/cm}^2 = 29.43 \times 10^4\text{ N/m}^2$
Velocity,	$v = 2.0\text{ m/s}$
Datum head,	$z = 5\text{ m}$
Total head	= pressure head + kinetic head + datum head

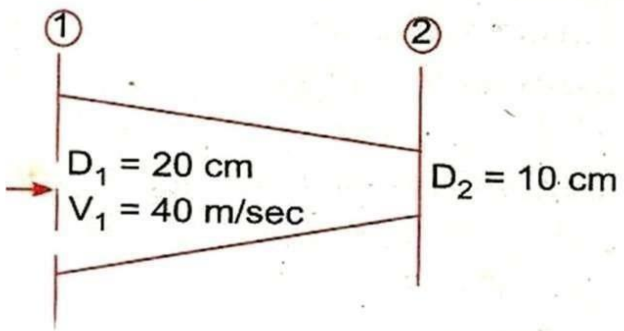
Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30\text{ m}$ $\left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204\text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem:-6

A pipe, through which water is flowing, is having diameters, 20cm and 10cm at the cross sections 1 and 2 respectively.The velocityofwateratsection1 is given4.0 m/s. Findthevelocityheadat sections 1 and 2 and also rate ofdischarge.



Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area,
$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

$\therefore$
$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \mathbf{0.815 \text{ m. Ans.}}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$\therefore$
$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$\therefore$ Velocity head at section 2 $= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} =$ **83.047 m. Ans.**

(*iii*) Rate of discharge
$$= A_1V_1 \quad \text{or} \quad A_2V_2$$
$$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$$

ApplicationofBernoulli’sequation:

Bernoulli’s equation is applied in all problems of incompressible fluid flow where energyconsideration are involved. Itisalsoappliedtofollowingmeasuringdevices

1. **Venturimeter**
2. **Orificemeter**
3. **Pitottube**

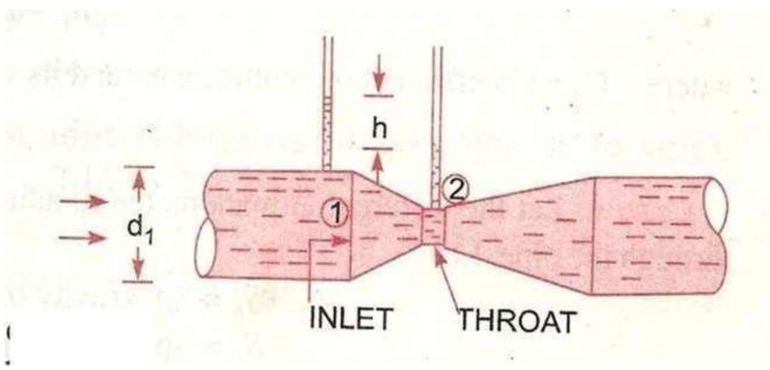
Venturimeter:

Aventurimeterisadeviceusedformeasuringtherateofaflowofa fluid flowing through a pipe it consists ofthree parts.

I. **Shortconvergingpart**
II. **Throat**
III. **Divergingpart**

Expressionforrateofflowthroughventurimeter:

Consideraventurimeterisfittedinahorizontalpipethroughwhicha fluid flowing



Letd$_1$=diameteratinletoratsection(i)-(ii)P$_1$= pressure

at section (1)-(1)

V$_1$=velocityoffluidatsection(1)–(1)

A$_1$=areaatsection(1)–(1)=$\frac{\pi}{4} d_1^2$

D$_2$,p$_2$,v$_2$,a$_2$arecorrespondingvaluesatsection2applyingBernouli's equation at sections 1 and 2 we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

~~But~~ $\frac{P_1-P_2}{\rho g}$ isthedifferenceofpressureheadsatsections1and2 anditisequaltoh

So,h=$\frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

Nowapplyingcontinuityequationatsections1&2a$_1$v$_1$=a$_2$v$_2$

Orv$_1$ =$\frac{a_2 v_2}{a_1}$

Substitutingthisvalue

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2{}^2 = 2gh\,\frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$$Q = a_2 v_2$$

$$= a_2\,\frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

WhereQ=Theoreticaldischarge

Actualdischargewillbelessthantheoreticaldischarge

$$Q_{\text{act}} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

WhereC_d=co-efficientofventurimetreandvalueislessthan1

Valueof'h'givenbydifferentialU-tubemanometer:Case-i:

Letthedifferentialmanometercontainsaliquidwhichisheavierthan the liquid flowing through the pipe

LetS_h=Sp.Gravityoftheheavierliquid

S_0=Sp.Gravityoftheliquidflowingthroughpipe

x=differenceoftheheavierliquidcolumninU-tube$P_A - P_B$=

$gx(\rho_g - \rho_0)$

$$\frac{P_A - P_B}{\rho_0 g} = x\left(\frac{\rho_g}{\rho_0} - 1\right)$$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case-ii

Ifthedifferentialmanometercontainsaliquidlighterthantheliquid flowingthroughthepipe

WhereS_l=SpecificgravityoflighterliquidinU-tubenanometre

So=SpecificgravityoffluidflowingthroughinU-tubenanometrex= Difference of lighter liquid columns in U- tube

Thevalueofhisgivenby

$$h = x\left[1 - \frac{S_l}{S_0}\right]$$

Case-iii:

InclinedventurimetrewithdifferentialU-tubemanometreLet the differential manometer contains heavier liquid Thenhisgivenas

$$h = \left[\frac{P_1}{\rho g} + z_1\right] - \left[\frac{P_2}{\rho g} + z_2\right]$$

$$= x\left[\frac{S_h}{S_0} - 1\right]$$

Case-iv:

Similarlyforinclinedventurimetreinwhichdifferentialmanometer contaoinsaliquidwhichiskighterthantheliquidflowingthroughthepipe. Then

$$\mathbf{h} = \left[\frac{P_1}{\rho g} + z_1\right] - \left[\frac{P_2}{\rho g} + z_2\right]$$

$$\mathbf{h} = \mathbf{x}\left[1 - \frac{S_l}{S_0}\right]$$

Limitations:

- Bernoulli's equation has been derived underthe assumption that no external force except the gravity force is acting on the liquid. But in actual practice some external forces always acting ontheliquid when effect the flow of liquid
- If the liquid is flowingin a curved path the energy due to centrifugal force should also be taken into account.

Pitot-tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

It is based on the principle that if the velocity flow at a point becomes zero, the pressure there is increased due to conversion ofthe kinetic energy into pressure energy.

The pitot-tube consists of a glass tube, bent an right angles

Considertwo points 1 and 2 atte same level. Sucha aythat2 is atheinletofpitottubeandoneisthefarawayfromthetube

Let P_1=pressureatpoint1

V_1=velocityoffluidatpoint1P_2=

pressure at 2

V_2=velocityoffluidatpoint2

H=Depthoftubeinthe liquid

h=RiseoftheliquidinthetubeabovethefreesurfaceApplying

Bernoulli'stheorm

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{P_1}{\rho g}=H \qquad \frac{P_2}{\rho g}=(h+H)$$

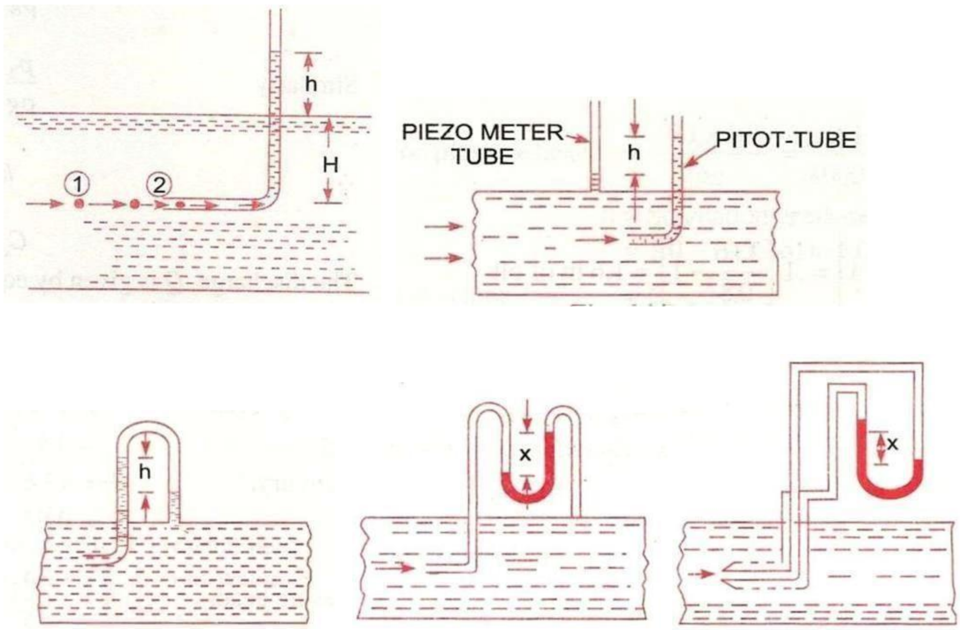
$$H+\frac{V_1^2}{2g}=h+H$$

$$V_1=\sqrt{2gh}$$

Actualvelocity, $V_{act}=C_v\sqrt{2gh}$

C_v=co-efficientofPitot-tube

DifferentArrangementofPitottubes



NumericalProblems:

Problem:-7

Waterisflowingthroughapipeof5cmdiameterunderapressureof 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution. Given :

Diameter of pipe = 5 cm = 0.5 m

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0$ m/s

Datum head, $z = 5$ m

Total head = pressure head + kinetic head + datum head

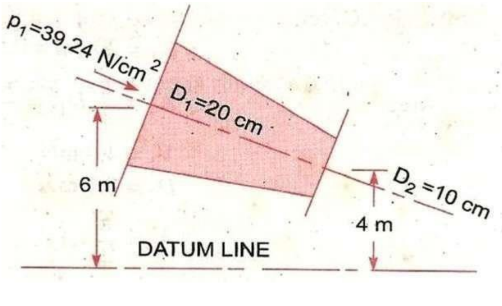
Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30$ m $\left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204$ m

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem:-8

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35lit/s. The section 1 is 6m above datum and sedction 2 is 4m aboved datum. If the pressure at section 1 is 39.24 N/cm^2. Find the intensity ofpressureat section2



Solution:

Given

At section 1,

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2$$
$$= 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2,

$$D_2 = 0.10 \text{ m}$$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow,

$$Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$$

Now

$$Q = A_1V_1 = A_2V_2$$

$\therefore$

$$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or

$$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or

$$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$

$$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

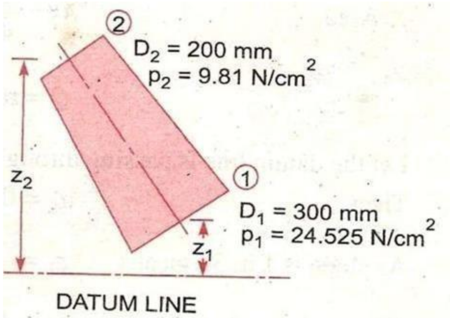
$\therefore$

$$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}.$$

Problem:-9

Water is flowing through a pipe having diameter 300mm and 200 mm at the buttom and upper end respectively. The intensity of pressure at the bottom end is 9.81N/m^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/s



Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$\therefore$ $$V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$\therefore$ $$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem:-10

A horizontal venturimetre with inlet and throat diameters 10cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20cmof mercury. Determine the rate of flow. Take C_d= 0.98

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ Area at inlet, $a_1 = \frac{\pi}{4} d_1{}^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

∴ Difference of pressure head is given by (6.9)

or $$h = x \left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_0 = Sp. gravity of water = 1

$$= 20 \left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s.}}$$

Problem:-11

An oil of Sp.gr. 0.8 is flowing through a horizontal venturimrtre having inlet diameter 20cm and throaty diameter 10 cm. The oil mercury differential manometer shows a readingof 25cm. Calculate the discharge of oil through the horizontal venturimetre. Take Cd = 0.98

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

∴ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

∴ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3\text{/s}$$

$= 70465 \text{ cm}^3\text{/s} =$ **70.465 litres/s. Ans.**

Problem:-12

A horizontal venturimrtre with inlet and throat diameters 20cm and10cm respectivelyisusedtomeasuretheflowof oilof Sp.gr.

The discharge of oil through venturimetre is 60lit/s . Find thereading of oil-mercury differential manometer. Take C_d = 0.98

Solution. Given :

$$d_1 = 20 \text{ cm}$$

$$\therefore \quad a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$$

Using the equation (6.8),

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or

$$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h}$$

$$= \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore$ $$h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$\therefore$ $$289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$\therefore$ $$x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm.**

Problem:-13

A static pitot-tube placed in the centre of a 300 mm pipe linehasone orifice pointing upstream and is perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipeif the pressure difference between the two orifices is 60mm of water. Take C_v= 0.98

Solution. Given :

Dia. of pipe, d = 300 mm = 0.30 m

Diff. of pressure head, h = 60 mm of water = .06 m of water

$C_v = 0.98$

Mean velocity, $\overline{V}$ = 0.80 × Central velocity

Central velocity is given by equation (6.14)

$$= C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore$ $$\overline{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$$

Discharge, Q = Area of pipe × $\overline{V}$

$$= \frac{\pi}{4}d^2 \times \overline{V} = \frac{\pi}{4}(.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Orifice:

Orifice is a small openingof any Cross-section (such as triangular, rectangularetc)onthesideoratthebottomofatank, thoughwhichafluid is flowing. Orifices are used for measuringthe rate of flow of fluid.

ApplyingBernoulli'stheoremat1and2

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$H+0=0+\frac{V_2^2}{2g}$$

$$V_2=\sqrt{2gh}$$

OrificeCo-efficients:

TheOrificeco-efficientsare

- **Co-efficientofvelocityC_v**
- **Co-efficientofcontractionC_c**
- **Co-efficientofdischargeC_d**

Co-efficientofvelocityC_v:

Itisdefinedastheratiobetweentheactual velocityofa jetofliquidatvena-contraandthetheoreticalvelocityofjet.Itisdenoted by C_v and Mathematically C_v is given as

$$C_v=\frac{Actual velocity of jet at vena-contra}{Theoretical velocity}$$

$$=\frac{V}{\sqrt{2gh}}$$

Where V=actualvelocity

$\sqrt{2gh}$=theoreticalvelocity

ThevalueofC$_v$variesfromo.95to0.99fordofferentorificial dependingontheshape, sizeoftheorifice.

Co-efficientofcontraction:

Itisdefinedastheratiooftheareaofthejetatvena-contratotheareaof the orifice.

ItisdenotedbyCca

= area of orifice

a_c=areaofjetatvena-contra

$$C_c = \frac{\text{areaofjetatvena} - \text{contra}}{\text{areaoforifice}}$$

$$= \frac{a_c}{a}$$

Thevalueofccvariesfrom0.61to0.69dependingonshapeandsizeof the orifice.

Co-efficientofDischarge:

Itistheratiooftheactualdischargefromanorificetothetheoreticaldischargefrom the orifice. It is denoted by C_d

IfQistheactualdischargeandQthisthetheoreticaldischarge then

$$C_d = \frac{Q_{act}}{Q_{th}}$$

$$= \frac{Actualvelocity \times Actualarea}{Theoreticalvelocity \times Theoreticalarea}$$

$$= C_c \times C_v$$

ThevalueofC$_d$arriesfrom0.610.65

ForgeneralpurposeCdis0.62

Classification

Orifficesareclassifiedonthebasisoftheirsize,shapeandnatureof discharge

Accordingtosize

- Smallorifice(Iftheheadofliquidabovethecentreoforificeismorethan5 times the depth of orifice)
- Largeorifice(Ifheadislessthan5timesthedepthoforiffice)

Accordingtoshape

1. **Circular**
2. **Triangular**
3. **Rectangular**
4. **Square**

Accordingtotheshapeofupstreamedge:

- Sharpedgedorifice
- Bellmouthedorifice

Accordingtonatureofdischarge:

- Freedischargeorifices
- Drownedorsubmergedorifices
    - Partiallysubmergedorifices
    - Fullysubmergedorifices

OroficeMeterorOrificePlate:

It is advice used for measuringthe rate flowofafluid througha pipe. It is a cheaper device as compare to venturimetre. It also works on the same principle as that of venturimetre . It consists iof a flat circular plate which has a circular sharp edge hole called orifice, which is concentric with the pipe. The orifice diameter iskept gene rally 0.5 times the diameter of the pipe, through it may vary 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section 1which is at a distance of about 1.5 to 2.0 times time pipe diameter of upstream of the orifice plate and at section 2., which at a distance about half the diameter of the orifice on the down stream side from the orifice plate

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and

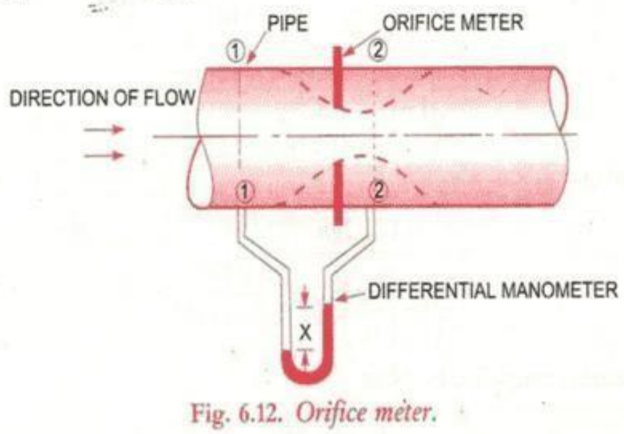


Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

or
$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=h=\text{Differential head}$$

$\therefore$
$$h=\frac{v_2^2}{2g}-\frac{v_1^2}{2g} \quad \text{or} \quad 2gh={v_2}^2-{v_1}^2$$

or
$$v_2=\sqrt{2gh+v_1^2} \qquad ...(i)$$

Now section (2) is at the vena contracta and a_2 represents the area at the vena contracta. If a_0 is the area of orifice then, we have

$$C_c=\frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$\therefore$
$$a_2=a_0\times C_c \qquad ...(ii)$$

By continuity equation, we have

$$a_1v_1=a_2v_2 \quad \text{or} \quad v_1=\frac{a_2}{a_1}\,v_2=\frac{a_0\,C_c}{a_1}\,v_2 \qquad ...(iii)$$

Substituting the value of v_1 in equation (*i*), we get

$$v_2=\sqrt{2gh+\frac{a_0^2C_c^2v_2^2}{a_1^2}}$$

or $$v_2^{\,2} = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^{\,2} v_2^{\,2} \text{ or } v_2^{\,2} = \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2hg$$

$\therefore$ $$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0\, C_c$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ $$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (*iv*), we get

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}.$$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

Chapter-5

NOTCHES&WEIRS

INTRODUCTION

A **notch** is a device used for measuring the rate of flow of a liquid through a small channel ora tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquidsurface in the tankor channelisbelowthe top edge ofthe opening.

A weir is a concrete or masonary structure, placed in anopen channel over which the flow occurs. It is generally in the form of vertical wall, witha sharp edge at the top, running all the way across the openchannel.Thenotchisofsmallsizewhiletheweirisof abiggersize.Thenotchisgenerallymade of metallic plate whileweir is made of concrete or masonarystructure.

1. **NappeorVein.**Thesheetofwaterflowingthroughanotchoroveraweiriscalledar NappeorVein.
2. **CrestorSill.**Thebottomedgeofanotchoratopofaweiroverwhichthewaterflows,isknown as thesill or crest.

CLASSIFCATIONOFNOTCHESANDWIEIRS

Thenotchesareclassifiedas:

1. Accordingtotheshapeoftheopening:
   (a) Rectangularnotch,
   (b) Triangularnotch,
   (c) Trapezoidalnotch,and
   (d) Steppednotch.
2. Accordingtotheeffectofthesidesonthenappe:
   (a) Notchwithendcontraction.
   (b) Notchwithoutendcontractionorsuppressednotch.

Weirsareclassifiedaccordingtotheshapeoftheopening,theshapeofthecrest,theeffectofthesidesonthe nappeandnatureofdischarge.Thefollowingareimportantclassifications.

(a) Accordingtotheshapeoftheopening:

(i) Rectangularweir, *(ii)* Triangularweir,and

(iii)Trapezoidalweir(Cipollettiweir)

(b) Accordingtotheshapeofthecrest:

(i) Sharp-crestedweir, *(ii)*Broad-crestedweir,

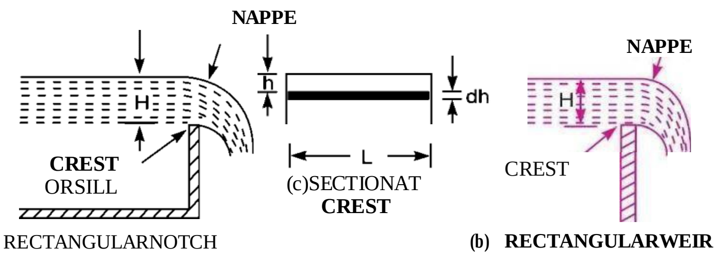
(iii)Narrow-crestedweir,and *(iv)*Ogee-shapedweir.

(c) Accordingtotheeffectofsidesontheemergingnappe:

(i) Weirwithendcontraction,and (ii)Weirwithoutendcontraction.

DISCHARGEOVERARECTANGULARNOTCHORWEIR

Theexpressionfordischargeoverarectangularnotchorweiristhesame.



Rectangularnotchandweir.

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1. Let*H* -- Head ofwater overthe crest

L=Lengthofthenotchorweir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness*dh* and lengthL at a depth /ifrom the free surface ofwater as shown in Fig.8.1(c).

Theareaofstrip =*Lxdh*

andtheoreticalvelocityofwaterflowingthroughstrip=$(2gh)^{0.5}$

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$
$$= C_d \times L \times dh \times \sqrt{2gh}$$

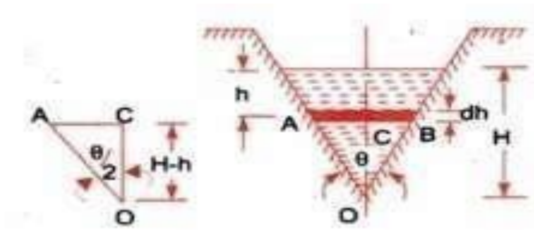
Total discharge i.e. Q over a rectangular notch or weir

$$Q = \int_0^H C_d \,.\, L \,.\, \sqrt{2gh}\ .\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1}\right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= \frac{2}{3}\, C_d \times L \times \sqrt{2g}\ [H]^{3/2}$$

DISCHARGE OVERATRIANGULARNOTCHORWEIR



Dischargethroughthestrip $dQ = C_d \times$ Areaofthestrip $\times$ Velocity(Theoritical)

$= C_d \times 2(H-h)\tan\theta/2 \times dh \times (2gh)^{1/2}$

TotaldischargeQ $= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$ =

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

For a right-angled *V*-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge $Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$

$= 1.417\ H^{5/2}$

Chapter-6

Flowthroughpipe

Pipe

Apipeisaclosedconduit,generallyofcircularcross-sectionused tocarrywatteroranyotherflui.

Whenthepipeisrunningfull,theflowisunderpressurebutifthepipeis not running full the flow is nit unsder pressure (culverts, sewer pipes)

<u>Lossoffluidfriction</u>:

Thefrictionalresisytanceofapipedependsupontheroughnessof the insidesurface of thepipemore the roughness more istheresistance. This frictionisknownasfluidfrictionandtheresistanceisknownasfrictional resistance

<u>AccordingFroude</u>

Thefrictionalresistancevarieswiththesquareofthevelocity.

Thefrictionresistancevarieswiththenaturalofthesurface.

Amongvarieslaws,theDarcy-weisbatchformula&Chezy'sformula.

<u>Lossofenergyinpipes</u>:

When a fluid is flowing through a pipe, the fluid experiences some, resistanceduetowhichsomeifenergyisloss.Energylosses:majorenergy losses - it is calculated by Darcy Weisbach formula andChezy's formula.

minorenergylossesduetofriction- 1-subbenexpansionofpipe

2- suddencontractionof

3- bendinpipe

4- pipefittingsetc

5- anobstructioninpipe.

Darcy-weisbatchformula:

Thelossofheadinpipesduetofrictioncalculatedfromdarcy-weisbath equation.

$$h_f = \frac{4FLV^2}{2gd}$$

h_f=lossofheadduetofriction

F=coefficientoffriction(functionofreylond'snumber)

$=\frac{16}{R_e}$ forR_e<2000(viscousflow)

$=\frac{0.079}{R_e^{\frac{1}{4}}}forR_e$varyingfrom4000to$10^6$

L=lengthofthepipe

V=meanvelocityofflowD

=diameterofthepipe.

Chezy'sformula:

$$h_f = \frac{f'}{\delta g} \times \frac{P}{A} \times L \times V^2$$

h_f=lossofheadduetofriction.P= wetted

perimeter of pipe A=C.Sareaofpipe L

=lengthof pipe

V=mmeanvelocityofflow.

$$M = \frac{A}{P} = \frac{areaofflow}{perimeter}$$

=hydraulicmeandepthorhydraulicradius

$$\Rightarrow M = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

Substituting $\frac{P}{A} = \frac{1}{M}$

$$h_f = \frac{f'}{\rho g} \times \frac{1}{M} \times L \times V^2$$

$$v^2 = h_f \times \frac{\rho g}{f'} \times M \times \frac{1}{L}$$

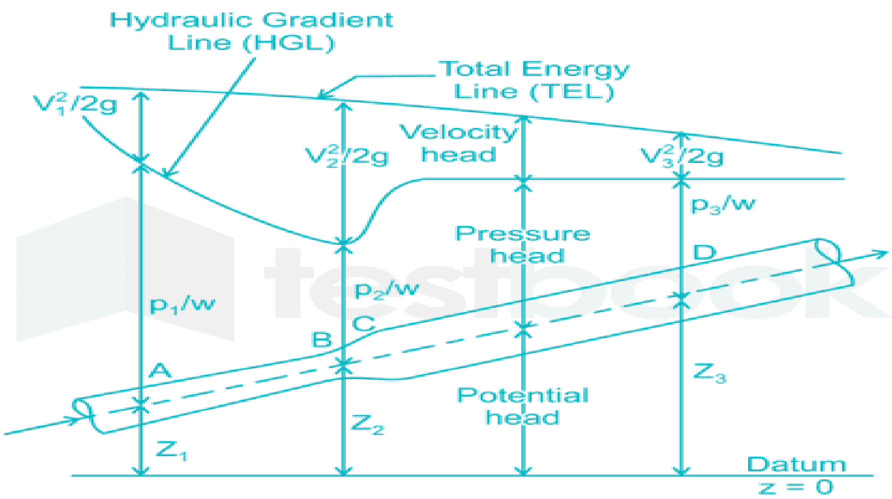
$$V = C(MI)^{1/2}$$

Hydraulicgradientline:

It is defined as the line which gives the sum of pressure head P/W & datumhead (Z) if a flowingfluid in a pipe with respect to the reference line or it is the line which is obtained by joining of the top of all vertical ordinates showing pressure head (P/W)of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L .

Totalenergyline:

It is defined as the line which gives the sum of pressure head, dutum head & kinetic head of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the tops of all vertical orbinates showingthe sum of pressure head & kinetic head fromthe centre of thepipe. It is also written as T.E.L



Hydraulic gradient and total energy line

CHAPTER-7

Impactofjets

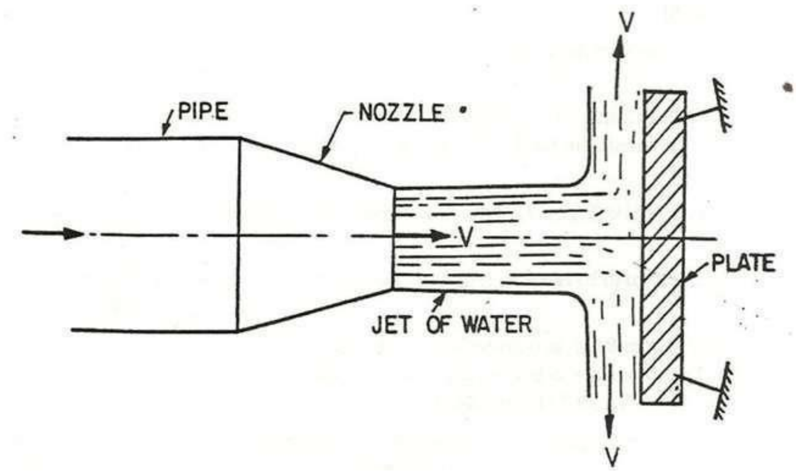
Introduction:

Impactofjetmeanstheforceexertedbythejetonaplatewhichmaybe stationary or moving

Thevariouscasesofimpactofjetare:

1. forceexertedbythejetonastaticnaryplatewhen
   1) plateisverticaltothejet
   2) plateisinclinedtothejet
   3) plateiscurved
2. forceexertedbythejetonamovingplatewhen-
   1. plateisverticaltojet
   2. plateisinclinedtothejet
   3. plateiscurved

Impactofjetflatsurface:



Forceexertedbyjetonfixedverticalplate

Considerajetofwatercomingoutfromthenoiilestrikesaflatvertical plate

Let v=velocityofthejetd

= diameter of jet a=areaofcross–

sectionofthejet

$$= \frac{\pi}{4} d^2$$

Astheplateisfixed,thejetafterstrikingwillgetdeflectedthrough 90^0

Hencethecomponentofthevelocityofjet,inthedirectionofjet, afterstrikingwillbezero.

Theforceexertedbythejetontheplateinthedirectionofjet F_x=rate of

change of momentumin the direction of force

$$= \frac{\text{initialmomentum} - \text{finalmomentum}}{Time}$$

$$= \frac{\text{mass} \times \text{initialvelocity} - \text{mass} \times \text{Finalvelocity}}{Time}$$

$$= \frac{\text{mass}}{Time} (\text{Initialvelocity} - \text{Finalvelocity})$$

$= \frac{\text{mass}}{sec}$ (velocityofjetbeforestriking–Finalvelocityofjetafter striking)

$$= \rho av[v-0]$$

$$= \rho a v^2$$

NOTE: Intheaboveequationininitial velocity minusfinalvelocityistakenas because force exerted by the jet on the plate is is calculated if force exerted on the jet is to be calculated then final velocity is taken.

NUMERICALPROBLEMS

Problem-1

Findtheforceexertedbyajetofwaterofdiameter75mmona stationaryflatplatewhenthejetstrikestheplatenormallywithavelocityof20m/s.

Solution.

Given:

Diameterofjet=d=75mm

= 0.075m

Velocityofjet=20m/s

Area=a=$\frac{\pi}{4}d^2$

=$\frac{\pi}{4}(0.075)^2$

=0.004417m^2

Theforceexertedbythejetofwateronastationaryverticalplateis given by

F=$\rho a v^2$

=1000×0.004417×20^2

=1766.8N

Problem-2

Waterisflowingthroughapipeattheendofwhichanozzleisfitted . the diameter of the nozzle is 100m and the head of water at the centre of nozzle 100m . find the force exerted by the jet of water on a fined vertical plate . the co-efficient of velocity is given as 0.95

SOLUTION:

Ggiven:

Diameterofnozzle=d=100mm=0.1mHead

of water , H = 100m

Co-efficientofvelocity,Cv=0.95

Areaofnozzlea=$\pi/4d^2$=$\pi/4(0.1)^2$ =0.007854m^2

Theoreticalvelocityofjetofwaterisgivenas $V_{th}=\sqrt{2gH}$
$=\sqrt{2\times9.81\times100}=44.294$ M/S

But, $C_v=\frac{\text{actualvelocity}}{\text{Theoreticalvelocity}}$

∴Actualvelocityofjetofwater(v) $=c_v\times v_{th}$

$V=0.95\times44.294$

$=42.08$ M/S

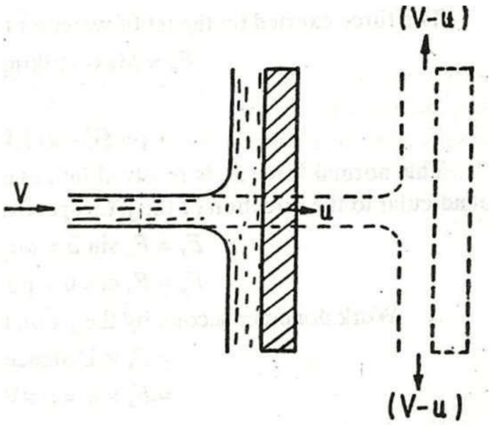
ForceexertedonafieldverticalplateisgivenbyF=

ρav^2

$=1000\times0.07854\times(42.08)^2$ $(\rho=1000\text{kg/m})$

$=13907.2$N$=13.9$KNans

Impactofjetonmovingflatplate:



Jetstrikingaflatverticalmovingplate

Considerajetofwaterstrikingaflatverticalplatemovingwithauniform velocity away from the jet ..

LetV=velocityofthejet(absolute

)A=areaofcross–sectionofthejetU

=velocityofflat plate

Inthiscasethejetstrikestheplatewitharelativevelocity,whichis equaltotheabsolutevelocityofjetofwaterminusthevelocityoftheplate.

Hencerelativevelocityofthejetwithrespecttoplate=v-uMassof water striking the plate per sec

$=\rho \times areaofjet \times velocity(relative)$

$=\rho a \times [v-u]$

Forceexertedbythejetonthemovingflatplatein thedirectionof motion of jet

Fx=massofwaterstriking/sec$\times [initialvelocity-finalvelocity]$

$=\rho a(v-u)[(v-u)-0]$

$=\rho a(v-u)^2$(finalvelocityinthedirectionofjetiszero)

Inthiscase,theworkwillbedonebythejetonplateastheplateismovingWorkdone per second by the jet on theplate

$$=\text{force}\times \frac{distanceinthecirectionofforce}{time}$$

$=f_x \times u$

$=\rho a(v-u)^2 \times u$

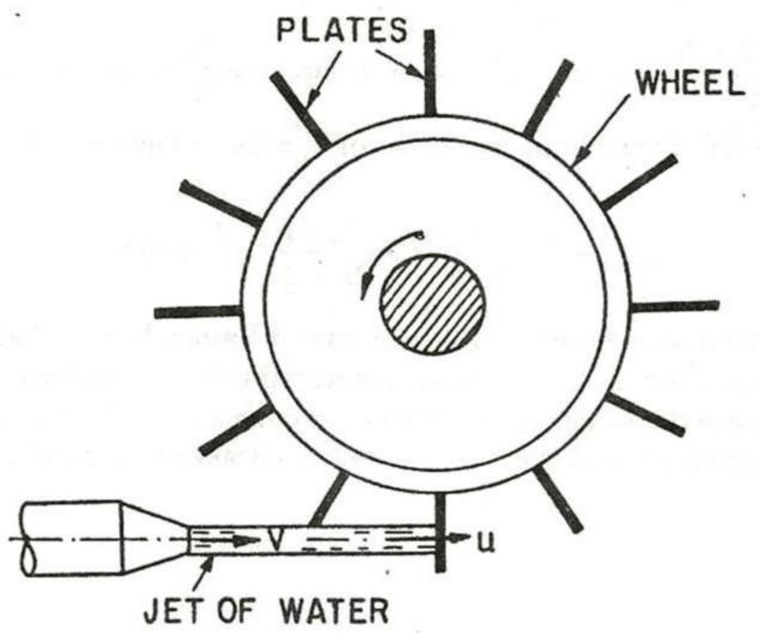
Jetstrikingaseriesofplates

In this case, a large number of flat platus aremounted on the rimof a wheel fixed distance apart. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mountedonthewheelappearsbeforethejet, whichagainexertstheforceon the 2nd plate.

Thus each plate appears successively be for the jet s the jet exerts force on each plate. The wheel states movingat a constant speed

LateV = velocity of jetD=diameterofjet

A=cross–sectionalareaofjet u = velocity of plate



In this case themass of water coming outfrom the nozzleper second is alwaysinconnect with the plates when all the plate are considered

Hencemassofwater/sec=ρav

Thejetstrikestheplkatewithvelocity=v–u

TheForceexetedbythejetinthedirectionofthemotionofplateF_x=$\frac{\text{mass}}{Time}$(Initialvelocity–Finalvelocity)

=ρav[(v–u)–0]

$= \rho av(v-u)$

Workdonepersecondbythejetontheseriesoftheplatepersec

$= F x u$

$= \rho av(v-u)\, u$

Kineticenergyofthejetpersecond

$= \frac{1}{2} m v^2$

$= \frac{1}{2} (\rho a v) v^2$

$= \frac{1}{2} \rho a v^3$

Efficiency, $= \frac{Workdone/sec}{Kineticenergy/sec}$

$$= \frac{\rho av(v-u)u}{\frac{1}{2}\rho a v^3}$$

$$= \frac{2u(v-u)}{v^2}$$

ConditionformaximumEfficiency

Foragivenjetvelocityv,theefficiencywillbemaximumwhen

$$\frac{d}{du} = 0$$

$$\Rightarrow \frac{d\left[\frac{2u(v-u)}{v2}\right]}{du} = 0$$

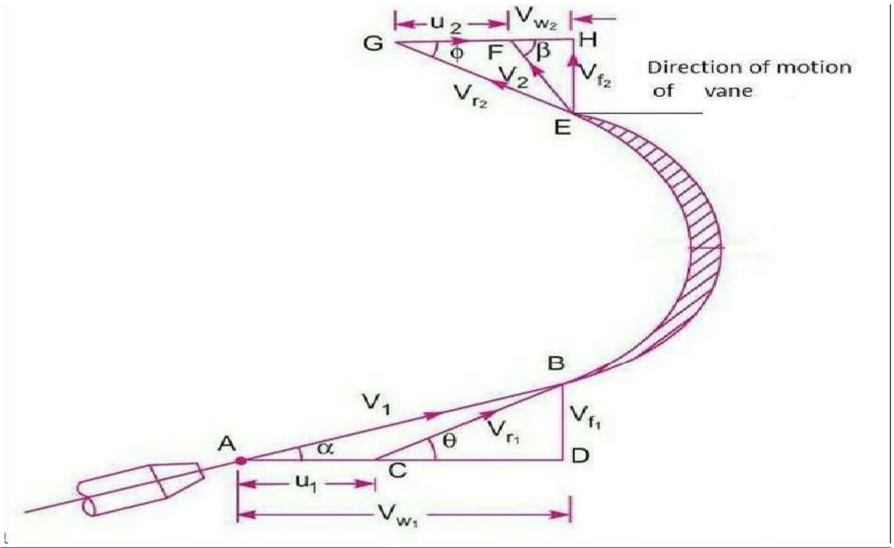
$$\Rightarrow \frac{d\left[\frac{2uv-u^2)}{v2}\right]}{du} = 0$$

$$\Rightarrow \frac{2v-4u}{v^2} = 0$$

$$\Rightarrow 2v-4u=0$$

$$\Rightarrow u = \frac{v}{2}$$

IMPACTOFJETONAMOVINGCURVEDPLATEWHENJETSTRIKESTANGENTISLY AT ONE OF THE TIPS:



Considera jetofwaterstrikinga movingcuirved vane tangentialy at one of itstips. In this caseasplateismoving, the velocity with which jet of aaterisequaltotherelativevelocityofthejeytwithrerspectgtotheplate Letv_1= velocity of the jet at inlet

u_1=velocityoftheplateat inlet

V_{r1}=Relativevelocityofthejet&plateatinlet

α=guidebladeangle

θ=vaneanglemadebyrelativevelocityV_{r1}withthedirectionofmotion ofinlet

V_{w1}&v_{f1}`=ComponentsofV_1inthedirectionofmotion&

perpendiculartothedirectionofmotionofvanerespectivelyV_{w1}=

Whirl veloicity at inlet

V_{f1}=velocityatinletr

V_2=velocityofthejetatoutlet

u_2=velocityofvaneatoutlet

V_{r2}=relativeofthejetatoutlet

β=Anglemadebythevelocityv2withthedirectionofthemotion of vane at outlet

$\emptyset$ =vaneangleatoutlet

V_{w2}=velocityofwhirlatoputlet

V_{f2}=velocityofwhirlatoputlet

ThetrianglesABD&EGHarecalledthevelocitytriangleatinlet& outlet

Ifthevaneissmooth&havingvelocityinthedirectionofmotion at inlet&outletequalthenwehaveu_1=

u_2= u

V_{r1}=v_{r2}

Butinitialvelocitywithwhichjetstrikesthgevane=v_{r1}

Thecomponentofthisvelocityinthedirectionofmotion

$= v_{r1}\cos\theta$

$=(V_{w1}-u_1)$

Similarlythecomponentofrelativevelocityvr2atoutletinthedirectionof motion = $- v_{r2}\cos\emptyset$

$=-[u_2+v_{w2}]$

(-vesignistakenasthecomponentofvr2isinoppositedirection)Substitutingthese values in the above equation

$$F_x = \rho aVr_1[(v_{w1}-u_1)-\{-(u_2+v_{w2})\}]$$

$$=\rho aVr_1[v_{w1}-u_1+u_2-v_{w2}]$$

$$=avr1(v_{w1}+v_{w2})$$

Thisequationistureonlywhenβisactuewhen

$\beta=90^0$

$V_{w2}=0$

$F_x=av_{r1}(v_{w1})$

When$\beta>90^0$(obtuse)

$F_x=av_{r1}(v_{w1} \quad -v_{w2})$

Inequationfxiswrittenas

$F_x=av_{r1}(v_{w1}\pm v_{w2})$

Workdown/seconthevanebythejet

$=Fx\times u$

$=\rho av_{r1}(v_{w1}\pm v_{w2})\times u$

Workdone/sec/unitweightoffluidstriking/sec

$$=\frac{\rho av_{r1}[v_{w1}\pm v_{w2}]\times u}{\rho av_{r1}\times g}$$

$$=\frac{[v_{w1}\pm v_{w2}]\times u}{g}$$

Workdone/sec/unitmassofwaterstriking/sec

$$=\frac{\rho av_{r1}[v_{w1}\pm v_{w2}]\times u}{\rho av_{r1}}$$

$=[v_{w1}\pm v_{w2}]\times u$

Efficiencyofjet

$$= \frac{\text{Workdonepersecondonthevane}}{InitialKineticenergy/\text{sec}ofthejet}$$

$$= \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{\frac{1}{2} m v^2}$$

$$= \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{\frac{1}{2} \rho a v_1 \times v^2}$$